# HAND-BOOK

OF THE

# SLIDE RULE,

SHEWING ITS APPLICABILITY TO

I.—ARITHMETIC
(INCLUDING INTEREST AND ANNUITIES).

II.—MENSURATION
(SUPERFICIAL AND SOLID, INCLUDING LAND SURVEYING).

With Numerous Examples and Useful Tables.

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H.M. EAST INDIA CIVIL SERVICE.



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# PREFACE.

The first edition of this book was published in 1861, and has been long out of print. A New Edition having been asked for by friends, and some professional men, this is issued with sundry improvements.

The chief objects in view have been to illustrate the use of the Slide Rule by a great number of Examples,—to shew the use of the line D in Superficial and Solid Mensuration, for the use of practical men,—and to explain the utility of *inverting* or reversing the Slide.

Attention is requested to the "Table of Contents," which will shew to what a variety of uses the Slide Rule may be applied. The article under the head of "Slide Rule" (by the late Professor De Morgan) in Knight's Encyclopedia, vol. "Arts and Sciences," may also be consulted.

It would be well to cut out *the Plate*, and mount it on linen, as better suited for reference. Several "Examples" refer to this Plate in elucidation.

# TABLE OF CONTENTS.

## PART I.

### (LINES "A" AND "B.")



a 2

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## PART II.

### (LINES "D" AND "E.")

*Mensuration (Superficial and Solid)—Compound Interest—Annuities, &c.*

## APPENDIX A.



## APPENDIX B.

# ERRATA.

| PAGE | |
|---|---|
| 12. | lines 10, 18, 20, 22, *for* "Exs. 212, 250, 254, and 261," *read* "211, 251, 255, and 262." |
| 27. | Ex. 44, *for* "$4^3$," *read* "43." |
| 41. | Line 8, *for* "a mile," *read* "1 inch." |
| 60. | Ex. 182, *for* "91," *read* "$\frac{91}{365}$". |
| 72. | (XI.) *for* "1·1604," *read* "·1604." |
| 83. | 2nd line from bottom, *for* "·0," *read* "·01." |
| 92. | Line 11, *for* "35," *read* "57." |
| 118. | 5 lines from bottom, *for* "12," *read* "$\sqrt{12}$." |
| 132. | Line 21, *for* "CV," *read* "CS." |
| 133. | Line 14, *for* "$\frac{4\cdot8}{4}$", *read* "$\frac{4\cdot8}{8}$". |
| 143. | Last line, *for* "(XIX)" *read* "XV½." (See *Addenda*, for p. 142.) |
| 147. | Erase Formula XXIV., and substitute |

| C | ·8284 | 1 | Area of included octagon |
|---|---|---|---|
| D | 1 | 1·1 | Side of square |

or Area = Side $^2 \times$ ·8284, and Side $= \sqrt{\frac{\text{area}}{\cdot8284}}$.

| | |
|---|---|
| 165. | Line 12, *for* "2892," *read* "$289^2$." |
| 167. | Footnote, *for* "p. 165," *read* "p. 170." |
| 168. | Line 7, *for* "3·568," *read* "4·75." |
| " | Line 6 from bottom, *for* "426," *read* "424." |
| 178. | Line 1, *for* "$\frac{d^2 \times h}{6\cdot14}$", *read* "$\frac{c^2 \times h}{6\cdot14}$". |
| 194. | Line 7 from bottom, *for* "1077·0," *read* "10,770." |

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213. Line 19, *for* "353·036," *read* "35·036."
226. Two lines from bottom, and 6 lines from bottom, *for* "43560," *read* "4356."
240. In the Figure, the "first parallel to the left" (1,000 yards), has been omitted.
242. Line 4, *for* "32·5," *read* "3·25."
243. Line 4, *for* "651′ = ·39," *read* "·651′ = 39."
254. Line 13, *after* "INVOLUTION," *add* "and EVOLUTION."
281. Line 18, for "3 ft." read "2 ft."
299. In the first Example, erase the final "0," of each of the last 5 lines.

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# ADDENDA.

PAGE

45. Ex. 126, *add* "See p. 303, nine lines from bottom."

89. Ex. 246, *add* "See also footnote to Ex. 254."

118. *Add* Ex. 312½ − $x = \cdot25 \times \sqrt[3]{320}$, solved as follows

$$\frac{\text{E}\quad 1 \qquad\qquad\qquad\qquad 320}{\text{D}\quad \cdot25 \qquad\qquad\qquad\qquad 1\cdot71}.$$

119. *Add* Ex. 319½ $x = \frac{7\cdot2^3}{3^2}$, solved as follows $x = \frac{7\cdot2^2 . \times 7\cdot2}{3^2}$, (as in Ex. 256) = 41·47.

119. *Add* Ex. 320½ $x = \sqrt[3]{75 \times 8^2}$, solved as follows :—

$$\frac{\text{E}\quad 9 \qquad\qquad\qquad\qquad 75}{\text{D}\quad 9 \qquad\qquad\qquad\qquad 8}.$$

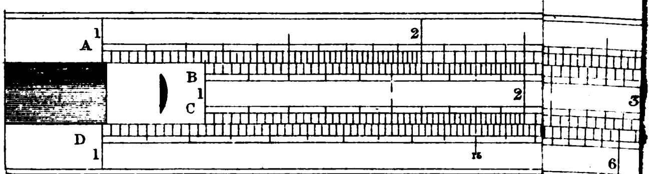
123. *Add* $\frac{\text{C}\quad 1 \qquad\qquad\qquad\qquad 3}{\text{D}\quad \text{Side of cube} \qquad\qquad \text{Diag. of cube}}.$

139. *Add* $\frac{\text{A}\quad \text{Area in Sq. feet} \qquad\qquad \text{Feet long}}{\text{B}\quad \text{Inches wide} \qquad\qquad 12}.$

142. *Add* another Formula XV½

$$\frac{\text{C}\quad \cdot433 \qquad 3\cdot9 \qquad \text{Area of } \textit{equilateral} \text{ triangle}}{\text{D}\quad 1 \qquad 3 \qquad \text{Length of any one side}}$$

or Area = Side² × ·433, and Side = $\sqrt{\frac{\text{area}}{\cdot433}}$.


A
B
C
D

# PART I.

# ARITHMETIC.

## (LINES A AND B.)

# THE SLIDE RULE.

## DESCRIPTION OF THE INSTRUMENT.

THE *Plate* is a fac-simile of the face of the Rule; the Slide being partly drawn out.

The line A on the upper stock, and the line B on the Slide, are identical. They are used together, for the solution of such questions as do *not* involve Squares, Square Roots, Cubes, or Cube Roots.

The line C is identical with A and B, but is used only in connection with the line D.

The line D on the lower stock (sometimes called the Square line), is used in conjunction with the line C, for the solution of questions which *do* involve Squares, Square Roots, and some cases of Cubes and Cube Roots.

The Slide takes out, and can be put in with the other face of it upwards. On this other face are :—

(1) Line E (sometimes called the Cube line), which in conjunction with D, is used for questions involving Cubes and Cube Roots.

(2) Line *Log.* (or Num.), which, if set even with D, the numbers on it will be the logarithms to three places, of the numbers underneath on D.

On the *back* of the instrument are often printed or engraved various useful Formulæ, referred to in APPENDIX L.

Many of the ordinary *Carpenter's Rules* have a brass Slide Rule let into the wood. The lines A, B, C, D, thereon, are the same as those of the Slide Rule above described, except that the square line D begins with 4 instead of A (see APPENDIX H); but they do not possess

the lines E, and "Log." Any one possessing such a Rule, may profit by the teaching of the present work, but as the divisions on such Rules are not always carefully graduated, the results obtained from them cannot be depended upon. This fact, as well as their being in brass, which makes it difficult for the eye to see the divisions, renders them almost useless except to *learn* upon.

There are various makers of Slide Rules; but the writer of this Handbook, from his own experience of many years, can recommend those made by Messrs. Elliot, of 449, Strand. The accuracy with which they are graduated is remarkable.

[Messrs. Elliot also make the "Double" Slide Rule, so called from its having another Slide on the back, divided into a scale of Natural Sines and Tangents, most handy in Surveying, Navigation, and Plane Trigonometry in general. A small work elucidating the use of this Slide is in preparation.]

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## HOW TO READ THE DIVISIONS.

Half-an-hour's *viva voce* teaching, is better than pages of writing; but in default of this, the learner must make up his mind to exercise a little patience in going through the next few pages, *Slide Rule in hand;* or he may buy a *Plate* like that at the beginning of this book, mount it on linen, and carefully cut out the Slide. He will soon be induced to purchase the instrument itself. A person accustomed to figures, would soon learn to read the Slide Rule by attention to the "Examples" in page 17.

The lines A, B, C, are identical (but C need not be looked at till we come to the use of the line D). They are lines of double radii; that is, from **1** to **10** comes twice over.

It will be seen that from **4** to **5**,—**5** to **6**,—**6** to **7**,—**7** to **8**,—**8** to **9**, —**9** to **10**, the sub-divisions are to *tenths.* From **2** to **3**, and from **3** to **4**, are tenths and *half-tenths.* From **1** to **2**, are tenths and *fifths of tenths.*

The values assigned to the figures engraved on the Rule are arbitrary; that is, the **3**, for instance, may be considered 3, or 30, or

300, or 3000, &c., or it may be considered ·3, ·03, ·003, &c., according to the value of the numbers involved in the question to be solved.

For instance, suppose we take the **4** (on either radius) to represent 40. Since the space between it and the following **5** is sub-divided into ten parts, the first division will be 41, next 42, and so on to the **5**, which will be 50.

Now let us again assume the **4** to be 40, and count *back*. It will be seen that the space between **4** and **3** is sub-divided into ten long divisions, and each of these again into two. The first *long* division to the left of **4** will be 39, but the first *short* division to the left of **4** will be 39·5 ;* so that reading back from **4** (as 40) we read 39·5, 39, 38·5, 38, 37·5, 37, &c., till we get back to the **3** as 30.

Now let us assume the **1** (either at the extreme left or that near the middle) of the line A, or B, to be 100. The first *long* division to the right will be 110, the next *long* division will be 120, then 130, and so on to the **2**, which will be 200. But the first *short* division after the **1** (considered as 100) will be 102, then 104, 106, 108, and then 110.

Now let us assume the **6** on either radius to be 6. The first division to the right will be 6·1, then 6·2, 6·3, &c., till we come to the **7**, which will be 7.

Now let us assume the **6** on the *first* radius to be 600—the **6** on the *second* radius will be 6000 ; for the two radii always bear their *decennary proportion.* So if the **2** on the *second* radius be considered ·02, the **2** on the first radius will be ·002.

### *To find* 125.

Assume the **1** at the extreme left of the line A, or that near the middle of the line, to be 100. The next *long* division to the right will be 110 ; the next *long* division will be 120. Two *short* divisions to the right of the 120 will be 124 ; and *half way* (to be estimated by the eye) from this to the next *short* division (which would be 126) is

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* The reader is supposed to know that half = ·5 ; one-quarter = ·25 ; and three-quarters = ·75. (APPENDIX G.)

125.* (If the **1** had been considered as 10, the place found would be 12·5.) *See the Plate.*

*To find* 3325.

Assume the **3** on either radius to be 3000; the third *long* division will be 33000, and *half way* between this and the next *short* division (which would be 3350) is 3325. (It may be ·03325, ·325, 3·25, &c., according as the **1** is assumed to be ·03, ·3, 3, &c.)

*To find* 2·875.

Assume the **2** on either radius to be 2. Eight *long* divisions forward make 2·8. The next *short* division is 2·85; and *half way* between this short division and the next *long* one (which would be 2·9) is 2·875.

*To find* 314.

Assume **3** on either radius to be 300. The next *long* division is 310. The next *short* division would be 315. Then 314 lies $\frac{4}{5}$ of the distance from 310 to 315; or $\frac{1}{5}$ of the distance short. The eye soon gets accustomed to this *estimation* of distances between cut divisions.

*To find* 1035.

Assume **1** on either radius to be 1000. The next short division is 1020. Half way to the next would be 1030, but three-quarters of the way would be 1035. The next division would be 1040.

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* In *pointing out* the place required, it is better not to use any kind of pointer, but to use the slide. If the place sought is on A, draw out the slide till **1** on B comes under the place sought. If the place sought is on B, draw out the slide till the **1** on A comes over the place sought. The instrument can then be laid down and time taken to consider if the right place is really picked out. Thus in the above example, if the *Plate* is referred to, it will be seen that the **1** on B is under 125, or 1·25, or 12·5, or ·125, according as we value the **1** on A.

## To use the lines A and B together.

We have hitherto treated each line separately and independently; but when A and B are worked *together*, it does not follow that the engraved figure on the one, need be of the same value as the engraved figure on the other; still each line retains *its own* decennary increase or decrease.

*Refer to the Plate.* Let the **1** in the middle of the line A, represent 10, but let the **1** at the extreme left of B represent 100. Then over this 100 on B, will be 1·25 on A, and under the **1** on A which we have taken to represent 10, will be 800 on B, as under :—

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| A | 1·25 | 2 | 10 | 15 | 50 | 56·25 | 100 |
| B | 100 | 160 | 800 | 1200 | 4000 | 4500 | 8000 |

Again, instead of referring to the Plate, set the instrument itself. Assume **2** on the first radius of B to be ·2, and set it under the **5** of the first radius of A, which assume to be 50. Then we shall have :—

| | First radius. | | | Second radius. | | | |
|---|---|---|---|---|---|---|---|
| A | 25 | 4·75 | 50 | 200 | 400 | 750 | 1000 |
| B | ·1 | ·19 | ·2 | ·8 | 1·6 | **3** | 4 |

Again, set ·45 of the second radius of A, over 21 of the second radius of B. Then we have :—

| | First radius. | | | Second radius. | | | |
|---|---|---|---|---|---|---|---|
| A | ·0643 | ·09 | ·1 | ·225 | ·289 | ·45 | ·75 |
| B | 3 | 4·2 | 4·67 | 10·5 | 13·5 | 21 | 35 |

The learner will not fail to observe the constant *proportion* between the numbers on the lines A, and those on B. In the first case, the numbers on B are 80 times as great as those over them on A. In the second, the numbers on A are 250 times as great as those under them on B. [It is this *proportion* between the whole series of numbers on A and those on B, *place the Slide as we will*, that indicates the rationale of the Slide Rule as an instrument of computation.]

Keeping this element of *proportion* in view, the learner (if he does not mind time) can *test* his own accuracy of setting, by selecting three

such numbers as are well defined on the instrument, and finding from them a fourth proportional.

Take, for instance, the *Plate* (or a Slide Rule set as the Plate is), where we see 4 on B exactly under 5 on A. Then what on B, is under 31 on A? The learner may hesitate between 24·7 and 24·9; but if he wants to test his accuracy by calculating with pen and paper, he takes the *proportion* 5 : 4 : : 31 : answer. Or, answer $= \frac{4 \times 31}{5} = 24{\cdot}8$. A person accustomed to the instrument, would read 24·8 without hesitation. [Of course the paper Plate, though excellently engraved, may not be *quite* so dependable as the wooden Rule, as it is liable to expand unequally.]

Take a Slide Rule, and set 1 on A over 75 on B.

| | | |
|---|---|---|
| A | 1 | 14·2 |
| B | 75 | $x$ |

What is the value of $x$? An experienced eye would see that the 14·2 is over a place a very little to the right of 1060, but not as far as 1070, and would probably call it 1065. By the *test* given above, we have 1 : 75 : : 14·2 : 1065.

It has been already mentioned (p. 5) that the numbers on the second radius are ten times the value of the same numbers on the first radius; but there are cases where a beginner is apt to be puzzled, especially with *decimals*.

Thus, if the Rule is set as follows:—

Ex. (*a*)

| | | |
|---|---|---|
| A | ·3 | $x$ |
| B | 24 | 36800 |

and $x$ is required. If we read straight on from 24 on the first radius, we come only as far as to 368 on the second. In all such cases we multiply the lesser couple, or divide the greater couple, by 10, 100, or 1000. In the Ex. (*a*) above $\frac{\text{A} \quad \cdot 3}{\text{B} \quad 24}$ is the same as $\frac{\text{A} \quad 30}{\text{B} \quad 2400}$ and *then* we can read straight on on B, and over 36800 we find 460 on A.

As these are cases constantly occurring in Multiplication and Division of *Decimals*, it will be well to give a few more Examples, as follows:—

Ex. (*b*)

| | | |
|---|---|---|
| A | ·384 | $x$ |
| B | 1 | 62·7 |

What is the value of $x$?

Read as if it were $\frac{A \quad 3{\cdot}84}{B \quad 10}$, and we easily see that over 62·7 is 24·1 nearly. See Ex. 7.

Ex. (*c*) $\frac{A \quad {\cdot}0604 \quad\quad\quad x}{B \quad 1 \quad\quad\quad 543{\cdot}25}$. What is the value of $x$?
Here we multiply the 1 and the ·0604 by 100, and get

$$\frac{A \quad 6{\cdot}04 \quad\quad\quad x}{B \quad 100 \quad\quad\quad 543{\cdot}25} \quad x = 32{\cdot}8, \text{ as in Ex. 8.}$$

Ex. (*d*) $\frac{A \quad x \quad\quad\quad 1}{B \quad {\cdot}864 \quad\quad\quad 36}$. What is the value of $x$?
Here we divide the 1 and the 36 by 100, and get

$$\frac{A \quad {\cdot}01 \quad\quad\quad x}{B \quad {\cdot}36 \quad\quad\quad {\cdot}864} \quad x = {\cdot}024, \text{ as in Ex. 20.}$$

Ex. (*e*) $\frac{A \quad 1 \quad\quad\quad x}{B \quad {\cdot}09 \quad\quad\quad 441}$. What is the value of $x$?
Here we multiply the 1 and the ·09 by 1000, and get

$$\frac{A \quad 1000 \quad\quad\quad x}{B \quad 90 \quad\quad\quad 441} \quad x = 4900, \text{ as in Ex. 22.}$$

Ex. (*f*) $\frac{A \quad x \quad\quad\quad 1}{B \quad {\cdot}00222 \quad\quad\quad {\cdot}185}$. What is the value of $x$?
Here we divide the 1 and the ·185 by 100, and get

$$\frac{A \quad {\cdot}01 \quad\quad\quad x}{B \quad {\cdot}00185 \quad\quad\quad {\cdot}00222} \quad x = {\cdot}012, \text{ as in Ex. 23.}$$

Ex. (*g*) $\frac{A \quad x \quad\quad\quad 5}{B \quad {\cdot}125 \quad\quad\quad 9}$. What is the value of $x$?
Here we divide 5 and 9 by 10, and we get

$$\frac{A \quad x \quad\quad\quad {\cdot}5}{B \quad {\cdot}125 \quad\quad\quad {\cdot}9} \quad x = {\cdot}0694, \text{ as in Ex. 58.}$$

## THE SPECIAL USE OF THE SLIDE RULE.

The special use of the Slide Rule is to solve *quickly* (within certain limits of accuracy) all Arithmetical questions involving Multiplication or Division, or both combined,—Squares and Square Roots, Cube and Cube Roots, not excepted.*

It is evident, therefore, that it embraces all cases of Proportion (Rule of Three), Reduction, Per-centages, Simple Interest, Fellowship, &c., as also Mensuration, superficial and solid, including Land and Timber-measuring, Gauging, &c. The additional line of Logarithms on the back of the Slide assists in working out approximately, Compound Interest, Annuities, Leases, &c.

On many occasions (as will be noticed hereafter) the Slide Rule, once set, is in itself a "Table" for converting Moneys, Measures, Weights, &c., of one nation into those of another; and it is a "Ready Reckoner" in the ordinary computations of civilized life.

The variety of cases to which the Slide Rule may be applied, will be seen by reference to the "Index of Contents;" and if its utility were more generally known few accountants, artizans, engineers, merchants, tradesmen, or housekeepers would be without one.

And now is the proper place to check the exorbitant expectations some persons indulge in when they see the instrument used by an experienced hand. They think that if they also can get hold of a Slide Rule they will be able at once to apply it. Let it be understood, then, that little can be done without some acquaintance with *Decimals*, and the ordinary processes of Arithmetic. For instance, if the possessor of a Slide Rule does not know *how to state* a Rule of Three sum, the instrument will no more show him *how to do it*, than a pen put into a child's hand will teach it how to write a letter.

The chief difficulty to a beginner, but which soon wears off in practice, is the fact that the Slide Rule does not deal with *Fractions*, as such. All parts of wholes must be taken as *Decimals*. For instance, in "Division," 267 divided by 15 would not give the answer as $17\frac{12}{15}$ but as 17·8. Again, in "Multiplication," if we have to multiply $13\frac{1}{2}$ by $8\frac{3}{4}$ (as in Ex. 2) we must set it as 13·5 × 8·75. Again, 4*£* 2*s*. 6*d*.

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* It has nothing to do with Addition or Subtraction.

would be considered 4·125£. So 26lbs. 8oz. would be 26·5lbs. *Hence it is especially valuable for the "Metric system."*

The decimal equivalents for the fractions of most frequent occurrence, as also of *s. d.* in Decimals (to three places) of 1£, and *vice versa*, should be committed to memory from APPENDIX G. The Examples 80 to 85, under the head of "Reduction," will show that the Slide Rule itself will easily give the value of decimal equivalents quite near enough for all practical purposes. A knowledge of "Tables" of Weights and Measures, English and Foreign, (APPENDIX M) will be very useful.

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## DEGREE OF ACCURACY.

IT is not to be expected of the Slide Rule, that the answers taken out should always be *absolutely exact.* It is sufficient that they should be near enough for all practical purposes, where *quickness* is the first object. After a little experience, the greatest errors should not exceed $\frac{1}{400}$th part of the exact answer. The average error should not exceed $\frac{1}{750}$ or, say 1*d.* on every 3*l.* Very much depends on whether the numbers employed fall on a graduated division, or on a space between two divisions, as in taking out 314 in page 6. In very many cases the error is only $\frac{1}{1000}$, *as will be seen from the following fifteen Examples*, none of which occupied above a minute—some only a few seconds—in solving. These Examples will also show the variety of circumstances where the Slide Rule may be advantageously employed.

(*a*) Multiply 245 by 125. Answer, 30600. Error, $\frac{1}{1225}$, the exact product being 30625. (Ex. 3.)

(*b*) Divide 2360 by 8. Answer, 295. Quite correct. (Ex. 14.)

(*c*) Price in pence of 13½lbs. at 8¾*d.* per lb. Answer, 118*d.* Error, $\frac{1}{945}$; the exact answer being 118⅛*d.* (Ex. 2.)

(*d*) Reduce 47lbs. to the decimal of a cwt. Answer, ·42. Error, $\frac{1}{1175}$, the exact decimal being ·419643. (Examples 5, and 93.)

(*e*) If a chest of tea weighing 84lbs. cost 19£ 4*s.*, what is the quantity (in lbs.) that I should get for 1£ 8*s.*? Answer, 6·12lbs. Error, $\frac{1}{1225}$, the exact quantity being 6·125lbs. (Ex. 100.)

(*f*) How many Pounds are equivalent to 560 Rupees, when the exchange is 1*s.* 10$\frac{1}{2}$*d.* per Rupee? Answer, 52·5£. Error, none. (Ex. 49.)

(*g*) What remains of 690£ after deducting 17$\frac{1}{2}$ per cent. discount? Answer, 569£. Error, $\frac{1}{2277}$, the exact answer being 569·25£. (Ex. 114.)

(*h*) I buy Stock at 118 in a Railway paying 6$\frac{1}{4}$ per cent. dividend. What interest do I get for my money? Answer, 5·3 per cent. Error, $\frac{1}{1360}$, the exact answer being 5·29961. (Ex. 115.)

(*i*) Which is the greatest and which is the least fraction of $\frac{8}{13}$, $\frac{11}{18}$, $\frac{18}{29}$, or $\frac{35}{58}$. Answer, $\frac{18}{29}$ is the greatest, and $\frac{11}{18}$ the least.* (Ex. 212.)

(*k*) There are three maps drawn to the respective scales of 200, 220, and 320 feet to an inch. Given 5280 feet to a mile, and say how many inches in each map correspond to 1 mile. Answers, 26·4, 24, 16 All quite correct, and done in a few seconds. (Ex. 35.)

(*l*) How much is $\frac{28}{35}$ of 21, of 45, and of 80? (See Ex. 64.) Answers, 16·8, 36, 64. All exact, and with one set of the Slide.

(*m*) What is the Square Root of 6$\frac{7}{8}$? Answer, 2·62, in a few seconds. Error, $\frac{1}{1311}$, the exact answer being 2·622. (See Ex. 250.)

(*n*) Divide 5 by the Square Root of 20. Answer, read in a few seconds, 1·118. Exact answer is 1·118034. (Ex. 254.)

(*o*) Solve $\sqrt{\frac{7 \times 162}{56}}$. Answer, read off in a few seconds, 4·5. The answer is exact. (Ex. 261.)

(*p*) Solve $\sqrt{\frac{21 \times 180}{168}}$, $\sqrt{\frac{21 \times 320}{168}}$, $\sqrt{\frac{21 \times 6200}{168}}$, without moving the Slide when once set. Answer, in one minute, 15, 20, and 88. Exact answers are 15, 20, and 88·034. (Ex. 295.)†

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* These answers were obtained in a few seconds. Query, the time it would take by the usual process? (See Examples 212 to 221.)

† It is when there is a "series" containing certain *constant* numbers, that the use of the Slide Rule is shown to most advantage, as in (*k*), (*l*), (*p*): also Examples 11, 29, 34, 66, 75, 96, 134, 155, 217, 225, 292, 304, &c.

## PROPERTIES OF THE SLIDE RULE.

Let the Slide be drawn out to *any* extent right or left—little or much—and the following "Proportion" will hold good :—

As any number on B,
Is to any number over it on A ;
So is any other number on B,
To the number over *it* on A.

| | | |
|---|---|---|
| A | 5 | 10 |
| B | 4 | 8 |

Here 4 : 5 :: 8 : 10—or 8 : 10 :: 4 : 5, as seen in the *Plate.**
Or set 8 on A, over 2 on B : then

| | | | | | |
|---|---|---|---|---|---|
| A | 8 | 16 | 20 | 24 | 28 |
| B | 2 | 4 | 5 | 6 | 7 |

Where 2 : 8 :: 4 : 16.—Or 20 : 5 :: 28 : 7.—Or 6 : 24 :: 5 : 20.

*It may be asked, "If every sum is to be worked as a 'Proportion,' how do you manage with Multiplication or Division?"*

As regards *Multiplication,* the reply is as follows: If $x = 125 \times 245$, it is the same as 1 : 125 :: 245 : $x$, as shown in Ex. 3.

As regards *Division,* the reply is as follows: If $x = \frac{2360}{8}$, it is the same as 8 : 1 :: 2360 : $x$, as shown in Ex. 14.

Hence every case of Multiplication and Division can be set as a "Rule of Three," and every case that can be set in the form

$$x = \frac{b \times c}{a}$$

is adapted to the Slide Rule, and can be solved *at once,* as shown in Examples 37 to 61.

---

* Besides the two readings given above, there are *six* others equally available, but the learner had better keep to the two above given. The other six are :—

| | | |
|---|---|---|
| (1) 5 : 4 :: 10 : 8 | (3) 4 : 8 :: 5 : 10 | (5) 8 : 4 :: 10 : 5 |
| (2) 10 : 8 :: 5 : 4 | (4) 5 : 10 :: 4 : 8 | (6) 10 : 5 :: 8 : 4 |

*Theoretically,* as explained in Appendix A, the four last are the more in accordance with the principle on which the instrument is constructed: but *practically* the two given, and those marked (1) and (2) in this note, are more convenient to the eye.

N.B. 1. If *any three* terms in a "Proportion" are given, the fourth may be found. The first and last terms are considered as "partners," and the two middle terms as "partners." This being settled, the arrangement is easy. Suppose we have $m : s :: o : r$, with $m, o, r$, given; to find $s$. Here $s$ is made the last term, and its "partner" $o$ the first term; and the two others must be the middle terms. Hence we have $o : m :: r : s$. Or $o : r :: m : s$. The setting on the Slide Rule would be :—

$$\frac{\text{A} \qquad m \qquad\qquad s}{\text{B} \qquad o \qquad\qquad r} \text{ or else } \frac{\text{A} \qquad r \qquad\qquad s}{\text{B} \qquad o \qquad\qquad m}$$

N.B. 2. It will also be observed, that set the Slide as we please, the numbers over or under each other on the lines A and B represent a "series of EQUAL FRACTIONS." Thus, if we look at the *Plate*, we see $\frac{2}{16} = \frac{4}{24} = \frac{5}{40} = \frac{6}{48} = \frac{27}{216} = \frac{45}{360}$ &c. Or, if we like, we may consider the numbers on B to be the Numerators, and those over them on A, the Denominators, and we have $\frac{4}{5} = \frac{12}{15} = \frac{20}{25} = \frac{75}{60}$ &c.* It is this property that makes the Slide Rule so useful as a "Table" of conversion. For instance, if we know that 1 Mètre = 3·28 Feet, we have only to set 1 on A over 3·28 on B, and all the numbers on B will give the Feet equivalent to the numbers of Mètres over them on A. See Ex. 12, and the "Formulæ"† that precede Ex. 222.

N.B. 3. The Slide Rule will not directly solve a case where *three* numbers have to be multiplied together, but as to divide by a *reciprocal*, is the same as to multiply by its integral number, we can directly solve three multipliers if the reciprocal of one is known. Thus $x = 12 \times 13 \times 17$. The reciprocal of 12 ought to be known‡ as it so frequently occurs in mensuration. Hence we have $x = \frac{13 \times 1{\cdot}7}{{\cdot}08333}$ easily solvable, as shown in Example 59. So in Division: the Slide Rule will not at once solve $x = \frac{2016}{7 \times 16}$, but if we know the reciprocal of 16 to be ·0625, we have $x = \frac{2016 \times {\cdot}0625}{7}$ as in Ex. 60.

---

* See Examples 205 to 221.

† The line D worked with C, or with A, B, C, can also be used for many most useful Formulæ, as seen in Part II. of this work.

‡ The "Reciprocals" of the most commonly occurring numbers may be committed to memory, from Appendix G. See also Ex. 24.

## Inversion of the Slide.

The Slide may be taken out, and put in again with the figures on C turned upside down, as follows :—

| | | |
|---|---|---|
| A | 7·2 | 12 |
| C | 5 | 3 |

in which case the "Proportion" reads cross-ways, *i.e.* 3 : 7·2 :: 5 : 12, or, 5 : 12 :: 3 : 7·2, or, $3 \times 12 = 7 \cdot 2 \times 5$.

The beginner, however, need not trouble himself about this, till he comes to the cases where one constant number,—or two constant numbers to be multiplied together,—has to be divided by a "series" of varying Divisions, as in Examples 33 to 36, and Examples 75 to 79.

In using the line D (Part II.) this inversion of the Slide is also useful.

---

## MULTIPLICATION.

[The answers will be given as near as they would be read off by an experienced hand. In some cases there is no error at all.]

### The set of the Rule is as under:

| | | |
|---|---|---|
| A | Multiplier | Product |
| B | 1 | Multiplier * |

**Ex. 1.**—Multiply 15 by 7 ;—or 7 by 15.*

| | | |
|---|---|---|
| A | 15 | 85 = Ans. |
| B | 1 | 7 |

or

| | | |
|---|---|---|
| A | 7 | 85 = Ans. |
| B | 1 | 15 |

It is in fact a "Rule of Three" sum, as shown in page 13. Here 1 : 15 :: 7 : $x$, or 1 : 7 :: 15 : $x$.

**Ex. 2.**—Multiply $13\frac{1}{2}$ by $8\frac{3}{4}$ ; or $8\frac{3}{4}$ by $13\frac{1}{2}$.*

---

* The distinction between "Multiplier" and "Multiplicand" is needless, as they are *both* Multipliers.

| | | |
|---|---|---|
| A | 13·5 | 118 = Ans. |
| B | 1 | 8·75 |

or

| | | |
|---|---|---|
| A | 8·75 | 118 = Ans. |
| B | 1 | 13·5 |

Here 1 : 13·5 :: 8·75 : $x$, or 1 : 8·75 :: 13·5 : $x$. The *exact* product is 118·125. Error $\frac{1}{945}$.

**Ex. 3.**—Multiply 125 by 245. (See *Plate.*)

| | | |
|---|---|---|
| A | 125 | 30600 = Ans. |
| B | 1 | 245 |

The *exact* product is 30625. Error $\frac{1}{1225}$. To know how many figures there will be in the Product, see page 17.

**Ex. 4.**—Multiply 31·75 by 69·5.

| | | |
|---|---|---|
| A | 31·75 | 2210 = Ans. |
| B | 1 | 69·5 |

The *exact* product is 2206·625. Error $\frac{1}{654}$. To know how many figures there will be in the Product, see page 18.

***Ex. 5.**—What is the total weight, *in Cwt*, of 14 packages, each weighing 2 cwt. 1 qr. 19lbs.? (2·42 × 14).

| | | |
|---|---|---|
| A | 2·42 | 33·9 cwt. = Ans. |
| B | 1 | 14 |

The *exact* answer is 33·875 cwt. Error = $\frac{1}{1355}$.

**Ex. 6.**—What would be the rent of 8 ac. 2 ro. 18 po. at 5£ 15*s.* per acre? ( $x$ = 8·61 × 5·75.)†

| | | |
|---|---|---|
| A | 8·61 | 49·5£. = Ans. |
| B | 1 | 5·75 |

* In Ex. 5, the 1 qr. 19lbs, or 47lbs., is reduced to decimals of a Cwt. in an instant, by the Slide Rule, as in Ex. 18, quite near enough for any Slide Rule work. It gives ·42 cwt. ; whereas the exact decimal is ·419643 cwt. All these Reductions to a decimal, are very quickly shown by the Slide Rule. The mental Rules in Appendix G may also be referred to.

† As in Ex. 18 we divide $\frac{47}{112}$, so here 2 ro. 18 po. = $\frac{98}{160}$ acre = ·612.

The *exact* answer is 8·6125 × 5·75 = 49·521875£ or 5*d.* more than the Slide Rule gives.

**Ex. 7.**—Multiply 62·7 by ·384. Ex. (*b*) p. 9.

| | | |
|---|---|---|
| A | ·384 | 24·1 = Ans. |
| B | 1 | 62·7 |

The *exact* answer is 24·0768. Error = $\frac{1}{1040}$.

**Ex. 8.**—Multiply ·0604 by 543·25.

| | | |
|---|---|---|
| A | ·0604 | 32·8 = Ans. |
| B | 1 | 543·25 |

Ex. (*c*) p. 9.

The *exact* answer is 32·8123. Error = $\frac{1}{2567}$.

**Ex. 9.**—Multiply 12 × 13 × 1·7. See Ex. 59.

**Ex. 10.**—Multiply 119 by $3\frac{1}{7}$. See Ex. 43.

---

## To find how many figures in the Product.

RULE.—As soon as the Slide is set for the Multiplication, look at *the first figure of the Product,* and see if it is less than the first figure of either * Multiplier. If it is,—the "number of figures in the Product" is *equal to the sum* of the figures in the two Multipliers. But if it is greater,*—the number of figures in the Product will be *one less than the sum* of the figures in the two Multipliers.

For instance 985 × 1923. As soon as the Slide is set, we see that the first figure of the Product is 1. Now though this is equal to the first figure of one Multiplier, it is *less* than the first figure of the other Multiplier: therefore the number of figures in the Product is 3 + 4, or seven. (1894155).

Again, 3213 × 1122. As soon as the Slide is set, we see that the first figure of the Product is 3 Now though this is equal to the first figure of one Multiplier, it is *greater* than the first figure of the other

* No case can occur where the first figure of the Product is greater than the first figure of one Multiplier, and less than the first figure of the other Multiplier.

Multiplier. Therefore the number of figures in the Product is $(4 + 4) - 1$, or seven. (3604986).

But it sometimes happens, that the first figure of the Product, is *the same* as the first figure of *both* Multipliers. In this case we see what is the *second* figure of the Product, and comparing it with the second figures of the two Multipliers, proceed as before. Thus, if we have $950 + 960$, the first figure of the Product is also 9; but the Slide Rule shows us with certainty the *second* figure to be 1; hence the Product has $3 + 3$, or six figures. (912000).

Again in $112 \times 12100$, the first figure of the Product is also 1; but the second figure is seen to be 3; hence the Product has $(3 + 5) - 1$, or seven figures. (1355200).

---

## If there are Decimals

Proceed as if there were no decimal point, but all were integers; and *afterwards* cut off as many decimals from the Product as equals the sum of the decimals in the two Multipliers. Thus in Ex. 6, or $8{\cdot}61 \times 5{\cdot}75$, we proceed as if it were $861 \times 575$. We see, as soon as the Slide is set, that the first figure of the Product is 4. This by the preceding Rule would show that the Product has $3 + 3$, or six figures, or 495000. *Then* cut off four for decimals, and the answer is 49·5.

Again, if we have $151 \times {\cdot}0604$, we proceed as if it were $151 \times 604$; and as we see the first figure of the Product to be 9, we know that the number of figures in the Product will be $(3 + 3) - 1$, or five. It reads 91200, but cutting off four for decimals, we have 9·1200.

---

## To Multiply a "series" of numbers by a constant Multiplier; or to Multiply a constant number by a "series" of Multipliers.

The set of the Rule is:

| | | | | |
|---|---|---|---|---|
| A | "Constant number" | Product | Product | Product, &c. |
| B | 1 | Multiplier | Multiplier | Multiplier, &c. |

**Ex. 11.**—Multiply $12\frac{1}{2}$ by 4, by 5·6, by 7·6, and by 12 (as shown in the *Plate*).

| | | | | | |
|---|---|---|---|---|---|
| A | 12·5 | 50 | 70 | 95 | 150 |
| B | 1 | 4 | 5·6 | 7·6 | 12 |

N.B. Of course this is the same as multiplying 4, 5·6, 7·6, and 12, each by the "constant" 12½.

**Ex. 12.**—Suppose we know the English inch to be equal to 25·4 French millemètres, what are the values in millemètres of 3½ inches, 6·7 inches, and 12 inches?

| | | | | |
|---|---|---|---|---|
| A | 25·4 | 88·9 | 170 | 305 millem. |
| B | 1 | 3·5 | 6·7 | 12 inches |

**Ex. 13.**—If we know the Russian "Verst" to be equal to ·663 miles, how many miles are equivalent to 300, 800, and 1880 versts?

| | | | | |
|---|---|---|---|---|
| A | ·663 | 19·9 | 530 | 1246 miles |
| B | 1 | 300 | 800 | 1880 versts |

N.B. 1. Examples 12 and 13 indicate how "Tables of Conversion" are formed by the Slide Rule, as will be shown more fully further on under the head of FORMULÆ, and Examples 222, 225, 232, &c.

N.B. 2. When *a Fraction* has to be multiplied by a "series" of multipliers, see Ex. 64.

---

## DIVISION.

[The answers will be given as near as they would be read off by an experienced hand. In some cases,—as in Ex. 14,—there would be no error at all.]

### The set of the Rule is as under: *

| | | |
|---|---|---|
| A | 1 | *Quotient* |
| B | Divisor | Dividend |

---

* The Rule may also be set

| | | | |
|---|---|---|---|
| A | Divisor | Dividend |
| B | 1 | *Quotient* |

or

| | | |
|---|---|---|
| A | Dividend | *Quotient* |
| B | Divisor | 1 |

; but though the latter of these is often used, the set given in the text is preferable, especially in "Long" Division, page 23.

**Ex. 14.**—Divide 2360 by 8 (as in the *Plate*).

| | | |
|---|---|---|
| A | 1 | 295 = Ans. |
| B | 8 | 2360 |

It is in fact a "Rule of Three" sum, as shown in page 13. Here 8 : 1 :: 2360 : $x$.

**Ex. 15.**—Divide 88493568 by 256.

| | | |
|---|---|---|
| A | 1 | 346000 = Ans. |
| B | 256 | 88493568 |

Of course we cannot profess to set 88493568, but we can set very near 885; and with this we get an answer with only an error of $\frac{1}{1073}$. The *exact* quotient being 345678, as in Ex. 28.

N.B. To know before we take up the Slide Rule, how many figures there will be in the Quotient, see page 22.

***Ex. 16.**—Divide 15£ 13*s.* 9½*d.* by 7.

| | | |
|---|---|---|
| A | 1 | 2·24£ = Ans. |
| B | 7 | 15·69 |

The error is less than ½*d.*

**Ex. 17.**—Divide 5592·65 by 40·6.

| | | |
|---|---|---|
| A | 1 | 138 = Ans. |
| B | 40·6 | 5592·65 |

*Exact* quotient is 137·75. Error $= \frac{1}{5510}$.

N.B. To know beforehand how many figures there will be in the quotient, see page 22.

**Ex. 18.**—Divide 47 by 112. $\left(x = \frac{47}{112}.\right)$

| | | |
|---|---|---|
| A | ·42 = Ans. | 1 |
| B | 47 | 112 |

---

* In Ex. 16, the learner is supposed to know (from Appendix G) how to reduce 13*s.* 9½*d.* into decimals of 1£, to *three* places of decimals. The *exact* solution is 15·68958$\dot{3}$£ ÷ 7 = 2·24137£; or 2£ 4*s.* 9·9*d.*

***Ex. 19.**—Divide 36·67 by 482·5. $\left(x = \frac{36·67}{482·5}.\right)$

| | | |
|---|---|---|
| A | ·076 = Ans. | 1 |
| B | 36·67 | 482·5 |

***Ex. 20.**—Divide ·864 by 36. $\left(x = \frac{·864}{36}.\right)$

| | | |
|---|---|---|
| A | ·024 = Ans. | 1 |
| B | ·864 | 36 |

***Ex. 21.**—Divide ·184 by 2¾. $\left(x = \frac{·184}{2·75}.\right)$

| | | |
|---|---|---|
| A | ·067 = Ans. | 1 |
| B | ·184 | 2·75 |

***Ex. 22.**—Divide 441 by ·09. $\left(x = \frac{441}{·09}.\right)$

| | | |
|---|---|---|
| A | 1 | 4900 = Ans. |
| B | ·09 | 441 |

***Ex. 23.**—Divide ·00222 by ·185. $\left(x = \frac{·00222}{·185}.\right)$

| | | |
|---|---|---|
| A | .021 = Ans. | 1 |
| B | ·00222 | ·185 |

**Ex. 24.**—Find the "Reciprocal" of 112. $\left(x = \frac{1}{112}\right)$

| | | |
|---|---|---|
| A | ·0089 = Ans. | 1 |
| B | 1 | 112 |

The *exact* answer is ·00893857 (*i.e.* 1lb. reduced to the decimal of 1cwt).

N.B. There is no difficulty in knowing where the decimal point will come, in the case of "Reciprocals" of whole numbers ; for the dot

---

* In such Examples as 18 to 24, the remarks in page 9 will assist the learner in placing the decimal point in the Quotient : otherwise APPENDIX C must be referred to.

and the 0's *together,* always equal in number the number of figures in the divisor (or denominator).

**Ex. 25.**—Find the "Reciprocal" of ·7854. $\left(x = \frac{1}{\cdot 7854}.\right)$

| | | |
|---|---|---|
| A | 1 | 1·273 = Ans. |
| B | ·7854 | 1 |

The *exact* answer is 1·27324.

**Ex. 26.**—Divide 350 by $6\frac{1}{7}$. See Ex. $59\frac{1}{2}$.

**Ex. 27.**—Divide 2016 by (7 × 16). See Ex. 60.

---

## To find beforehand, how many figures there will be in the Quotient.

Rule.—When the first figure of the Divisor is *greater* than the first figure of the Dividend, the number of figures in the Quotient will be *equal to the difference* between the number of figures in the Divisor, and those in the Dividend. If the first figure of the Divisor is *less*, the number of figures in the Quotient will be *one more than the difference.*

If the Divisor and the Dividend both begin with the same figure, be guided by their second figures.—If their two first figures are the same, be guided by their third figures—and so on.

For instance, in Ex. 15, the difference in the number of figures in the Divisor and Dividend is *five*; but as we see the first figure of the Divisor to be *less* than the first figure of the Dividend, we know before we begin work, that the number of figures in the Quotient will be *six*.

---

## Where there are Decimals.

In such cases as $\frac{554\cdot85}{40\cdot5}$, $\frac{26727}{442\cdot5}$, $\frac{731\cdot43}{81}$, where the *integral* part of the divisor is less than the integral part of the dividend, the number of *integral* figures in the Quotient will be just the same as there

would be if the decimals were erased. The three Quotients being 13·7, 60·4, 9·03. In all *other cases* of "Division of Decimals," if the notation of the Rule itself is not considered sufficient by the learner, from what has been explained in page 9, he can find where the decimal point will come, before commencing work, by the rules in APPENDIX C.

---

## Long Division.

Since two figures of the Quotient at a time, can be taken out with certainty, much guessing and rubbing out is avoided.

**Ex. 28.**—Divide 88493568 by 256.

Leave the instrument set as follows, *throughout the whole* operation :

| | | |
|---|---|---|
| A | 1 | |
| B | 256 | 885 (near enough) |

In the first place we know by the Rule given in page 22, that the Quotient will consist of *six* figures ; and as an approximation, we may, if we like, read the Quotient as 346000, as in Ex. 15. But in working out the detail, we first take out the two first figures, 34, and go on as far as "*rem.*" with pen or pencil.

```
256 ) 88493568 ( 34
      768
      ----
      1169
      1024
      ----
"rem." 145
```

Leaving the Rule still untouched, and bringing down the remaining figures of the Dividend, we have 56 given us as the *two next* figures of the Quotient.

```
256 ) 1453568 ( 56
      1280
      ----
       1735
       1536
       ----
        199
```

Leaving the Rule still untouched, and bringing down the remaining figures of the Dividend, we have, over 19968 on B, the two last figures of the Quotient, namely 78, which need not be worked out. So the full Quotient is 345678. (Ex. 15.)

---

**[I.] To divide a "series" of numbers by a "Constant Divisor;"** or $\frac{b}{a}$, where $b$ varies, but $a$ is constant.

| | | | | |
|---|---|---|---|---|
| A | 1 | Quotient | Quotient | Quotient |
| B | Constant Divisor | Dividend | Dividend | Dividend |

**Ex. 29.**—Divide 27, 58, 345, 610, by 8. (See *Plate.*)

| | | | | | |
|---|---|---|---|---|---|
| A | 1 | 3·37 | 7·25 | 43·1 | 76·4 |
| B | 8 | 27 | 58 | 345 | 610 |

**Ex. 30.**—Divide 88·9,—170,—305,—each by 25·4 (*i.e.* Reduce 88·9 &c. millemètres to English inches, as in Ex. 12).

| | | | | |
|---|---|---|---|---|
| A | 1 | 3·5 | 6·7 | 12 inches |
| B | 25·4 | 88·9 | 170 | 305 millemètres |

**Ex. 31.**—What is the weight in lbs. avoir. of 1000 Shillings and of 1000 Sovereigns? the Shilling weighing 87·273 grains, and the Sovereign 123·274 grains. (Here 87273 and 123274 are each to be divided by 7000).

| | | | |
|---|---|---|---|
| A | 1 | 12·47lbs. | 17·61lbs. |
| B | 7000 | 87273 | 123274 |

**Ex. 32.**—The fineness of Standard silver in England is 222dwts. In India 220dwts. In France 216dwts. Required the "touch," or percentage of *pure* silver, in each. (Here $\frac{216}{240}$, $\frac{220}{240}$, $\frac{222}{240}$, there being 240dwts. in 1lb. Troy).

| | | | | |
|---|---|---|---|---|
| A | ·900 | ·91$\dot{6}$ | ·925 | 1 |
| B | 216 | 220 | 222 | 240 |

---

* The learner is supposed to know that there are 7000 grains in 1lb. Avoir. See APPENDIX M.

### [II.] To divide one constant number by a "series" of varying Divisors.

or $\frac{b}{a}$, where $b$ is constant, but $a$ varies.

(*Invert the slide*, as in page 15.)

| | | | | |
|---|---|---|---|---|
| A | Quotient | Quotient | Quotient | Constant Dividend |
| Ↄ | Divisor | Divisor | Divisor | 1 |

**Ex. 33.**—Divide 225 by 26, by 87, and by 126.

$$\left(\text{Here } \frac{225}{26}, \frac{225}{87}, \frac{225}{126}.\right)^*$$

| | | | | |
|---|---|---|---|---|
| A | 1·79 | 2·59 | 8·66 | 225 |
| Ↄ | 126 | 87 | 26 | 1 |

**Ex. 34.**—Required a "series" of reciprocals, for $3\frac{3}{4}$, $5\frac{1}{4}$, $8\frac{3}{4}$ and 12. (Here the "Constant dividend" is 1, and it has to be divided by 3·75, 5·25, 8·75, 12.)

| | | | | | |
|---|---|---|---|---|---|
| A | ·0833 | ·114 | ·190 | ·266 | 1 |
| Ↄ | 12 | 8·75 | 5·25 | 3·75 | 1 |

†**Ex. 35.**—How many *inches to a mile* are each of three Plans drawn to the respective scales of 200, 220, and 330 feet to an inch?

$$\left(\text{Here } \frac{5280}{200}, \frac{5280}{220}, \frac{5280}{330}.\right)$$

| | | | | |
|---|---|---|---|---|
| A | 16 | 24 | 26·4 | 5280 |
| Ↄ | 330 | 220 | 200 | 1 |

†**Ex. 36.**—A person at three different times, invests in a Railway when it is at 12, 10, and 6 discount, but which pays regularly

---

* Or the Rule may be set as under:

| | | | | |
|---|---|---|---|---|
| A | 1 | 26 | 87 | 126 |
| Ↄ | 225 | 8·66 | 2·59 | 1·79 |

† In all cases where the Slide is inverted, as the numbers on A decrease back by tens, the numbers on Ↄ under them increase forward by tens, and *vice versâ*.

6 per cent. How much per cent. interest does he get on each investment? $\left(\frac{600}{88}, \frac{600}{90}, \frac{600}{94}\right)$

| A | 6·383 | 6·6̇6 | 6·817 | 600 |
|---|---|---|---|---|
| D | 94 | 90 | 88 | 1 |

N.B. This is more properly solved as in Ex. 77.

---

## To multiply two numbers, and at the same time to divide their Product by a third number, or

$$x = \frac{b \times c}{a}.$$

*This is the "specialite" of the Slide Rule*, expediting an endless variety of computations. Anyone reading the above heading, will see that it is in fact a "Rule of Three sum," for $x = \frac{b \times c}{a}$, is the same as $a : b :: c : x$.

The Rule is set as under:

| A | Either multiplier | *Answer* |
|---|---|---|
| B | Divisor | Other multiplier |

**Ex. 37.**—Given $x = \frac{37{\cdot}5 \times 7{\cdot}6}{3}$. Required the value of $x$

| A | 37·5 | 95 = $x$ |
|---|---|---|
| B | 3 | 7·6 |

N.B. The Rule, as set in the *Plate*, will show this.

**Ex. 38.**—How many yards in a minute, does a person advance, who walks $3\frac{1}{2}$ miles in an hour? $\left(x = \frac{3{\cdot}5 \times 1760}{60}.\right)$

| A | 3·5 | 103 yds. |
|---|---|---|
| B | 60 | 1760 |

**Ex. 39.**—Since a shilling weighs 87·273 grains, what weight in lbs. Avoir. is 20 shillings? $\left(x = \frac{20 \times 87{\cdot}273}{7000}.\right)$

| | | |
|---|---|---|
| A | ·25 lbs. | 20 |
| B | 87·3 | 7000 |

The *exact* answer is ·24935lb.; or 4½ grains less than ¼lb.

**Ex. 40.**—The Mint price of Standard Gold is £3 17*s.* 10½*d.* per Troy oz. What is the value of one grain, *in pence?* $\left(x = \frac{3{\cdot}89375 \times 240}{480}\right)$

| | | |
|---|---|---|
| A | 1·94*d.* | 3·89 |
| B | 240 | 480 |

**Ex. 41.**—What is $\frac{13}{17}$ of 112? (or $\frac{13}{17}$ of 1 cwt. *in lbs.*) $x = \frac{13 \times 112}{17}$.

| | | |
|---|---|---|
| A | 13 | 85·65 = Ans. |
| B | 17 | 112 |

**Ex. 42.**—Multiply 119 by $3\frac{1}{7}$, (*i.e.* if Radius = 119, what is the Circumference?) $x = \frac{119 \times 22}{7}$.

| | | |
|---|---|---|
| A | 347 | 22 |
| B | 119 | 7 |

**Ex. 43.**—Divide $24^2$ by 8. $\left(x = \frac{24 \times 24}{1}.\right)$

| | | |
|---|---|---|
| A | 24 | 72 = Ans. |
| B | 1 | 24 |

**Ex. 44.**—Divide 350 by $6\frac{1}{7}$. $\left(x = \frac{350 \times 7}{4^3}.\right)$

| | | |
|---|---|---|
| A | 57 = Ans. | 350 |
| B | 7 | 43 |

**Ex. 45.**—Half a guinea a day, is how many pounds a year? $\left(x = \frac{10{\cdot}5 \times 365}{20}.\right)$

| | | |
|---|---|---|
| A | 10·5 | 191·5 £ |
| B | 20 | 365 |

The *exact* answer is 191·625, or $\frac{1}{1533}$ error.

**Ex. 46.**—18½lbs. of meat at 7¼ per lb., is how many shillings? $\left(x = \frac{18{\cdot}5 \times 7{\cdot}25}{12}.\right)$

| | | |
|---|---|---|
| A | 11·2*s.* | 18·5 |
| B | 7·25 | 12 |

**Ex. 47.**—At 17 yards for 15*s.* what is the price per yard *in pence?* $\left(x = \frac{15 \times 12}{17}.\right)$

| | | |
|---|---|---|
| A | 10·6*d.* | 15 |
| B | 12 | 17 |

**Ex. 48.**—What is the price in £, of 96 yards of Calico, at 6¾*d.* per yard? $\left(x = \frac{96 \times 6{\cdot}75}{240}.\right)$

| | | |
|---|---|---|
| A | 2·7 £ | 6·75 |
| B | 96 | 240 |

**Ex. 49.**—What is the equivalent in £, of 560 Rupees, when the exchange is 1*s.* 10½*d.* per Rupee? $\left(x = \frac{560 \times 22{\cdot}5}{240}.\right)$

| | | |
|---|---|---|
| A | 52·5 £ | 560 |
| B | 22·5 | 240 |

**Ex. 50.**—680 articles were sold for £4 13*s.* 6*d.* Required the average price of each, in *farthings.* $\left(x = \frac{4{\cdot}67 \times 960}{680}.\right)$

| | | |
|---|---|---|
| A | 4·67 | 6·6*f.* |
| B | 680 | 960 |

**Ex. 51.**—How many Square yards are there in a rectangular plot of ground 152 ft. long, and $44\frac{1}{2}$ ft. broad? $\left(x = \frac{152 \times 44\cdot5}{9}.\right)$

| | | |
|---|---|---|
| A | 152 | 752 = Ans. |
| B | 9 | 44·5 |

**Ex. 52.**—How many inches must be cut off a board $9\frac{1}{2}$ inches wide, to obtain a piece equal to three Square feet? $\left(x = \frac{3 \times 144}{9\cdot5}.\right)$

| | | |
|---|---|---|
| A | 3 | 4·5 in. |
| B | 9·5 | 144 |

**Ex. 53.**—How many feet length of matting 2 ft. 3 in. wide, will cover a floor that measures 21 ft. by 14 ft. 8 in.? $\left(x = \frac{21 \times 14\cdot67}{2\cdot25}.\right)$

| | | |
|---|---|---|
| A | 21 | 136 ft. |
| B | 2·25 | 14·67 |

**Ex. 54.**—What remains of £690, after deducting $17\frac{1}{2}$ per cent? $\left(x = \frac{82\frac{1}{2}}{100} \text{ of } 690.\right)$ See Examples 41, and 114.

| | | |
|---|---|---|
| A | 82·5 | 569£ |
| B | 100 | 690 |

*Exact* answer, 569·25. Error, $\frac{1}{2277}$.

**Ex. 55.**—How many gallons of water fall on every acre, for every inch of rain-fall? Take 277·274 cubic inches to a Gallon, and 43560 Square feet to an acre. $\left(x = \frac{43560 \times 12 \times 12}{277\cdot274}.\right)$

| | | |
|---|---|---|
| A | 144 | 22622 gall. |
| B | 277·274 | 43560 |

N.B. This, at 224 gallons to a Ton, is 101 Tons *weight.*

**Ex. 56.**—Increase 384 by $\frac{3}{4}$ of itself. $(x = \frac{7}{4} \text{ of } 384.)$

| | | |
|---|---|---|
| A | 7 | 672 = Ans. |
| B | 4 | 384 |

**Ex. 57.**—Decrease $8\frac{3}{8}$ by $\frac{2}{13}$ of itself. $\left(x = \frac{11}{13} \text{ of } 8{\cdot}375.\right)$

| | | |
|---|---|---|
| A | 7·1 = Ans. | 11 |
| B | 8·375 | 13 |

***Ex. 58.**—Divide $\frac{5}{9}$ by 8. $\left(x = \frac{5}{9 \times 8} = \frac{5 \times {\cdot}125}{9}.\right)$

| | | |
|---|---|---|
| A | ·0694 = Ans. | 5 |
| B | ·125 | 9 |

**Ex. 58½.**—Divide 350 by $6\frac{1}{7}$. This is the same as $350 \div \frac{43}{7}$, or $x = \frac{350 \times 7}{43}$.

| | | |
|---|---|---|
| A | 57 = Ans. | 350 |
| B | 7 | 43 |

***Ex. 59.**—Multiply $12 \times 13 \times 1{\cdot}7$. $\left(x = \frac{13 \times 1{\cdot}7}{{\cdot}0833}.\right)$

| | | |
|---|---|---|
| A | 13 | 265·2 = Ans. |
| B | ·0833 | 17 |

**Ex. 60.**—Divide 2016 by $(7 \times 16)$ : or $x = \frac{2016 \times 0625}{7}$.

| | | |
|---|---|---|
| A | ·0625 | 18 = Ans. |
| B | 1 | 2016 |

***Ex. 61.**—What is the price, in £, of 15 pieces of linen, each containing 27 yards, at 4*s.* 6*d.* per yard? $\left(x = \frac{27 \times {\cdot}225}{{\cdot}0666}.\right)$

---

* In Examples 58, 59, 60, 61, it is supposed that the learner has made himself acquainted with the most common "reciprocals;" such as those of 8, 12, 16. See Ex. 34. (See APPENDIX G.)

| | | |
|---|---|---|
| A | 27 | 91·1£ |
| B | ·0667 | ·225 |

**Ex. 62.**—Since in any Triangle, the *area* $= \dfrac{\text{Base} \times \text{Perp.}}{2}$, what setting of the Slide Rule is adapted to this Formula?

Answer,

| | | |
|---|---|---|
| A | Base | Area |
| B | 2 | Perp.' |

or

| | | |
|---|---|---|
| A | Perp. | Area |
| B | 2 | Base |

N.B. With the above set of the Rule, we can find the Area, when the Base and Perp. are given;—and also the Base, when the Area and Perp. are given;—and also the Perp. when the Area and Base are given. For instance, in a Triangle whose Base is 45 inches, and Perp. 33·6 inches, its Area in Square inches is 755.

**Ex. 63.**—Put the Formula $x = \dfrac{R^2 - r^2}{\cdot 3183}$ into a setting adapted to the Slide Rule.

Answer,

| | | |
|---|---|---|
| A | $(R - r)$ | $x$ |
| B | ·3183 | ·R + r |

N.B. The above is the Formula for finding the area of the outer of two concentric plane Rings, whose radii are $R$ for the outer, and $r$ for the inner.

Many more Examples of $x = \dfrac{b \times c}{a}$ will be found under the head of "Reduction."

---

## Case of $x = \dfrac{b \times c}{a}$ in a "Series," where either $b$ or $c$ varies, but $a$ the Divisor, remains Constant.

The Rule is, to keep whichever of $b$ or $c$ is "constant," over or under $a$. (Compare Ex. 11.)

**Ex. 64.**—Multiply $\dfrac{28}{35}$ by 21, by 45, and by 80. (This is the

same as $x = \frac{28 \times 21}{35}, \frac{28 \times 45}{35}, \frac{28 \times 80}{35}$; where 28 and 35 are "constant.")*

| A | 16·8 | 28 | 36 | 64 |
|---|---|---|---|---|
| B | 21 | 35 | 45 | 80 |

**Ex. 65.**—What is $\frac{2}{13}$ of 689, $\frac{5}{13}$ of 689, and $\frac{6}{13}$ of 689? (Here $\frac{2 \times 689}{13}, \frac{5 \times 689}{13}, \frac{6 \times 689}{13}$, as in Ex. 64.)

| A | 106 | 265 | 318 | 689 |
|---|---|---|---|---|
| B | 2 | 5 | 6 | 13 |

**Ex. 66.**—Divide 555 into three parts, bearing the relative proportions of 5, $6\frac{1}{2}$, and 7. (Here $x = \frac{555 \times 5}{18{\cdot}5}, \frac{555 \times 6{\cdot}5}{18{\cdot}5}, \frac{555 \times 7}{18{\cdot}5}$; where $18{\cdot}5 = 5 + 6\frac{1}{2} + 7$.)

| A | 150 | 195 | 210 | 555 |
|---|---|---|---|---|
| B | 5 | 6·5 | 7 | 18·5 |

For other Examples like this, see under "Distributive Proportion."

**Ex. 67.**—If a Steamer consumes 80 tons of coals in 9 days, how many days will 76 tons,—112 tons,—and 150 tons,—last? $\left(\frac{9 \times 112}{80}, \frac{9 \times 150}{80}, \frac{9 \times 76}{80}.\right)$

| A | 8·55 | 9 | 12·6 | 16·9 |
|---|---|---|---|---|
| B | 76 | 80 | 112 | 150 tons |

**Ex. 68.**—What number of yards in a minute, does each of three persons advance, whose rates of walking are respectively 3, $3\frac{1}{2}$, and 4 miles an hour? $\left(\frac{3 \times 1760}{60}, \frac{3{\cdot}5 \times 1760}{60}, \frac{4 \times 1760}{60}.\right)$

* In Ex. 64, 28 may be on B, and 35 on A, as shown in the *Plate*:

| A | 21 | 35 | 45 | 80 |
|---|---|---|---|---|
| B | 16·8 | 28 | 36 | 64 |

| | | | | |
|---|---|---|---|---|
| A | 88 yds. | 103 yds. | 117 yds. | 1760 |
| B | 3 | 3·5 | 4 | 60 |

**Ex. 69.**—At $3\frac{1}{4}d.$ per Cubic yard, what will be the cost in £, of excavating 470, 580, and 800, Cubic yards?

$$\left(\frac{3{\cdot}25 \times 470}{240}, \frac{3{\cdot}25 \times 580}{240}, \frac{3{\cdot}25 \times 800}{240}.\right)$$

| | | | | |
|---|---|---|---|---|
| A | 3·25 | 6·364£ | 7·854£ | 10·83£ |
| B | 240 | 470 | 580 | 800 c. yds. |

**Ex. 70.**—A rate is levied on a row of houses, at 1*s.* 6*d.* per foot of frontage. What is the *frontage in feet*, of three houses, whose payments are £1 17*s.* 6*d.*, £2 0*s.* 6*d.*, and £2 6*s.* 6*d.*?

$$\left(\frac{37{\cdot}5 \times 12}{18}, \frac{40{\cdot}5 \times 12}{18}, \frac{46{\cdot}5 \times 12}{18}.\right)$$

| | | | | |
|---|---|---|---|---|
| A | 12 | 25 | 27 | 31 t. |
| B | 18 | 37·5 | 40·5 | 46·5£ |

**Ex. 71.**—If a Gunpowder manufacturer uses 76 per cent. of Nitre, 14 per cent. of Charcoal, and 10 per cent. of Sulphur, how many lbs. of each ingredient will there be in 112 lbs. of powder?

$$\left(\frac{112 \times 76}{100}, \frac{112 \times 14}{100}, \frac{112 \times 10}{100}.\right)$$

| | | | | |
|---|---|---|---|---|
| A | 11·2 = S. | 15·68 = C. | 85·12 = N. | 112 |
| B | 10 | 14 | 76 | 100 |

**Ex. 72.**—Subtract 16 per cent. from 230, 245, and 265.

$$\left(\frac{84 \times 230}{100}, \frac{84 \times 245}{100}, \frac{84 \times 265}{100}.\right)$$

| | | | | |
|---|---|---|---|---|
| A | 84 | 193 | 206 | 223 |
| B | 100 | 230 | 245 | 265 |

**Ex. 73.**—How many £ are 845 Rs. equivalent to, when the exchange varies from 1*s.* $10\frac{1}{2}d.$, to 1*s.* 11*d.*, and 1*s.* $11\frac{1}{2}d.$?

$$\left(\frac{845 \times 22{\cdot}5}{240}, \frac{845 \times 23}{240}, \frac{845 \times 23{\cdot}5}{240}.\right)$$

| | | | | |
|---|---|---|---|---|
| A | 79·2£ | 81·0£ | 83·0£ | 845 |
| B | 22·5*d.* | 23*d.* | 23·5*d.* | 240 |

**Ex. 74.**—If £2500 are invested in a 4 per cent Loan, at 18 discount, what will be (I.) the interest, and (II.) the annual income? $\left(\frac{2500 \times 4}{82}, \frac{100 \times 4}{82}.\right)$

| | | | |
|---|---|---|---|
| A | 4 | 4·88 Interest | 122£ Income |
| B | 82 | 100 | 2500 |

**Ex. 74½.**—How many shillings, for 7 days, is £16, £22, and £46, a year? $\left(\frac{16 \times 20 \times 7}{365}, \frac{22 \times 20 \times 7}{365}, \frac{46 \times 20 \times 7}{365}\right.$; where $20 \times 7 = 140$ is constant.)

| | | | | |
|---|---|---|---|---|
| A | 6·1*s.* | 8·4*s.* | 17·6*s.* | 140 |
| B | 16 | 22 | 46 | 365 |

---

### Case of $x = \frac{b \times c}{a}$ in a "Series," where both $b$ and $c$ remain constant, but $a$ varies.

(*Invert the Slide.* See p. 15.)

**Ex. 75.**—Divide (37·5 × 7·6) by 3, 4, and 5.

| | | | | |
|---|---|---|---|---|
| A | 95 | 57 | 71·25 | 37·5 |
| Ɔ | 3 | 5 | 4 | 7·6 |

or

| | | | | |
|---|---|---|---|---|
| A | 3 | 5 | 4 | 7·6 |
| Ɔ | 95 | 57 | 71·25 | 37·5 |

***Ex. 76.**—Five pumps empty a cistern in 8 hours. How many similar pumps must be employed to empty it in 2½, 4, and 4½ hours? $\left(\frac{5 \times 8}{2 \cdot 5}, \frac{5 \times 8}{4}, \frac{5 \times 8}{4 \cdot 5}.\right)$

---

* It will be seen from these Examples of the "Inverted Slide," that if the numbers on A are read from left to right, those under them on Ɔ, are read from right to left; and *vice versâ.*

| A | 5 | 8·9 | 10 | 16 hours |
|---|---|---|---|---|
| D | 8 | 4·5 | 4 | 2·5 |

**Ex. 77.**—If purchases are made of Railway stock, where the regular dividend is $6\frac{1}{2}$ per cent., at the rate of 8, 12, and 14 discount, what Interest obtained by each purchase?

$$\left(\frac{6{\cdot}5 \times 100}{92}, \frac{6{\cdot}5 \times 100}{88}, \frac{6{\cdot}5 \times 100}{86}.\right)$$ See Ex. 36.

| A | 7·56 | 7·39 | 7·07 | 6·5 |
|---|---|---|---|---|
| D | 86 | 88 | 92 | 100 |

**Ex. 78.**—What should the price of £100 Stock in an 8 per cent. Loan be, to obtain, $5\frac{1}{2}$, 6, and $6\frac{1}{2}$ per cent. interest?

$$\left(\frac{8 \times 100}{5\frac{1}{2}}, \frac{8 \times 100}{6}, \frac{8 \times 100}{6\frac{1}{2}}.\right)$$

| A | 100 | 124£ | 133£ | 145£ |
|---|---|---|---|---|
| D | 8 | 6·5 | 6 | 5·5 |

**Ex. 79.**—In Surveying, the Scale of 4 chains to an inch is equivalent to 20 inches to a mile. What are 3, $4\frac{1}{2}$, and 5 chains to an inch equivalent to, in inches per mile?

$$\left(\frac{4 \times 20}{3}, \frac{4 \times 20}{4{\cdot}5}, \frac{4 \times 20}{5}.\right)$$

| A | 26·7 | 20 | 17·8 | 16 inches |
|---|---|---|---|---|
| D | 3 | 4 | 4·5 | 5. |

or

| A | 16 | 17·8 | 20 | 26·7 inches |
|---|---|---|---|---|
| D | 5 | 4·5 | 4 | |

See also the two last Examples (133, 134) under "Rule of Three."

## REDUCTION.*

To Reduce from greater to lower denomination, is simply "multiplication;" from lower to higher, is "division."

**Ex. 80.**—Reduce $12\frac{1}{2}$ miles, to yards. (*Multiply* 12·5 × 1760, as in the *Plate.*)

| | | |
|---|---|---|
| A | 12·5 | 22000 yds. |
| B | 1 | 1760 |

**Ex. 81.**—Reduce $30\frac{1}{4}$ Square yards, *i.e.* a Rod, Pole, or Perch,—to Square feet. (*Multiply* 30·25 × 9.)

| | | |
|---|---|---|
| A | 30·25 | 272·25 Sq. Ft. |
| B | 1 | 9 |

**Ex. 82.**—Reduce 2080 Acres, to Square miles.

$\left(\textit{Divide } \frac{2080}{640}.\right)$

| | | |
|---|---|---|
| A | 640 | 3·25 Sq. M. |
| B | 1 | 2080 |

**Ex. 83.**—Reduce $26\frac{2}{3}$ Fluid ounces, to Imperial Pints.

$\left(\textit{Divide } \frac{26{\cdot}67}{20}.\right)$

| | | |
|---|---|---|
| A | 20 | 1·33 Pints |
| B | 1 | 26·67 |

**Ex. 84.**—Reduce $\frac{98}{160}$ to a Decimal. (i.e. *Divide* 98 by 160. See note to Ex. 6.)

| | | |
|---|---|---|
| A | ·612 = Ans. | 1 |
| B | 98 | 160 |

---

* *Reduction* of *s. d.* to £, and *vice versâ*, is soon attained *mentally*, quite near enough for all Slide Rule purposes. See APPENDIX G.

***Ex. 85.**—Reduce $\frac{47}{112}$ to a Decimal. (*i.e.* *Divide* 47 by 112. See Examples 6 and 93.)

$$\frac{A\quad \cdot 42 = \text{Ans.} \qquad\qquad 1}{B\quad 47 \qquad\qquad 112}$$

The *exact* decimal is ·419643. Error, $\frac{1}{1176}$.

**Ex. 86.**—Reduce $\frac{17}{19}d.$ to farthings. $\left(x = \frac{17 \times 4}{19}.\right)$

$$\frac{A\quad 3{\cdot}58\,f. \qquad\qquad 17}{B\quad 4 \qquad\qquad 19}$$

**Ex. 87.**—Reduce $\frac{8}{15}$ of Half-a-crown, to shillings. $\left(x = \frac{8}{15} \times 2{\cdot}5.\right)$

$$\frac{A\quad 1{\cdot}3s. \qquad\qquad 8}{B\quad 2{\cdot}5 \qquad\qquad 15}$$

Or we may take it as $\frac{8}{15}$ of $30d. = 16d.$

**Ex. 88.**—Reduce $\frac{25}{33}$ of an inch, to the decimal of a Foot. (Or $\frac{25}{33}$ of an Oz. Troy to the decimal of a lb. Troy). Here $x = \frac{25}{33} \div 12$; and such cases, unless we are acquainted with a few leading "Reciprocals," (see footnote ‡ p. 14), cannot be solved in *one setting*; but it is supposed that the reciprocal of 12 *is* known to be ·0833, and then we have $x = \frac{25 \times \cdot 0833}{33}$.

---

*In Examples 84 and 85, and others like them, we may remember that the Slide Rule exhibits *equal fractions*, (N.B. II., p. 14), hence $\frac{A\quad 98}{B\quad 160} = \frac{612}{1000}$, and $\frac{A\quad 47}{B\quad 112} = \frac{42}{100}$.

| | | |
|---|---|---|
| A | ·0631 Ft. | 25 |
| B | ·0833 = $\frac{1}{12}$ | 33 |

**Ex. 89.**—Reduce $\frac{7}{8}$ of an Oz. Avoir., to the decimal of 1lb. (Here $\frac{7}{8} \div 16 = \frac{7 \times \cdot0625}{8}$. See remarks to preceding Example.)

| | | |
|---|---|---|
| A | ·0547 lb. | 7 |
| B | ·0625 = $\frac{1}{16}$ | 8 |

**Ex. 90.**—Reduce ·62 Acre, to Square feet. (*Multiply* ·62 × 43560.)

| | | |
|---|---|---|
| A | ·62 | 27000 Sq. ft. |
| B | 1 | 43560 |

The *exact* answer is 27007·2.

**Ex. 91.**—Reduce 58 days, to the decimal of a year. $\left(i.e.\ Divide\ \frac{58}{365}.\right)$

| | | |
|---|---|---|
| A | ·159 year | 1 |
| B | 58 | 365 |

**Ex. 92.**—Reduce 80° 14′ 19·5″ (the Longitude of the Madras Observatory) to *Time*. (Here we first reduce the seconds of arc to dec. of a minute; secondly, minutes of arc to dec. of a degree; and lastly, degrees of arc to Hours of time, by dividing by 15.)

| | | | | |
|---|---|---|---|---|
| 1st. | $\frac{19·5''}{60}$ | A | ·325′ | 119·5″ |
| | | B | 1 | 60 |
| 2nd. | $\frac{14·3'}{60}$ | A | ·24° | 14·325′ |
| | | B | 1 | 60 |
| 3rd. | $\frac{80·24°}{15}$ | A | 5·35 h. | 80·24° |
| | | B | 1 | 15 |

The *exact* answer is 80·23875°, or 5·34925*h*. = 5*h*. 20*m*. 57·3*sec*.

**Ex. 93.**—Reduce the 1 Qr. 19lbs. of Ex. 5 to the decimal of a Cwt.

$$\left(x = \frac{47}{112}, \text{ solved as in Ex. 85.}\right)$$

**Ex. 94.**—Reduce the 2 ro. 18 po. of Ex. 6 to the decimal of an Acre.

$$\left(x = \frac{98}{160}, \text{ solved as in Ex. 84.}\right)$$

**Ex. 95.**—Reduce at *one setting* of the Slide, ·475 Ac.—·55 Ac.—and ·85 Ac. to Poles. (Here we have to *multiply* a "*Series*" of numbers, by a constant Multiplier 160, as in Ex. 11.)

| | | | | |
|---|---|---|---|---|
| A | 76 po. | 88 po. | 136 po. | 160 |
| B | ·475 | ·55 | ·85 | 1 |

*__Ex. 96.__—Reduce at *one setting* of the Slide, 1 qr. 24 lbs.—2 qr. 18 lbs.—and 3 qr. 10 lbs., to decimals of a Cwt. (Here we have to *divide* a "*Series*" of numbers, by a constant Divisor 112, as in Ex. 29.)

| | | | | |
|---|---|---|---|---|
| A | ·464 cwt. | ·661 cwt. | ·84 cwt. | 1 |
| B | 52 | 74 | 94 | 112 |

The decimals to four places, are ·4643, ·6607, ·8393, which shows how near the instrument will read.

---

## RULE OF THREE.†

| | | | |
|---|---|---|---|
| either | A | Second term | Fourth term |
| | B | First term | Third term |
| or | A | First term | Third term |
| | B | Second term | Fourth term |

---

* In Ex. 96, it is supposed that the learner sees at once that 2 qr. 18 lbs. = 52 lbs., 3 qr. 10 lbs. = 94 lbs., &c.

† It has not been thought desirable to make any distinction as to Rule of Three *inverse*, a term that ought to be exploded. (Examples 106, 129.)

**Ex. 97.**—3 : $37\frac{1}{2}$ :: 7·6 : $x$. What is the value of $x$? (See *Plate.*)

| | | |
|---|---|---|
| A | 37·5 | 95 = $x$ |
| B | 3 | 7·6 |

N.B. It will be observed that this is exactly the same as $x = \frac{37·5 \times 7·6}{3}$ in Ex. 37. Either the 37·5, or the 7·6, may be made the second term.

**Ex. 98.**—$x$ : 12·6 :: 150 : 16·9. Required the value of $x$.

| | | | |
|---|---|---|---|
| either | A | 12·6 | 16·9 |
| | B | 112 = $x$ | 150 |

| | | | |
|---|---|---|---|
| or | A | 112 = $x$ | 150 |
| | B | 12·6 | 16·9 |

**Ex. 99.**—If 19 cwt. of sugar cost £57, what will 37 cwt. cost? (19 : 57 :: 37 : $x$.)

| | | |
|---|---|---|
| A | 57 | 111 £ |
| B | 19 | 37 |

**Ex. 100.**—If a chest of tea weighing 84 lbs. cost £19 4*s.* how many lbs. do I get for £1 8*s.*? (19·2 : 1·4 :: 84 : $x$.)

| | | |
|---|---|---|
| A | 1·4£ | 6·12 lbs. |
| B | 19·2£ | 84 lbs. |

The *exact* answer is 6·125 lbs. or 6 lb. 2 oz.

**Ex. 101.**—If one acre of land is rented for £5 15*s.*, what would 8 Ac. 2 r. 18 po. rent for? (Simple "multiplication" 1 : 8·61 :: 57·5 : $x$. See Examples 6 and 94.)

| | | |
|---|---|---|
| A | 8·61 | 49 51 £ |
| B | 1 | 5·75 |

***Ex. 102.**—If one ounce of Gold plate is sold for £3 11*s.*, what will 5 oz. 17 dwts. 14 grains sell for? (This is simply to *multiply* 5·88 by 3·55.)

| | | |
|---|---|---|
| A | 3·55 | 20·9£ |
| B | 1 | 5·88 |

**Ex. 103.**—If the side of a Square acre is 208·7 feet, what is the length *in inches*, of the side of a Square acre, on a Map drawn to the scale of 330 feet to a mile? (330 : 208·7 :: 1 : $x$, or $\frac{208 \cdot 7}{330}$ as in Ex. 17, &c.)

| | | |
|---|---|---|
| A | ·6324 in. | 1 |
| B | 208·7 | 330 |

**Ex. 104.**—On a protractor engraved to the scale of 4 inches to a mile, what is the measure in *decimals of an inch*, of a space equivalent to 100 yards? (1760 : 4 :: 100 : $x$.)

| | | |
|---|---|---|
| A | ·2273 in. | 4 |
| B | 100 | 1760 |

**Ex. 105.**—What are the Parochial Rates, *in pence per £*, if they amount to £37 on a house rated at £194? (194 : 37 :: 240 : $x$.)

| | | |
|---|---|---|
| A | 37 | 45·8*d.* |
| B | 194 | 240 |

**Ex. 106.**—If 39 men can do a work in 168 days, in how many days can 72 men do it?

| | | |
|---|---|---|
| A | 39 | 91 days |
| B | 72 | 168 |

**Ex. 107.**—A bankrupt whose debts are £6400, realizes assets to the amount of £5600. How many shillings in the £ will his creditors get? (560 : 640 :: 20 : $x$.)

* Ex. 102. The ounce decimal ·88, for 17 dwts. 14 grains, is the same as 17*s.* 7*d.* would be of £1; namely $\cdot 8791\dot{6}$. See APPENDIX G, for mental Reduction to dec. of 1 oz. Troy.

| | | |
|---|---|---|
| A | 17·5s. | 640 Debts |
| B | 20 | 560 Assets |

N.B. The Rule set as above, will give any required term, if the other three are known. Thus if the assets of £560 are enough to pay 17*s.* 6*d.* in the £, the debts are £640. A creditor to whom £640 are due, will at 17*s.* 6*d.* in the £. get £560.

**Ex. 108.**—How much is 16 per cent. of 330? (100 : 16 :: 330 : $x$.) Or 330 × ·016, as in Examples 7 and 8.

| | | |
|---|---|---|
| A | 100 | 330 |
| B | 16 | 52·8 = Ans. |

**Ex. 109.**—How much per cent. is 7½*d.* in the £? (240*d.* : 7·5*d.* :: 100 : $x$.)

| | | |
|---|---|---|
| A | 100 | 240 |
| B | 3·125 = Ans. | 7·5 |

**Ex. 110.**—If 12*s.* 6*d.* is $\frac{5}{7}$ the hire of a cart, what is the whole hire? (5 : 7 :: 12·5 : $x$.)

| | | |
|---|---|---|
| A | 7 | 17·5 |
| B | 5 | 12·5 |

**Ex. 111.**—What is 45, three sixteenths of? (3 : 16 :: 45 : $x$.)

| | | |
|---|---|---|
| A | 16 | 240 = Ans. |
| B | 3 | 45 |

**Ex. 112.**—What *Capital* at 4 per cent. will an annual expenditure of £2320 represent? (4 : 100 :: 2320 : $x$.)

| | | |
|---|---|---|
| A | 100 | 58000£ |
| B | 4 | 2320 |

**Ex. 113.**—If a bookseller takes off 2*d.* in the shilling for ready money, and my *cash* payment comes to 7*s.* 6*d.*, what would have been the ordinary price? (10 : 12 :: 7·5 : $x$.)

| | | |
|---|---|---|
| A | 7·5*s.* | 10 |
| B | 9 | 12 |

**Ex. 114.**—What remains of £690, after deducting $17\frac{1}{2}$ per cent.? (100 : 82·5 :: 690 : *x*.) See Ex. 54.

| | | |
|---|---|---|
| A | 82·5 | 569£ |
| B | 100 | 690 |

The *exact* answer is £569 5*s.* Error $\frac{1}{2277}$.

**Ex. 115.**—If I buy £100 stock in a Railway paying $6\frac{1}{4}$ per cent. at £118, what interest do I get for my money?

(118 : 6·25 :: 100 : *x*.)

| | | |
|---|---|---|
| A | 5·3 = Ans. | 6·25 |
| B | 100 | 118 |

**Ex. 116.**—If after paying 7*d.* in the £ Income Tax, a person's net income is £1747 10*s.*, what was his *gross* income?

(233*d.* : 240*d.* :: 1747·5 : *x*.)

| | | |
|---|---|---|
| A | 240*d.* | 1800£ |
| B | 233*d.* | 1747·5£ |

**Ex. 117.**—When the Income Tax was 9*d.* in the £, a person's *net* income was £481 5*s.* What would it be when the Income Tax was 1*s.* 4*d.*; his *gross* income remaining the same?

(231*d.* : 224*d.* :: 481·5 : *x*.)

| | | |
|---|---|---|
| A | 224*d.* | 467£ |
| B | 231*d.* | 481·5£ |

**Ex. 118.**—By selling goods for £25, I lost $\frac{1}{6}$ of what they cost me. What did I pay for them? (5 : 6 :: 25 : *x*.)

| | | |
|---|---|---|
| A | 6 | 30£ |
| B | 5 | 25£ |

**Ex. 119.**—Lost 5 per cent. by selling goods for £40. How much *per cent.* should I have gained had I sold them for £44? (40 : 44 :: 95 : *x*.) A "Double Rule of Three" sum, as in Ex. 122.

| | | |
|---|---|---|
| A | 44£ | 104 5£ |
| B | 40£ | 95 |

This gives the answer = $4\frac{1}{2}$ per cent.

**Ex. 120.**—A person sold goods for £36, making a profit of 17 per cent. Required the original cost. (117 : 100 :: 36 : $x$.)

| | | |
|---|---|---|
| A | 30·77 £ | 100 |
| B | 36 | 117 |

**Ex. 121.**—A person sold a watch for £32, being 22 per cent. below its original cost. What was that cost? (78 : 100 :: 32 : $x$.)

| | | |
|---|---|---|
| A | 41 £ | 100 |
| B | 32 | 78 |

The exact answer is £41 0*s*. 6*d*.

***Ex. 122.**—By selling wine at 15*s*. a gallon, I lose 6 per cent. At what price per gallon ought I to have sold it, to gain $17\frac{1}{2}$ per cent.? (94 : 117·5 :: 15 : $x$.)

| | | |
|---|---|---|
| A | 18·75*s*. | 117·5 |
| B | 15 | 94 |

**Ex. 123.**—A Square whose perimeter is 44 feet, has an Area of 121 Square feet; but a Circle with the *same perimeter*, has an Area of 154 Square feet. How much *per cent.* greater is the Area of a Circle than that of a Square, having the same perimeter?

(121 : 154 :: 100 : $x$.)

| | | |
|---|---|---|
| A | 127·3 | 154 |
| B | 100 | 121 |

The answer therefore is 27·3 per cent.

---

* Ex 122. Properly, $x = \frac{100 \times 117{\cdot}5 \times 15}{100 \times 94}$, but the two 100's may be mentally rejected. It is, in fact, a "Double" Rule of Three. (1) 94 : 100 :: 15 : Original cost. (2) 100 : 117·5 :: Original cost : $x$.

**Ex. 124.**—In Timber measuring, the "Customary" content is 21·46 per cent. too little. How much per cent. must *be added* to a given "Customary" content, to know the "True" content?

(78·54 : 100 :: 100 : *x*.)

| | | |
|---|---|---|
| A | 100 | 127·324 |
| B | 78·54 | 100 |

Therefore the amount to be added is 27·324 per cent.

**Ex. 125.**—What discount should a 5 per cent. Loan be at, to obtain interest at 6 per cent.? (6 : 5 :: 100 : *x*.)

| | | |
|---|---|---|
| A | 5 | 83·33 |
| B | 6 | 100 |

This gives the "answer" 16$\frac{2}{3}$ Discount, (or 100 − 88$\frac{1}{3}$).

***Ex. 126.**—How many £ of Stock can be bought for £3625, when £100 Stock sells for 90$\frac{5}{8}$? (In cases like this it is better to consider 90$\frac{5}{8}$ as $\frac{725}{8}$. Then we have $\frac{725}{8}$ : 100 :: 3625. Or 725 : 8 :: 362500 : *x*.)

| | | |
|---|---|---|
| A | 4000£ | 362500 |
| B | 8 | 725 |

**Ex. 127.**—What length in *yards*, of papering 21 inches wide, will cover a surface of 73$\frac{1}{3}$ Square yards? (21 : 36 :: 73·33 : *x*.)

| | | |
|---|---|---|
| A | 36 | 125·7 yards |
| B | 21 | 73 33 |

**Ex. 128.**—What is the price *in shillings* of 17 dozen, at 11*s.* 4$\frac{1}{2}$*d.* per Gross? (12 : 17 :: 11·375 : *x*.)

---

* Ex. 126. As the learner is supposed to know (Appendix G) the decimal equivalent of $\frac{5}{8}$ to be ·625, the Rule might be set as follows:

| | | |
|---|---|---|
| A | 100 | 4000 |
| B | 90·625 | 3625, |

but it is not easy to set 90·625 exactly; so the setting given above is better.

† The learner will see that the original "proportion" is

144 : 11·375 :: 17 × 12 : *x*,—

which is equal to 12 : 11·375 :: 17 : *x*. The "reduction" of 4$\frac{1}{2}$*d.* to the

| | | |
|---|---|---|
| A | 16·1*s.* | 17 |
| B | 11·375 | 12 |

The *exact* answer is 16*s.* $1\frac{3}{8}$*d.*

**Ex. 129.**—If I lend a person £334 for 7 months, for how many months ought he to lend me £672? (672 : 384 :: 7 : $x$.)

| | | |
|---|---|---|
| A | 4 months | 384 |
| B | 7 | 672 |

**Ex. 130.**—If a yard of cloth costs 14*s.* 6*d.*, how many £ will 28, 32, and 36 yards cost, respectively? (Here we have a "series" of Rule of Three sums, with a first Term = 1; and thus it becomes a series of *Multiplications* as in Ex. 11.)

| | | | | |
|---|---|---|---|---|
| A | ·725£ | 20·3£ | 23·2£ | 26·1£ |
| B | 1 | 28 | 32 | 36 |

**Ex. 131.**—If the average rate of an Express train is 45 miles an hour, how many hours will it take to run 185, 270, and 306 miles? (Here we have a series of *Divisions*, as in Ex. 29.)

| | | | | |
|---|---|---|---|---|
| A | 1 | 4·11h. | 6h. | 6·8h. |
| B | 45 | 185 | 270 | 306 |

**Ex. 132.**—Three persons had each an 84 lb. chest of tea given them to divide ~~equally among~~ friends. One had to divide his among 14 persons; the second among 18; and the third among 24. In what quantities did they respectively make up their packages? (Here 14 : 1 :: 84 : $x$.—18 : 1 :: 84 : $y$.—24 : 1 :: 84 : $z$; or a constant number 84, to be divided by a "series" of Divisors as in Ex. 33. *Invert the Slide.*

| | | | | |
|---|---|---|---|---|
| A | 3·5 | 4·67 lbs. | 6 lbs. | 84 lbs. |
| C | 24 | 18 | 14 | 1 |

---

decimal of a shilling, is done *instantly*, as in Ex. 85. (see note) for $\frac{4\cdot5}{12} = \frac{375}{1000}$ths.

**Ex. 133.**—Five pumps empty a cistern in 8 hours. How many similar pumps must be employed to empty it in $2\frac{1}{2}$ h., 4 h., and $4\frac{1}{2}$ h.?

$$\left.\begin{array}{l} 2{\cdot}5 : 5 :: 8 : x \\ 4 : 5 :: 8 : y \\ 4{\cdot}5 : 5 :: 8 : z \end{array}\right\}$$ Here the 1st Term or "Divisor" varies, and the other Terms remain constant. See Ex. 76. *Invert the Slide.*

| A | 5 | 8·9 h. | 10 h. | 16 h. |
|---|---|---|---|---|
| Ɔ | 8 | 4·5 | 4 | 2·5 |

**Ex. 134.**—What should the price of £100 Stock be, in an 8 per cent. Loan, to obtain $5\frac{1}{2}$, 6, and $6\frac{1}{2}$ per cent. interest?

$$\left.\begin{array}{l} 5{\cdot}5 : 8 :: 100 : x \\ 6 : 8 :: 100 : y \\ 6{\cdot}5 : 8 :; 100 : z \end{array}\right\}$$ Here the 1st Term or "Divisor" varies, whilst the other two Terms remain constant. *Invert the Slide* as in Ex. 78.

| A | 100 | 124£ | 133£ | 145£ |
|---|---|---|---|---|
| Ɔ | 8 | 6·5 | 6 | 5·5 |

## DOUBLE RULE OF THREE.

The Slide Rule is not of so much use here, except in expediting the multiplications, especially when the decimal equivalents for usual fractions are known. See N.B. 3, p. 14.

**Ex. 135.**—A Steamer has fuel for 12 days when steaming 13 h. 25 m. per day. How *many times as much* fuel will be required for a voyage of 14 days, steaming 17 h. 15 m. per day? (Here we are supposed to know that the Reciprocal of 12 is ·0833, and that 25 m. = ·417 h. See first footnote p. 30, and footnote p. 27.

$$\left.\begin{array}{l} 12 : 14 \\ 13\tfrac{5}{12} : 17\tfrac{1}{4} \end{array}\right\} :: 1. \quad \text{Or } x = \frac{14 \times 17{\cdot}25 \times 1}{12 \times 13{\cdot}417}.$$

(I.) $14 \times 17{\cdot}25$

| A | 14 | 241·5 |
|---|---|---|
| B | 1 | 17·25 |

(II.) $\dfrac{241{\cdot}5 \times {\cdot}0833}{13{\cdot}417}$

| A | 1·5 times = Ans. | 241·5 |
|---|---|---|
| B | ·0833 | 13·417 |

**Ex. 136.**—If 40 bushels of corn serve 12 horses for 37 days, how many days would 195 bushels serve 9 horses?

$$\left.\begin{matrix} 40 : 195 \\ 9 : 12 \end{matrix}\right\} :: 37 : x, \text{ or } x = \frac{195 \times 12 \times 37}{40 \times 9} = \frac{195 \times 12 \times 37}{360}.$$

1st. $12 \times 37$

| | | |
|---|---|---|
| A | 12 | 444 |
| B | 1 | 37 |

2d. $\dfrac{444 \times 195}{360}$

| | | |
|---|---|---|
| A | 240 days = Ans. | 444 |
| B | 195 | 37 |

**Ex. 137.**—If 5 men can do a piece of work, in 27 days of 9 hours each, how many days would it take 6 men to do double the quantity, provided they worked 10 hours per day?

$$\left.\begin{matrix} 6 : 5 \\ 10 : 9 \\ 1 : 2 \end{matrix}\right\} 27 : x, \text{ or } x = \frac{5 \times 9 \times 2 \times 27}{60} = \frac{90 \times 27}{60}.$$

| | | |
|---|---|---|
| A | 90 | 40·5 day = Ans. |
| B | 60 | 27 |

**Ex. 138.**—If 7 men can mow 83 acres, in 12 days of $8\frac{1}{4}$ hours each, how many acres can be mowed by 20 men in 11 days of $7\frac{4}{5}$ hours each?

$$\left.\begin{matrix} 7 : 20 \\ 12 : 11 \\ 8\frac{1}{4} : 7\frac{4}{5} \end{matrix}\right\} :: 83 : x. \quad \text{Or } x = \frac{20 \times 11 \times 7{\cdot}8 \times 83}{7 \times 12 \times 8{\cdot}25}.$$

1st. $220 \times 7{\cdot}8$

| | | |
|---|---|---|
| A | 7·8 | 1720 |
| B | 1 | 220 |

2d. $84 \times 8{\cdot}25$

| | | |
|---|---|---|
| A | 8·25 | 693 |
| B | 1 | 84 |

3d. $\dfrac{1720 \times 83}{693}$

| | | |
|---|---|---|
| A | 206 Ac. = Ans | 83 |
| B | 1720 | 693 |

The correct answer is 205·53.

**Ex. 139.**—In 1852, when the Income Tax was 7*d.* in 1£, a person's *net* income was £500. What would his net income be in 1855, when the Income Tax was 1*s.* 4*d.* in 1£; his *gross* income remaining the same?

$$\left.\begin{array}{l} 233 : 240 \\ 240 : 224 \end{array}\right\} \begin{array}{l} :: \\ :: \end{array} 500 : x. \text{ Or } x = \frac{240 \times 224 \times 500}{240 \times 233} = \frac{224 \times 500}{233}.$$

| | | |
|---|---|---|
| A | 224 | 481 £ |
| B | 233 | 500 |

The exact answer is £480·676. Error $= \frac{1}{1490}$.

**Ex. 140.**—If 7 masons take 12 days to build a wall 21 ft. high, 4 ft. thick, and 30 ft. long; how many feet in length can be built by 12 masons in 30 days, if the wall is to be 30 ft. high, and 4½ ft. thick?

$$x = \frac{30 \times 12 \times 21 \times 4 \times 30}{7 \times 30 \times 4{\cdot}5 \times 12} \text{ cancelled to } \frac{12 \times 30}{4{\cdot}5}$$

| | | |
|---|---|---|
| A | 30 | 80 ft. long |
| B | 4·5 | 12 |

**Ex. 141.**—Required the "commercial par" value of £1, *in francs*,—by the following "chain."

| | | |
|---|---|---|
| $x$ Francs | = | 20 Shillings |
| 77·875 Shillings | = | 1 Oz. Troy of Standard gold |
| 1 Oz. Troy | = | 31·1 Grammes |
| 1000 Grammes | = | 3150·58 Francs * |

$$x = \frac{20 \times 1 \times 31{\cdot}1 \times 3150{\cdot}58}{77{\cdot}875 \times 1 \times 1000} = \frac{622 \times 3150{\cdot}58}{77875}$$

| | | |
|---|---|---|
| A | 25·2 francs = Ans. | 3150·58 |
| B | 622 | 77875 |

An experienced eye would see that the answer was something less than 25·2. The exact answer is 25·1642 francs.

---

* See Appendix M for the French coinage.

## DISTRIBUTIVE PROPORTION.

[Sometimes known as "Single," or "Double," *Fellowship.*]

THIS is simply a "Series" of Rule-of-Three sums, where the Slide Rule is very useful, because the first, and one of the middle Terms are *constant:* as shown in page 31, where in the equation $x = \frac{b \times c}{a}$, $a$, and either $b$ or $c$, continue constant throughout the "Series."

**Ex. 142.**—Three persons put in respectively £5, £6 10*s.*, and £7, into a purchase which yielded a profit of £55 10*s.* How ought they to share this profit? (Here $5 + 6\frac{1}{2} + 7 = 18\frac{1}{2}$. Then as in Ex. 66.)

$$18{\cdot}5 : 55{\cdot}5 :: 5 : x, \text{ or } \frac{55{\cdot}5 \times 5}{18{\cdot}5}$$

$$18{\cdot}5 : 55{\cdot}5 :: 6{\cdot}5 : y, \text{ or } \frac{55{\cdot}5 \times 6{\cdot}5}{18{\cdot}5}$$

$$18{\cdot}5 : 55{\cdot}5 :: 7 : z, \text{ or } \frac{55{\cdot}5 \times 7}{18{\cdot}5}$$

| A | 15 | 19·5 | 21 | 55·5 |
|---|---|---|---|---|
| B | 5 | 6·5 | 7 | 18·5 |

Whence, *as a general setting of the Slide, we have:*

| A | Share of gain or loss | Total to be divided |
|---|---|---|
| B | Share of amount subscribed | Total subscribed |

**Ex. 143.**—It is required to divide the number 34 into two parts, which shall be to each other as 11 to 6. (*i.e.* $\frac{11}{17}$ of 34, and $\frac{6}{17}$ of 34 as in Ex. 65.)

| A | 12 | 22 | 34 |
|---|---|---|---|
| B | 6 | 11 | 17 |

**Ex. 144.**—Divide a profit of £689 among A, B, and C, where A has invested $\frac{2}{13}$ths, B $\frac{5}{13}$ths, and C $\frac{6}{13}$ths. (Ex. 65.)

| A | 106£ | 265£ | 318£ | 689 |
|---|---|---|---|---|
| B | 2 | 5 | 6 | 13 |

**Ex. 145.**—Three persons rent some grass land for £70. A puts in 50 cattle for 4 months; B puts in 40 for 7 months; and C puts in 30 for 12 months. What part of the rent should each pay? (Here 50 × 4 = 200, 40 × 7 = 280, 30 × 12 = 360. Total 840.)

| | | | | |
|---|---|---|---|---|
| A | 16·6£ | 23·3£ | 30£ | 70 |
| B | 200 | 280 | 360 | 840 |

**Ex. 146.**—A bankrupt owes to 3 creditors as follows: £312 10*s.*, £418 6*s.* 8*d.*, and £596 3*s.* 4*d.* His effects sell for £870 16*s.* 10½*d.** What will each receive?

| | | | | |
|---|---|---|---|---|
| A | 205£ | 275£ | 391£ | 870·84 |
| B | 312·5 | 418·6 | 596·3 | 1327·5 |

N.B. The above answers were read off in half a minute. The *exact* answers are £205 1*s.* 6¾*d.*—£274 10*s.* 7½*d.*—£391 4*s.* 8¼*d.*

**Ex. 147.**—17 persons agreed to dine together every day during the month of September, but they did not all attend regularly. One dined 17 times: six dined 21 times: nine dined 27 times: and one dined every day. At the end of the month, the Bill came to £91 10*s.* How should they divide?

| | | | | | | |
|---|---|---|---|---|---|---|
| A | 4·4*s.* | 74·6*s* | 92·3*s.* | 118·8*s.* | 132*s.* | 1830*s.* |
| B | 1 | 17 | 21 | 27 | 30 | 416 |

---

## EQUATION OF PAYMENTS.

**Ex. 148.**—A person is indebted to another, £120; of which half is due in 3 months, a quarter in 6 months, and the rest in 9 months. If he wishes to pay the whole sum in one payment, after how many months should he make it?

---

* See line 3, page 11.

$$x = \frac{(60 \times 3) + (30 \times 6) + (30 \times 9)}{60 + 30 + 30} = \frac{630}{120}$$ which is a Sum in "Simple Division," p. 19.

| | | |
|---|---|---|
| A | 1 | 5·25 months |
| B | 120 | 630 |

---

## PROPORTIONING THE "DIFFERENCES" IN TABLES OF LOGARITHMS.

In those Tables which have a column of "*diff. to* 100," it is worth while for persons who have to use these Tables daily, to have a Slide Rule; even if it were only for the purpose of taking out the proportion of difference for the last two or three figures. It is a simple "Rule of Three" sum (p. 39).

**Ex. 149.**—What number answers to Log. ·104910, when ·104828 = 1·273, and the "*diff. to* 100" is 341? (Here we want to know the value of the "82" difference in excess of ·104828; or 341 : 100 :: 82 : $x$.)

| | | |
|---|---|---|
| A | 24 | 100 |
| B | 82 | 341 |

showing *instantly*, that 24 must be added to 1·273 to obtain the answer 1·27324.

**Ex. 150.**—Required the Log. Sine of 51° 28′ 38″, when the Table gives 51° 28′ = 9·8933433, and the "*diff. to* 10" is 168. (Here 10″ : 368 :: 38″ : $x$, or $x$ = 16·8 × 38.)

| | | |
|---|---|---|
| A | 168 | 638 |
| B | 10 | 38 |

Then add 638 to 9·8933433, and we have the answer 9·8934071.

**Ex. 151.**—Required the Log. of 15432·35, when the Log. cf 15432 = 4·1884222, and the "*diff. to* 100" = 281. (Here 100 : 281 :: 35 : $x$.)

| | | |
|---|---|---|
| A | 98 | 281 |
| B | 35 | 100 |

Then add 98 to 4·1884222, and we have the answer 4·1884320.

---

## UNIFORM MOTION.

**Ex. 152.**—Two trains start at the same time, in *opposite* directions, from places 180 miles apart; one running at the rate of 50, the other 22 miles, an hour. After what time will they meet, and at this moment how far will each be from the place it left? (All the answers can be read off at one setting of the Rule.)

| | | | | |
|---|---|---|---|---|
| A | 2·5 h. | 55 miles | 125 miles | 180 |
| B | 1 | 22 | 50 | 72 = (50 + 22) |

That is, they will meet in $2\frac{1}{2}$ hours from the time they start; and at this moment, the fast train will have travelled 125 miles, and the ordinary 55 miles.

**Ex. 153.**—A ship that sails 5 miles an hour to another's 8, both being bound in the *same direction*, has a start of 186 miles. In how many hours will the faster come up to the slower, and at this moment, how many miles will each have sailed?

| | | | | |
|---|---|---|---|---|
| A | 62·h. | 186 | 310 miles | 496 miles |
| B | 1 | 3 = (8 – 5) | 5 | 8 |

N.B. In Examples 152 and 153, the "time" can be found independent of the "distance;" or the "distance" independent of the "time."

It will be observed that when the movement is in *opposite* directions, the "time" = miles ÷ sum of miles per hour; but if in the *same* direction, the "time" = miles ÷ diff. of miles per hour. The "distance" travelled by each, is of course = time × miles per hour.

**Ex. 154.**—Suppose a body put in motion with a velocity of 24 feet per second, and three other bodies *b*, *c*, *d*, put in motion 3, 6, and 8, seconds after, all in the *same* direction. Required to find their

respective velocities, so that they may all reach the same place in 12 seconds from the time that the first was put in motion. (Here *b*, *c*, *d*, will be in motion respectively 9, 6, and 4 seconds; whence their respective velocities will be $\frac{24 \times 12}{4}$, $\frac{24 \times 12}{6}$, $\frac{24 \times 12}{9}$, as in Ex. 76. Invert the Slide.)

| A | 24 | 32 ft. | 48 ft. | 72 ft. |
|---|---|---|---|---|
| Ɔ | 12 | 9 | 6 | 4 |

**Ex. 155.**—During the time the *Short* hand of a clock revolves round the dial, at what times will it and the Long hand be together again after XII h.? (Add as many 11ths of an hour as hours elapsed since XII. Or multiply any given hour by 60, and divide by 11.)

| A | 5·454 | 21·816 | 32·724 | 43·636 | 54·545 | 60 |
|---|---|---|---|---|---|---|
| B | I | IV | VI | VIII | X | 11 |

So that after VIII o'clock the hands will be together at VIII h. 43 m. 37·9 s. (Observe that 5·454 m. or $\frac{1}{11}$ h. = 5 m. 27·24 s.)

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## PER-CENTAGES.

The Slide Rule is eminently useful in this sort of computation, as it *immediately* gives the answer to three figures, which is as close as required in most per-centage comparisons. [See Examples 32, 36, 54, 71, 72, 108, 109, and 114 to 125.]

**Ex. 156.**—How much per cent. is 6*d*. in the £? (Here 240 : 6 :: 100 : *x*.)

| A | 25 | 6 |
|---|---|---|
| B | 100 | 240 |

**Ex. 157.**—How much per cent. is $\frac{5}{6}$? (Here 6 : 5 :: 100 : *x*.)

| A | 5 | 83·$\dot{3}$ |
|---|---|---|
| B | 6 | 100 |

**Ex. 158.**—If Stock that I bought at 117, falls to 110, what is the loss per cent.? (117 : 100 :: 110 : $x$.)

| | | |
|---|---|---|
| A | 110 | 117 |
| B | 94 | 180 |

Giving the *Answer* = 6 per cent.

**Ex. 159.**—In "Timber measuring," the *customary* content is 21½ per cent. too little. How much per cent. must be *added* to a given customary content, to show the *true* content?

| | | |
|---|---|---|
| A | 100 | 127·3 |
| B | 78·5 | 100 |

*i.e.* 27·3 per cent. will have to be added.

**Ex. 160.**—Of 93498 Births registered, 47872 were males. Find the per-centage. (It will be near enough if we take 48 out of 93·5.)

| | | |
|---|---|---|
| A | 48 | 51·2 |
| B | 93·5 | 100 |

The exact answer is 51·201 per cent., showing that in these Statistical computations, the Slide Rule gives a result quite near enough, and *immediately.*

**Ex. 161.**—The population of Ireland in 1841 was 8175124, and in 1851 it was 6552385. Find the decrease per cent.

| | | |
|---|---|---|
| A | 80·1 | 655 |
| B | 100 | 817 |

*i.e.* a decrease of 19·9 per cent. The *exact* answer being 19·8497.

**Ex. 162.**—In 1856 the number of deaths in England and Wales was 391369; the decrease below 1855 being 8·18103 per cent. Find the number of deaths in 1855.

| | | |
|---|---|---|
| A | 91·8 | 391000 |
| B | 100 | 426000 |

The *exact* answer is 426240.

**Ex. 163.**—In four villages, the populations whereof were 1045, 756, 898, and 865, the deaths during the cholera were 205, 160, 220, and 159 respectively. Required the per-centage of deaths in each. (Here 4 settings of the Slide are required, but the *time taken to find the answers* should be compared with that by pen or pencil.)

| | | | |
|---|---|---|---|
| 1. | A | 19·6 per cent. | 205 |
| | B | 100 | 1045 |
| 2. | A | 21·2 per cent. | 160 |
| | B | 100 | 756 |
| 3. | A | 24·5 per cent. | 220 |
| | B | 100 | 898 |
| 4. | A | 18·4 per cent. | 159 |
| | B | 100 | 865 |

**Ex. 164.**—Shares that were originally 117, fell to 112, 110, and 108 successively. Find the decline per cent. *from the original price*, on each occasion.

| | | | | |
|---|---|---|---|---|
| A | 108 | 110 | 112 | 117 |
| B | 92·4 | 94 | 95·7 | 100 |

Giving the *answers* 4·3,—6,—and 7·6 per cent.

**Ex. 165.**—Shares that were at 66, rose successively to 68, 71, 73, and 75. Required the rise per cent. above *the original price*, on each occasion.

| | | | | | |
|---|---|---|---|---|---|
| A | 66 | 68 | 71 | 73 | 75 |
| B | 100 | 103 | 107 | 111 | 114 |

Giving the *answers*, 3,—7,—11,—and 14 per cent.

**Ex. 166.**—In five successive years there were respectively 55, 62, 66, 57, and 52 very rainy days. Required the per-centage each year, by way of comparison.

| | | | | | | |
|---|---|---|---|---|---|---|
| A | 52 | 55 | 57 | 62 | 66 | 365 |
| B | 14·2 | 15·1 | 15·6 | 16·9 | 18·0 | 100 |

**Ex. 167.**—In three samples of Bone manure, weighing respectively 195, 205, and 210 grains, the same quantity of Sulphate of lime, viz. 37 grains, was found in each sample. What was the *per-centage* of that ingredient in each sample? (Invert the Slide, as in Examples 75, 76, &c.)

| | | | | |
|---|---|---|---|---|
| A | 17·6 | 18·0 | 19 | 37 |
| Ɔ | 210 | 205 | 195 | 100 |

**Ex. 168.**—The "automic weights" of Chlorine and Silver, being 35·5, and 108·1, respectively; required the per-centage of Chlorine, and of Silver also, in any quantity of Chloride of Silver.

| | | | |
|---|---|---|---|
| A | 35·5 | 108·1 | 143·6 (= 108·1 + 35·5) |
| B | 24·72 C. | 75·28 S. | 100 |

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## STOCKS—INSURANCE—COMMISSION, etc.

**Ex. 169.**—If £3400 be invested in the $3\frac{1}{2}$ per cents. at 96, what will be the annual dividend? (Here 96 : 3·5 :: 3400 : *x*.)

| | | |
|---|---|---|
| A | 124£ | 3·5 |
| B | 3400 | 96 |

**Ex. 170.**—If a 4 per cent. Loan is at 14 discount, how much must be invested to produce an Income of £122? (Here 4 : 86 :: 122 : *x*.)

| | | |
|---|---|---|
| A | 86 | 2623£ |
| B | 4 | 122 |

**Ex. 171.**—What is the price of £100 Stock, when a person can purchase £2766 13*s*. 4*d*., for £2490? (Here $2766{\cdot}\dot{6}$ : 100 :: 2490 : *x*.)

| | | |
|---|---|---|
| A | 90£ | 100 |
| B | 2490 | $2766{\cdot}\dot{6}$ |

**Ex. 172.**—What interest do I get on Shares purchased at 62, the dividend on the original £40 shares being 8 per cent.? (Here 62 : 40 :: 8 : *x*.)

| | | |
|---|---|---|
| A | 5·16 per cent. | 40 |
| B | 8 | 62 |

**Ex. 173.**—How many "years purchase" are equivalent respectively, to $3\frac{1}{2}$, $4\frac{1}{2}$, 5, and 6 per cent.? (Here 100 is divided by a "series of divisors," as in Ex. 33.)

| | | | | | |
|---|---|---|---|---|---|
| A | 16·7 | 20 | 22·25 | 28·6 | 100 |
| Ↄ | 6 | 5 | 4·5 | 3·5 | 1 |

**Ex. 174.**—Which is the most advantageous, investing in a 4 per cent. Loan at 14 discount, or in a 5 per cent. Loan at 8 premium? (This requires two settings of the Slide, if we want to find the interest in each case; but if we only want to find which gives the highest interest, see Ex. 217.)

| | | | |
|---|---|---|---|
| 1. | A | 4 | 4·65 per cent. |
| | B | 86 | 100 |

| | | | |
|---|---|---|---|
| 2. | A | 5 | 4·63 per cent. |
| | B | 108 | 100 |

Showing that the 4 per cent. Loan is the best.

**Ex. 175.**—What is to be paid for insuring a vessel and cargo worth £2225 at $3\frac{1}{4}$ per cent? (Here 2225 × ·0325, as in Ex. 8, and (*c*), p. 9.)

| | | |
|---|---|---|
| A | ·0325 | 72·3£ |
| B | 1 | 2225 |

**Ex. 176.**—What is the Premium on a Policy of Insurance for £3750, the rate being £2 9*s.* per cent.? (Here 375 × ·245, as in Ex. 4.)

| | | |
|---|---|---|
| A | ·245 | 91·875£ |
| B | 1 | 3750 |

**Ex. 177.**—If I insure property valued at £3840, for £67 4*s.*, what is the rate per cent? (Here 3840 : 67·2 :: 1 : *x*.)

| | | |
|---|---|---|
| A | ·0175 | 67·2 |
| B | 1 | 3840 |

Giving an answer of 1·75 per cent.

**Ex. 178.**—What is the Commission on £713, at $2\frac{3}{4}$ per cent.? (Here 713 × ·0275.)

| | | |
|---|---|---|
| A | ·0275 | 19·61£ |
| B | 1 | 713 |

**Ex. 179.**—What is the Brokerage on £7680 at $\frac{1}{8}$, or 2*s*. 6*d*. per cent.? (Here 7680 × ·00125.)

| | | |
|---|---|---|
| A | ·00125 | 9·6£ |
| B | 1 | 7680 |

---

## SIMPLE INTEREST, OR DISCOUNT.

Let $p$ = Principal, in £.

$t$ = Time, in years.

$r$ = Rate; or interest on £1, in one year.

$i$ = Total amount of *interest alone.*

$m$ = Amount; or principal *plus* interest.

$$i = p \times r \times t$$

$$i = \frac{m \times r \times t}{1 + (r \times t)}$$

$$i = m - p$$

$$p = \frac{i}{r \times t}$$

$$p = \frac{m}{1 + rt}$$

$$m = p \times (1 + rt)$$

$$m = p + i$$

$$t = \frac{i}{p \times r}$$

$$t = \frac{m - p}{p \times r}$$

$$r = \frac{i}{p \times t}$$

$$r = \frac{m - p}{p \times t}$$

**Ex. 180.**—What is the Interest, Discount, or Dividend, on £7250, for one year, at $4\frac{1}{2}$ per cent.? ($i = 7250 \times \cdot 045 \times 1$.)

| | | |
|---|---|---|
| A | 326£ | 7250 |
| B | ·045 | 1 |

The *exact* answer is £326·25.

**Ex. 181.**—What is the Interest, or Discount, for 2 years and 9 months, on £326, at 6 per cent.?

$$\left(i = 326 \times 2{\cdot}75 \times {\cdot}06\,;\ \text{whence } i = \frac{326 \times 2{\cdot}75}{16{\cdot}66}.\right)$$

| | | |
|---|---|---|
| A | 53·8£ | 326 |
| B | 2·75 | 16·66 |

**Ex. 182.**—What is the Interest or Discount on £151 for 91 *days*, at 5 per cent. per annum?

$$\left(\text{Here } i = 151 \times {\cdot}05 \times 91 = \frac{151 \times 4{\cdot}55}{365}.\right)$$ See N.B. after Ex. 183.

| | | |
|---|---|---|
| A | 1·88£ | 4·55 |
| B | 151 | 365 |

N.B. At *Five per cent.*, the interest is a penny a £ *per month*, or nearly 1£ a week (19*s*. 2*d*.) *on every* £1000.

***Ex. 183.**—If I have received £45 as Interest (simple) in 3 years at $4\frac{1}{4}$ per cent., what was the Principal lent?

$$\left(p = \frac{45}{3 \times {\cdot}0425} = \frac{45 \times {\cdot}\dot{3}}{{\cdot}0425}.\right)$$

| | | |
|---|---|---|
| A | 45 | 353£ |
| B | ·0425 | ·3333 |

N.B. When it is *often* required to know the Interest for a number of *days*, it is worth while to use the "constants" mentioned in the "Supplementary note," farther on, and the Examples that follow it. See also footnote.†

---

* Examples 181 and 183, show the use of knowing some "reciprocals." See footnote ‡, page 14.

† 73 days = ·02 years; 146 days = ·04; 219 = ·06; 292 = ·08 year.

**Ex. 184.**—What sum lent out at 4 per cent. (simple interest), will obtain interest amounting to £57 12*s.* in $4\frac{1}{2}$ years? $\left(p = \frac{57 \cdot 6}{\cdot 04 \times 4 \cdot 5} = \frac{57 \cdot 6}{\cdot 18}.\right)$

| | | |
|---|---|---|
| A | 1 | 320£ |
| B | ·18 | 57·6 |

**Ex. 185.**—Find the present value (or Principal) of £2000 due 3 years hence, at 5 per cent. simple interest.

$\left(p = \frac{2000}{1 + (\cdot 05 \times 3)} = \frac{2000}{1 \cdot 15}.\right)$ See p. 63 and Ex. 197.

| | | |
|---|---|---|
| A | 1 | 1739£ |
| B | 1·15 | 2000 |

The *exact* answer is £1739 2*s.* 7*d.* This shows that this sum as Principal, will, with its 3 years' interest, £260·87, amount to £2000.

**Ex. 186.**—The present value of a sum of money due 6 months hence is £540. Find what that sum is, at 4 per cent. simple interest. $(m = 540 \times (1 + \cdot 04 \times \frac{1}{2}) = 540 \times 1 \cdot 02.)$

| | | |
|---|---|---|
| A | 540 | 551£ |
| B | 1 | 1·02 |

The exact answer is 550·8£.

**Ex. 187.**—In how many years' time will £425 amount to £539 15*s.*—*including* the accumulated interest (simple) at $4\frac{1}{2}$ per cent.? $\left(t = \frac{539 \cdot 75 - 425}{425 \times \cdot 045} = \frac{114 \cdot 75}{19 \cdot 125}.\right)$

| | | |
|---|---|---|
| A | 1 | 6 years |
| B | 19·125 | 114·75 |

Here £539·75 − £425 = 6 years' interest, or 114·75£.

**Ex. 188.**—In how many *days'* time will £600 be payable, on a present Principal, or value, of £580, carrying interest at 5 per cent.? $\left(t = \frac{600 - 580}{580 \times \cdot 05} \times 365.\ \text{Reduce to } \frac{20 \times 365}{29}.\right)$

$$\frac{A \quad 20 \qquad\qquad 252 \text{ days}}{B \quad 24 \qquad\qquad 365}$$

**Ex. 189.**—If a person receives £840 cash, for £1204 16*s.* due 7½ years hence, carrying simple interest, what is the rate allowed?

$$\left(r = \frac{1240{\cdot}8 - 840}{840 \times 7\frac{1}{2}} = \frac{364{\cdot}8}{6300}.\right)$$

$$\frac{A \quad {\cdot}0579 \text{ or } 5{\cdot}79 \text{ per cent.} \qquad 1}{B \quad 364{\cdot}8 \qquad\qquad 6300}$$

**Ex. 190.**—If a person returns £64 for the loan of £600 for one month, what rate of Annual Interest is charged?

$$\left(r = \frac{604 - 60}{600 \times \frac{1}{12}} = \frac{4}{50}.\right)$$

$$\frac{A \quad {\cdot}08, \text{ or } 8 \text{ per cent.} \qquad 1}{B \quad 4 \qquad\qquad 50}$$

**Ex. 191.**—A person puts out his whole capital, at 4½ per cent. interest. After paying Income Tax at 4*d.* in the £, he finds his net income for the year to be £1416. What is his capital?

$$\text{1st.}\quad \frac{A \quad 240 \qquad\qquad 1440 = \text{Gross Income}}{B \quad 236 \qquad\qquad 1416}$$

$$\text{2nd.}\quad \frac{A \quad 1 \qquad\qquad 32000 = \text{Capital}}{B \quad {\cdot}045 \qquad\qquad 1440}$$

## Supplementary Note.

IF the Time ($t$) is given in *days*, the Divisors in the margin will be useful to those who are in the habit of computing interest ($i$) or discount for *days:* for

$$i = \frac{p \times \text{No. of days}}{\text{Divisor}}.$$

| per cent. | Divisor |
|---|---|
| 3 | 12169 |
| 3½ | 10428 |
| 4 | 9125 |
| 4½ | 8111 |
| 5 | 7300 |
| 5½ | 6636 |
| 6 | 6083 |
| 6½ | 5615 |
| 7 | 5214 |
| 8 | 4562 |

(found by dividing 365 by the rate of interest.)

The set of the rule is:—

| | | |
|---|---|---|
| A | Principal in £ | Interest in £ |
| B | Divisor | No. of days |

**Ex. 192.**—What is the amount of interest on £151, for 91 days, at 5 per cent. per annum? (Ex. 182.)

| | | |
|---|---|---|
| A | 1·88£ | 151 |
| B | 91 | 7300 |

**Ex. 193.**—What is the amount of interest (or Discount) on £3842, for 80 days, at 6 per cent.?

| | | |
|---|---|---|
| A | 50·5£ | 3842 |
| B | 80 | 6083 (constant) |

The *exact* answer is 50·525£.

**Ex. 194.**—In how many days will the interest on £72 at 7 per cent., amount to 8 shillings?

| | | |
|---|---|---|
| A | ·4£ | 72 |
| B | 29 days | 5214 (constant) |

## PRESENT VALUE,* AND TRUE DISCOUNT.

If I have a Bill of Exchange for £750, due 6 months hence, and a person offers to discount it at 7 per cent. interest, the *usual* thing is, to calculate the interest for the 6 months, namely £26 5*s.*, and deducting this, to return me £723 15*s.* But strictly speaking, the sum to be returned ought to be the "*present value*," or such a sum as would with interest at 7 per cent. for 6 months amount to £750.† This "present value" calculated as in Ex. 185 would be

$$\frac{750}{1 + (\cdot 07 \times \cdot 5)} = \frac{750}{1\cdot 035} = £724\cdot 64.$$

* For "Present value" in *Compound* Interest, Annuities, &c., see Appendix B.

† In proof of this, it may be shown, that the 6 months' interest on £724·64 would be £25·36; which added to £724·64, would make up £750.

From the above, it will be seen that the "True" present value is more than the "Ordinary," and the "True" discount less than the "Ordinary." Still the "Ordinary" method is always adopted in practice, as being more easy to calculate, and as covering risk, &c.

In the above instance, the "True" present value is £724·64; the "Ordinary," £723·75. The "True" discount is £25·36; the Ordinary, £26·25.

**Ex. 195.**—What is the "True" present value of a Bill for £250, due 72 days hence; interest being taken at 4 per cent.?

$$\left(p = \frac{250}{1 + ({\cdot}04 \times \frac{72}{365})} = \frac{250}{1 + \frac{2{\cdot}88}{365}} = \frac{250 \times 365}{367{\cdot}88}.\right)$$

| | | |
|---|---|---|
| A | 248£ | 365 |
| B | 250 | 367·88 |

The *exact* answer is £248·043, which at 4 per cent. per annum for 72 days, would give £1·957 interest, making up the amount £250.

---

## True Discount

may be calculated in *two* ways:

$$\text{I.} = m - \frac{m}{1 + (r + t)} \text{ or II.} = \frac{m \times r \times t}{1 + (r \times t)}.$$

Method I. is first to find the "present value," as in Ex. 185, and deduct it from $m$, the amount of the Bill: the remainder being the True discount.

Method II. finds the True discount at once.

Method I. is generally adopted, as rather less trouble: but Method II. is better adapted to the Slide Rule. (Ex. 185.)

**Ex. 196.**—Find the "True" discount for £500, due 47 days hence, at 7 per cent. per annum, by *Method* I. (See Examples 193, and 203.)

$$m = \frac{500}{1 + ({\cdot}07 \times \frac{47}{365})} = \frac{500}{1 + \frac{3{\cdot}29}{365}} = \frac{500}{\frac{368{\cdot}29}{365}} = \frac{500 \times 365}{368{\cdot}29}$$

| | | |
|---|---|---|
| A | 495£ = Present value | 500 |
| B | 365 | 368·29 |

Then £500 − 495 = £5 the "True" discount. The *exact* answer is $500 - 495{\cdot}5\dot{3} = 4{\cdot}4\dot{6}$.

**Ex. 197.**—Try the above *by Method II.*

$$\text{Here } x = \frac{500 \times (\cdot 07 \times \frac{47}{365})}{1 + (\cdot 07 \times \frac{47}{365})} = \frac{500 \times \frac{3\cdot 29}{365}}{1 + \frac{3\cdot 29}{365}} =$$

$$\frac{500 \times 3\cdot 29}{365} \times \frac{365}{368\cdot 29} = \frac{500 \times 3\cdot 29}{368\cdot 29}$$

| | | |
|---|---|---|
| A | 4·47 = True discount | 500 |
| B | 3·29 | 368·29 |

The *exact* answer is $4{\cdot}4\dot{6}$£.

The following applies to *one year*, only :—

(I.) *Ordinary Discount* = (True Discount × $r$) + True Discount. Thus if the True Discount for *one year*, is known to be £35, and the rate of interest 5 per cent., the "Ordinary" Discount is (35 × ·05) + 35 = 1·75 + 35 = £36·75.

(II.) *True Discount* = Ordinary Discount *minus* $\dfrac{\text{Ord. Disc.} \times r}{1 + r}$. Thus if the Ordinary Discount for *one year*, is £58 12*s*., the rate of interest being 7 per cent., the "True" Discount is $58\cdot 6 - \dfrac{58\cdot 6 \times \cdot 07}{1\cdot 07} = 58\cdot 6 - 54\cdot 7 = £3\cdot 9$.

If a Bill for £100 due *one year* hence, is to be discounted at 5 per cent., the "Ordinary" discount is £5; and the "True" discount is £4·7619. The "present value" is $\dfrac{100}{1\cdot 05} = £95\cdot 2381$.

## EQUAL FRACTIONS.

It was observed in page 14, N.B. 2, that set the Slide as we will, the numbers on A with those respectively under them on B,* represent a series of *equal Fractions.*

Thus if we set

| | | | | |
|---|---|---|---|---|
| A | 2 | 4 | 6 | 10 &c. |
| B | 3 | 6 | 9 | 15 &c. |

we see that $\frac{2}{3} = \frac{4}{6} = \frac{6}{9} = \frac{10}{15}$, &c.

**Ex. 198.**—Find a fraction equal to $\frac{825}{900}$ but with fewer figures. We set 825 on A, over 900 on B, and look along till we see other coincidences; such as 6 over 7, 37 over 43, 55 over 64.

| | | | | |
|---|---|---|---|---|
| A | 6 | 37 | 55 | 825 |
| B | 7 | 43 | 64 | 900 |

Either of these is close enough for ordinary purposes; but if we want more exactness, we must test them with pen and paper. We find $\frac{825}{900}$ = ·85938; also $\frac{6}{7}$ = ·85714, and $\frac{37}{43}$ = ·86046, and $\frac{55}{64}$ = ·85938. This last happens to be *exact.*

**Ex. 199.**—Find a Fraction equal to $\frac{154}{715}$, but of a lower denomination, and one of which both numerator and denominator fall on *cut divisions* of the lines A and B.

| | | |
|---|---|---|
| A | 14 | 154 |
| B | 65 | 715 |

**Ex. 200.**—Under the same conditions as in Ex. 199, find a Fraction equivalent to $\frac{136}{289}$.

---

* We may consider the figures on B as Numerators, and those over them on A as Denominators, but the way shown above commends itself most to the eye.

| | | |
|---|---|---|
| A | 8 | 136 |
| B | 17 | 289 |

**Ex. 201.**—Under the same conditions as in Ex. 199, find a Fraction equivalent to $\frac{1715}{2695}$.

| | | |
|---|---|---|
| A | 7 | 1715 |
| B | 11 | 2695 |

**Ex. 202.**—Under the same conditions as in Ex. 199, find a Fraction equivalent to $\frac{3575}{4718}$.

| | | |
|---|---|---|
| A | 25 | 3575 |
| B | 33 | 4718 |

**Ex. 203.**—Under the same conditions as in Ex. 199, find a Fraction equivalent to $\frac{10759}{20405}$.

| | | |
|---|---|---|
| A | 29 | 10759 |
| B | 55 | 20405 |

The above use of the Slide Rule is constantly required in obtaining *Formulæ* * adapted to the Slide Rule.

**Ex. 204.**—Since 1 French mètre = 3·280899 feet, find two *easily read* numbers on A and B which will represent these Proportions :

| | | | | |
|---|---|---|---|---|
| A | 3·21 | 16·4 | 69 | 210 |
| B | 1 | 5 | 21 | 64 |

All are very near ; but if for special purposes we want to try which is nearest, we find $\frac{69}{21}$ = 3·28571, and $\frac{16·4}{5}$ = 3·2800 and $\frac{210}{64}$ = 3·28122.

We accordingly select the third or $\frac{210}{64}$ which shows that

* See under the head of FORMULÆ farther on, and APPENDIX I.

64 Mètres = 210 feet, or 70 yards. The Rule set as above, shows on line A, the feet corresponding to the mètres respectively under them on the line B.

**Ex. 205.**—Since there are 43560 square feet in an Acre, find two *easily read* numbers on A and B, which will represent these Proportions.

| | | |
|---|---|---|
| A | 27000 | 43560 Sq. ft. |
| B | ·62 | 1 Acre |

We find ·62 to be 27072 Square feet, so the Proportion is near enough for all purposes. All the numbers on A will show the equivalent in Square feet, to their respective Acres on the line B.

**Ex. 206.**—In one ounce Troy of standard silver (240 dwt.), there are 11 oz. 2 dwt. (222 dwt.) *pure* silver. Find two *easily read* numbers* on A and B which will show what quantities of pure silver are in any quantities of standard silver.

| | | |
|---|---|---|
| A | 37 | 222 pure |
| B | 40 | 240 stand. |

This $\frac{37}{40}$ is exactly equal to $\frac{222}{240}$.

**Ex. 207.**—The proportion of a Pound Troy to a Pound Avoirdupois is 5760 grains to 7000 grains. Find two *easily* read numbers* bearing the same relation.

| | | |
|---|---|---|
| A | 65 | 5760 |
| B | 79 | 7000 |

Here $\frac{5760}{7000}$ = ·82286 and $\frac{65}{79}$ is ·82278 : so that the numbers found (in a few seconds) are very close.

* It may be asked with reference to Examples 204—208, why not let $\frac{3·21}{1}$, $\frac{43560}{1}$, $\frac{222}{240}$, $\frac{4760}{7000}$ stand, without looking for other numbers? The reason is that 3·21, 4356, 222, 576, are not easily found *exactly* on the Slide Rule.

**Ex. 208.**—A Millegramme is ·0154 grains. Find two easily read numbers on A and B which will represent the equivalents of Millegrammes and Grains.

$$\frac{\text{A}\quad 65 \qquad 10000 \qquad 1 \text{ grain}}{\text{B}\quad 1 \qquad 154 \qquad \cdot0154 \text{ Millegr.}}$$

Here all the numbers on A, will represent the values of the numbers of Millegrammes under them on B. (A Millegramme $= \frac{1}{65}$ grain.)

**Ex. 209.**—Suppose the error in a result is 36 in 22500, how is this expressed as a Fraction having 1 for its Numerator?

$$\frac{\text{A}\quad 1 \qquad\qquad 36}{\text{B}\quad 625 \qquad\qquad 22500}\ \text{Ans.} = \frac{1}{625}.$$

---

## To find the greatest, and the least of any given number of Fractions.

Rule.—Set each fraction (as explained in page 66) one after the other, on the Rule, with the numerators on A. Whichever fraction requires the Slide to be drawn out most to the right, is the *greatest;* and whichever requires the Slide to be pushed back most to the left, is the *least.*

**Ex. 210.**—Which is the greater $\frac{3}{16}$, or $\frac{5}{27}$?

We find that $\frac{\text{A}\quad 3}{\text{B}\quad 16}$ requires the Slide to be drawn out more to the right, than $\frac{\text{A}\quad 5}{\text{B}\quad 27}$; hence $\frac{3}{16}$ is the greater fraction.

**Ex. 211.**—Which is the greatest, and which is the least fraction of the four following: $\frac{11}{18}, \frac{18}{29}, \frac{8}{13}, \frac{35}{58}$. We begin with $\frac{\text{A}\quad 11}{\text{B}\quad 18}$, and after going through them all, we find $\frac{18}{29}$ requires the

Slide to be drawn out most to the right, and $\frac{35}{58}$ requires the Slide to be pushed most to the left. Hence $\frac{18}{29}$ is the greatest, and $\frac{35}{58}$ the least.

**Ex. 212.**—Four persons had been shooting at a mark; a limited time being allowed. At its close, the score stood as follows:

A had 70 shots, and hit 64 times.
B had 75 shots, and hit 68 times.
C had 68 shots, and hit 63 times.
D had 73 shots, and hit 67 times.

Which was the most successful?

Beginning with $\frac{A}{B}\frac{64}{70}$, and going on to $\frac{68}{75}, \frac{63}{68}, \frac{67}{73}$, we find in half a minute, that $\frac{63}{68}$ is the greatest fraction, and therefore C is the winner. (The respective percentages are 91·43, 90·67, 92·65, 91·78.)

**Ex. 213.**—Which is the cheapest work? digging out 34 cubic feet for 4*d.*; or 36 cubic feet for 6½*d.* Here (I.) $\frac{34}{4}$ c. ft., and $\frac{56}{6\cdot5}$ c. ft. *per penny*; or (II.) $\frac{4}{34}d.$, and $\frac{6\cdot5}{56}d.$ per *cubic foot.* Calculating per penny, the *greatest* fraction would be cheapest; but calculating per cubic foot the *least* fraction would be the cheapest. In the former case, the Slide Rule instantly shows that $\frac{56}{6\cdot5}$ is the greatest fraction, or cheapest work, and it also shows that in calculating per cubic foot, $\frac{6\cdot5}{56}$ is less than $\frac{4}{34}$, or 36 cubic feet for 6½*d.* the cheapest.

N.B. In comparing dearer and cheaper in this way, remember that if the *money* is made the Denominator, it is so much per penny, or per shilling, and the *greatest* fraction shows the cheapest, or the most for the money.

**Ex. 214.**—Which is the cheapest, a hat that costs 15*s.*, and lasts 9 months, or a hat that costs 11*s.* 6*d.* and lasts 7 months? (See N.B. to preceding.) It is to find which is the greatest fraction $\frac{9}{15}$

or $\frac{7}{11 \cdot 5}$. The Slide Rule shows at once that $\frac{7}{11 \cdot 5}$ is the greatest fraction, or more wear for the money.

**Ex. 215.**—Which is the best and which is the worst investment of the following five: 5 per cent. at 8 premium; 4 per cent. at 14 discount; 7 per cent. at $51\frac{1}{2}$ premium; 6 per cent. at 28 premium; or $3\frac{1}{2}$ per cent. at 25 discount?

Here the interest or dividend *per cent.* is represented respectively by $\frac{500}{108}$, $\frac{400}{86}$, $\frac{700}{151 \cdot 5}$, $\frac{600}{128}$, $\frac{350}{75}$, and if we try these fractions as in the preceding Examples, we find $\frac{600}{128}$ to be the *greatest;* that is, the Quotient or interest per cent. is greatest; and $\frac{700}{151 \cdot 5}$ is the least: so that the fourth is the best, and the fifth the worst investment. (There is no need in the question to find the actual rate of dividend: but see Ex. 174.)

N.B. To appreciate the advantage of the Slide Rule in such questions as this and the preceding, they should be solved in the usual way with pen and paper, and the time noted, as against the time occupied in working out the same with the Slide Rule.

**Ex. 216.**—There are three lines of Railway, the second class fares of which are as follows: A 114 miles for 18*s.*; B 179 miles for £1 8*s.*; and 194 miles for £1 11*s.* Which is the cheapest and which the dearest? (See N.B. to Ex. 213.) This is to find which is the greatest fraction $\frac{114}{18}$, $\frac{179}{28}$, $\frac{194}{31}$; and the Slide Rule at once shows $\frac{179}{28}$ or B, is the most distance per shilling* or greatest fraction, and $\frac{194}{31}$ or C the least fraction, or dearest.

N.B. We might compare $\frac{18}{114}$, $\frac{28}{179}$, $\frac{31}{194}$, or shillings (decimals of a shilling)† per mile; in which case the *least* fraction would be the cheapest; giving B as before.

---

* A = ·157894*s.* B = ·156424*s.* C = ·159794*s.* per mile.
† A = 6·333. B = 6·393. C = 6·258 miles per penny.

[The difference of rates, is exceedingly slight, yet the Slide Rule shows them without difficulty. The rates are respectively 7·58, 7·51, 7·67 farthings per mile, or less than $\frac{1}{10}$ of a farthing a mile between the cheapest and the dearest.]

**Ex. 217.**—Of four Ellipses, whose axes are respectively 128 and 160; 18 and 21; 40 and 60; and 61·2 and 84; which approximates nearest to a Circle? *i.e.* which is the greatest fraction?

The Slide Rule at once shows $\frac{18}{21}$ to be the greatest fraction, (and $\frac{40}{60}$ the least fraction).

---

## USEFUL "FORMULÆ."

(See four pages on, for Examples, and also APPENDIX M.)

| | | | | | |
|---|---|---|---|---|---|
| (I.) | A | 12 | | | Pence per lb. |
| | B | 8 | | | Shillings per Stone of 8lb. |
| (II.) | A | 2·143 | | | Pence per lb. |
| | B | Cwt. | | | £ cost |
| *(III.) | A | 6 | | | Pence per lb. |
| | B | 56 | | | £ per Ton, or Stone per Cwt. |
| (IV.) | A | 3 | | | Pence per Oz. Avoir. |
| | B | 4 | | | Shillings per lb. Avoir. |
| (V.) | A | 1 | | | Shillings per Bushel |
| | B | 8 | | | Shillings per Quarter |
| (VI.) | A | 1 | 1·215 | 79 | lbs. Troy |
| | B | ·8229 | 1 | 65 | lbs. Avoir. |

---

* If the price is given in "Shillings per Cwt." and we want "Farthings per lb."; multiply the Shillings by $\frac{3}{7}$. N.B. A Penny a lb. is $9\frac{1}{3}$*s.* per Cwt., or $9\frac{1}{3}$£ per Ton.

| | | | | | |
|---|---|---|---|---|---|
| (VII.) | A | 1 | 1·097 | 79 | Oz. Avoir. |
| | B | ·9115 | 1 | 72 | Oz. Troy |
| (VIII.) | A | 33 | 277·274 | | "Imperial" Corn Measure |
| | B | 22 | 268·8 | | "Old" or Winchester Measure |
| (IX.) | A | 1 | 1·20 | 6 | Old "Wine" Measure |
| | B | ·8831 | 1 | 5 | "Imperial" Measure |
| (X.) | A | 1 | 1·284 | 9 | Cubic Feet |
| | B | ·7788 | 1 | 7 | Bushels |
| (XI.) | A | 1·1604 | 1 | 13 | Cubic Feet |
| | B | 1 | 6·232 | 81 | Gallons |
| (XII.) | A | 1 | 27000 | 43560 | Square Feet |
| | B | ·00002296 | ·62 | 1 | Acres |
| (XIII.) | A | Yards long | | | Square Feet |
| | B | 4 | | | Inches wide |
| (XIV.) | A | 88 | | | Feet per Second |
| | B | 1 | | | Miles per Minute |
| (XV.) | A | 22 | | | Feet per Second |
| | B | 15 | | | Miles per Hour |
| (XVI.) | A | 60 | | | Seconds per Mile |
| | C (inv.) | 60 | | | Miles per Hour |
| (XVII.) | A | 24 | | | Pence per Square Yard |
| | B | 484 | | | £ per Acre |
| *(XVIII.) | A | 1 | 1·521 | 73 | £ per Year of 365 Days |
| | B | ·657 | 1 | 48 | Pence per Day |

* With a year of 365 days, multiply £1 10*s.* 5*d.* by the pence per day.

| | | | | | |
|---|---|---|---|---|---|
| *(XIX.) | A | 1 | 1·304 | 60 | £ per Year of 315 Days |
| | B | ·767 | 1 | 46 | Pence per Day |

| | | | |
|---|---|---|---|
| (XX.) | A | 365 | No. of Days × 20 |
| | B | £ per Annum | Wages in Shillings |

| | | | |
|---|---|---|---|
| (XXI.) | A | Yards long | Square Feet |
| | B | 4 | Inches wide |

---

## French Weights, Measures, etc.

(See also APPENDIX M.)

| | | | | | |
|---|---|---|---|---|---|
| (XXII.) | A | ·9144 | 1 | 64 | Mètres |
| | B | 1 | 1·09363 | 70 | Yards |

| | | | | | | |
|---|---|---|---|---|---|---|
| (XXIII.) | A | A | ·3048 | 1 | 64 | Mètres |
| | B | B | 1 | 3·281 | 210 | Feet |

| | | | | | |
|---|---|---|---|---|---|
| (XXIV.) | A | 1 | 25·4 | 305 | Millimètres (30·5 Centim.) |
| | B | ·039371 | 1 | 12 | Inches |

| | | | | | |
|---|---|---|---|---|---|
| †(XXV.) | A | 1 | 1·6093 | 37 | Kilomètres |
| | B | ·62138 | 1 | 23 | Miles |

| | | | | | |
|---|---|---|---|---|---|
| (XXVI.) | A | ·0648 | 1 | 46 | Grammes |
| | B | 1 | 15·43235 | 710 | Grains |

| | | | | | |
|---|---|---|---|---|---|
| (XXVII.) | A | 1 | 28·349 | 85 | Grammes |
| | B | ·03528 | 1 | 3 | Oz. Avoir. |

| | | | | | |
|---|---|---|---|---|---|
| (XXVIII.) | A | 1 | 31·10 | 280 | Grammes |
| | B | ·03215 | 1 | 9 | Oz. Troy |

* With a year of 313 days, multiply £1 6*s.* 1*d.* by the pence per day.
† 1609·3 Mètres = 1 Mile.

| | | | | | |
|---|---|---|---|---|---|
| * (XXIX.) | A | ·4536 | 1 | 34 | Kilogrammes |
| | B | 1 | 2·2046 | 75 | lbs. Avoir. |
| (XXX.) | A | ·8361 | 1 | 30 | Square Mètres |
| | B | 1 | 1·196 | 36 | Square Yards |
| † (XXXI.) | A | ·028315 | 1 | 5·1 | Cubic Mètres |
| | B | 1 | 35·317 | 180 | Cubic Feet |
| ‡ (XXXII.) | A | ·40467 | 1 | 17 | Hectares |
| | B | 1 | 2·4711 | 42 | Acres |
| (XXXIII.) | A | 1 | 2·589 | 258·9 | Square Kilomètres |
| | B | ·3861 | 1 | 100 | Square Miles |
| § (XXXIV.) | A | ·568 | 1 | 5 | Litres |
| | B | 1 | 1·76077 | 8·8 | Pints |
| § (XXXV.) | A | 1 | 4·5434 | 100 | Litres |
| | B | ·2201 | 1 | 22 | Gallons |
| (XXXVI.) | A | 1 | 2·9078 | 32 | Hectolitres |
| | B | ·3439 | 1 | 11 | Quarters of 8 Bushels |

---

*The following* 12 ***Formulæ*** *at* 25 ***Francs*** *to* **£**1 *or* ·8*s.* = 1 *franc.*

| | | | | | |
|---|---|---|---|---|---|
| (XXXVII.) | A | 25 | 100 | | 1 Million Francs |
| | B | 1 | 4 | | 40000£ |
| (XXXVIII.) | A | 1 | 10416 | 25 | Francs |
| | B | 9·60 | 1 | 240 | Pence |

---

* 1 Cwt. = 50·8 Kilogrammes.
† 1 Cubic Mètre = 1·308 Cubic Yards.
‡ 1 Are = 119·6033 Square Yards or ·0247 Acres.
§ 1 Litre = 61·628 Cubic Inches.

| Formula | Line | | |
|---|---|---|---|
| *(XXXIX.) | A | Price in Francs | Shilling per Oz. Avoir. |
| | B | No. of Grammes | 22·68 |
| †(XL.) | A | Price in Francs [Sh. per cwt.] | Pence per lb. Avoir. |
| | B | No. of Kilogr. [40·642] | 4·355 |
| (XLI.) | A | Price in Francs | Shillings per Cwt. |
| | B | No. of Kilogr. | 40·642 |
| ‡(XLII.) | A | Price in Francs | Shillings per Quarter |
| | B | No. of Hectolitres | 2·326 |
| §(XLIII.) | A | Price in Francs | Shillings per Yard |
| | B | No. of Mètres | ·73152 |
| (XLIV.) | A | Francs per Gramme | Price in Shillings |
| | B | ·441 | No. of Oz. Avoir. |
| (XLV.) | A | Francs per Kilogramme | Price in Shillings |
| | B | ·23 | No. of lbs. Avoir. |
| (XLVI.) | A | 51 | Francs per 157 Kilogr. |
| | B | 33 | Shillings per sack of 280 lbs. |
| (XLVII.) | A | 31·5 | Francs per 100 Kilogr. |
| | B | 32 | Shillings per sack of 280 lbs. |
| (XLVIII.) | A | 17 | Francs per Hectare |
| | B | 8·1 | Shillings per Acre |

* Formula XXXIX. shows that 1 Franc per Gramme = 22·68*s.* per Oz. Avoir.

† Formula XL. shows that 1 Franc per Kilogramme = 4·355 pence per lb. Avoir.; or 40·642 Shillings per Cwt.

‡ Formula XLII. shows that 1 Franc per Hectolitre = 2·3263*s.* per Quarter.

§ Formula XLIII. shows that 1 Franc per Mètre = ·7315*s.* per Yard (at 25 francs to 1£). At 25·15 francs to 1£., it would be ·7272*s.*

| | | | | |
|---|---|---|---|---|
| (XLIX.) A | 10 | [25·15] | | Francs to 1£ |
| Ɔ (inv.) | 24 | [9·543] | | Pence to Franc |

| | | | | |
|---|---|---|---|---|
| *(L.) A | 25 | [25·15] | | Francs to 1£ |
| Ɔ (inv.) | ·8 | [·7952] | | Shillings to 1 Franc |

| | | | | |
|---|---|---|---|---|
| (LI.) A | 1 | 1·508 | 80 | Russian Versts |
| B | ·663 | 1 | 53 | English Miles |

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*Thermometers.*†

| | | |
|---|---|---|
| (LII.) A | 4 | Reaumur |
| B | 9 | Fahrenheit |

| | | |
|---|---|---|
| (LIII.) A | 5 | Centigrade |
| B | 9 | Fahrenheit |

*Many other Formulæ*, with lines A and B, will be found in *Part II., under "Specific Gravity," "Mensuration of Superficies," "Land-measuring,"* &c.

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## Examples in the above Formulæ.

**Ex. 218.**—What is the price in pence per lb. of meat selling wholesale at 4*s.* 3*d.*—4*s.* 6*d.*—4*s.* 9*d.* per Stone?

| | | | | |
|---|---|---|---|---|
| (I.) A | 6·37*d.* | 6·75*d.* | 7·12*d.* | 12 |
| B | 4·25 | 4·5 | 4·75 | 8 |

* At 25·15 francs to 1£, 49 francs = 39 shillings.

† The degrees of Fahrenheit thus found are to be added to or subtracte from) 32°. See Example 233.

**Ex. 219.**—If 15 Cwt. of Sugar cost £24 10*s.*, what is the price per lb. in pence?

| | | | |
|---|---|---|---|
| (II.) | A | 2·143 | 3·5*d.* |
| | B | 15 | 24·5£ |

**Ex. 220.**—$7\frac{1}{2}$*d.* an ounce, is how many shillings a pound?

| | | | |
|---|---|---|---|
| (IV.) | A | 3 | 7·5 |
| | B | 4 | 10*s.* |

**Ex. 221.**—In 1855, the average price of wheat was £3 15*s.* per Quarter. In 1856 it was £3 9*s.* In 1857, £2 17*s.* Required the respective prices in shillings per Bushel. (This is the same as dividing 57, 69, 75, by 8, as in Ex. 29.)

| | | | | | |
|---|---|---|---|---|---|
| (V.) | A | 1 | 7·125*d.* | 8·625*d.* | 9·375*d.* |
| | B | 8 | 57*s.* | 69*s.* | 7 |

**Ex. 222.**—Required the Acres in 143750 square feet, and in 570578 square feet.

| | | | | |
|---|---|---|---|---|
| (XII.) | A | 27000 | 143750 | 570578 |
| | B | ·62 | 3·3 ac. | 13·1 ac. |

**Ex. 223.**—A point on the Equator has a *diurnal* motion of 1040 miles an hour; and the Earth's motion round the Sun, is at the rate of 68500 miles an hour. Reduce these rates to *feet per second.*

| | | | | |
|---|---|---|---|---|
| (XV.) | A | 22 | 1524 | 100000 |
| | B | 15 | 1040 | 68500 |

**Ex. 224.**—During a Railway journey, the speed was taken on three occasions, and found to be respectively, 92, 75, and 180 seconds to one mile. What were the rates in *miles per hour?* (Invert the Slide.)

| | | | | | |
|---|---|---|---|---|---|
| (XVI.) | A | 60 | 20 miles | 39 miles | 48 miles |
| | B | 60 | 180 | 92 | 75 |

**Ex. 225.**—What *length* in yards of matting 27 inches wide, will cover a floor 33 ft. by 22 ft.? (or 660 square feet).

| | | | |
|---|---|---|---|
| (XXI.) | A | 97·7 yards | 660 |
| | B | 4 · | 27 |

**Ex. 226.**—The summit of the spire of Strasburg Cathedral is 144 mètres above the pavement. Required the height in feet.

| | | | |
|---|---|---|---|
| (XXIII.) | A | 64 | 144 |
| | B | 210 | 472 ft. |

**Ex. 227.**—The mean height of the Barometer in Paris, is 745 millimètres; and at St. Petersburg, 750 millimètres. Required the respective value in inches.

| | | | | |
|---|---|---|---|---|
| (XXIV.) | A | 305 | 745 | 750 millim. |
| | B | 12 | 29·3 millim. | 29·5 millim. |

**Ex. 228.**—At what height in centimètres would persons standing respectively 5 ft. 3 inches, 5 ft. 9½ inches, and 6 ft. 2 inches be registered in a French passport?

| | | | | | |
|---|---|---|---|---|---|
| (XXIV.) | A | 30·5 | 166 centim. | 177 centim. | 194 centim. |
| | B | 12 | 63 | 69·5 | 74 |

**Ex. 229.**—The "Champ de Mars" in Paris, is a plain of 331 Hectares. What is this in Acres?

| | | | |
|---|---|---|---|
| (XXXII.) | A | 17 | 331 |
| | B | 42 | 818 ac. |

***Ex. 230.**—The price of wheat in Paris varied in one year

---

* Where there is a "series" of prices per *single* Hectolitre, or Sack;—or as in Formula XLIII., per *single* mètre, or yard; we have either to multiply a series by a constant multiplier, or to divide a series by a constant divisor, as in Examples 11 and 29. In Ex. 230 we have to

from 16 francs per Hectolitre, to 17, 18, 19, and 20 francs. How would these prices be expressed in *shillings per* Quarter?

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| (XLII.) | A | 2·326 | 37·2 | 39·5 | 41·9 | 44·2 | 46·5s. |
| | B | 1 | 16 | 17 | 18 | 19 | 20 |

**Ex. 231.**—If 17 mètres of silk cost 221 francs, what is the price in shillings per yard?

| | | | |
|---|---|---|---|
| (XLIII.) | A | 221 | 9·5s. |
| | B | 17 | ·73152 |

***Ex. 232.**—The temperature of the mineral spring at Aix-la-Chapelle, is 42·7 Reaumer; and that of the "Kokbrunnen" at Wiesbaden is 56° Reaumer. Required these temperatures in Fahrenheit.

| | | | | |
|---|---|---|---|---|
| (LII.) | A | 4 | 42·7 | 56 |
| | B | 9 | 96 | 116 R |

Whence 96 + 32 = 125° Fahr. at Aix; and 116 + 32° Fahr. at Wiesbaden.

**Ex. 233.**—The Seine begins to freeze at − 10° Cent. What is this in Fahrenheit? (This is the mean winter temperature in Russia, and = − 8° Reaumer.)

| | | | |
|---|---|---|---|
| (LIII.) | A | 5 | 10 C. |
| | B | 9 | 18 F. |

Here, as the Centigrade is *minus* 10, the equivalent 18° is to be *deducted* from 32°, leaving 14° Fahrenheit as the answer.

---

multiply 2·326 by 17, 18, 19, and 20. If the prices had been given in shillings per Quarter, and we wanted to reduce to francs per Hectolitre, we should have to divide a series of *shillings* by 2·326 as in Ex. 29. This remark applies to Formulæ XL. to XLV.

* 8° Reaumur = 50° of Fahrenheit; and for every extra 4° of Reaumur, add 9° to Fahrenheit: so 16° R. = 68° F. *Zero* of Fahrenheit = ∸ 14·2 Reaumur, or 32° ÷ 2¼. If the temperature is given in Centigrade, double it, and subtract $\frac{1}{10}$; and to the remainder add 32°. Thus if 20° Cent. is given, Fahrenheit = (20 × 2) − 4 + 32 = 68°. If Fahrenheit is given, and Centigrade required, subtract 32°; add $\frac{1}{9}$, and halve the sum. Thus 86° F. = $\frac{(86 - 32) + 6}{2}$ = 30° Cent.

# PART II.

## (LINES D AND E.)

MENSURATION (SUPERFICIAL, SOLID, AND LAND), COMPOUND INTEREST, ANNUITIES, AND MISCELLANEOUS.

# PART II.

## USE OF THE SQUARE LINE D.

When the Slide is carefully shut in, so that the 1 on the extreme left of C, coincides with the 1 on the extreme left of D ; and also the 1 at the right of C, with the 1 at the right of D, the numbers on C are the *Squares of* the numbers under them on D, and the numbers on D are the *Square Roots* of the numbers over them on C.

For example, 16 on C is over 4 on D, and 9 on D is under 81 on C.

## INVOLUTION AND EVOLUTION.

The above remarks will show the great use of the Slide Rule in finding the Squares, or Square Roots of any given number ; but the following note should be attended to.

The 1 at the *extreme left* of the line C will represent 1 (not 10), or 100 (not 1000), or ·01 (not ·1) ; and the 1 in the *middle* of the line C will represent 1000, or 10, or ·1, &c. as below :—

| First radius. | Second radius. | |
|---|---|---|
| 10,000 | 100,000 | 1,000,000 |
| 100 | 1000 | 10,000 |
| **1** | **10** | **100** |
| ·0 | ·1 | 1 |
| ·0001 | ·001 | ·01 |

When closed in, even at the ends, we have :

| | | | | | |
|---|---|---|---|---|---|
| C | 1 | 4 | 10 | 49 | 100 |
| D | 1 | 2 | 3·1623 | 7 | 10 |

or,

| | | | | | |
|---|---|---|---|---|---|
| C | 100 | 729 | 1000 | 4900 | 10000 |
| D | 10 | 27 | 31·623 | 70 | 100 |

So that if we want to find the Square Root of 2200, we must look under the *second* radius of C, and find 46·9 on D. If we want to find the Square Root of ·07562, we must look under the 7562 of the *first* radius of C, and find ·275 on D. For $\sqrt{\cdot 00031}$, we must look under the 31 of the *first* radius of C, and find ·0176 on D. For $\sqrt{722{,}500}$, we must look under the 7225 of the *second* radius of C, and find 850 on D.*

---

The *number of figures* in the Square Root of a given number, may easily be known beforehand, by dotting off every alternate figure, beginning with the last, and counting the number of dots. We can at the same time tell what the *first figure* of such Square Root will be, by seeing what is the nearest (less) Square Root of the figure up to the first dot, as in ordinary work with pen or pencil.

Thus $\sqrt{7\dot{2}25\dot{0}0\dot{0}}$, will have *three* figures, and as the nearest (less) Square Root of 72 is 8, the first figure will be 8.

**Ex. 234.**—How many figures will there be in the Square Root of 435600 ? and what will be the first figure ? Here $\sqrt{4\dot{3}56\dot{0}0\dot{0}}$ shows that there will be *three* figures, and also, that the first of the three will be 6, because the nearest (less) Square Root of 43 is 6. (The answer is 660.)

---

* It may assist the beginner to write with a pen, ·01 on the *first* **1** of the line C ; and 1000 on the **1** on the *middle* of the line C.

**Ex. 235.**—How many figures will there be in the Square Root of 3463321? and what will be the first figure? Here $\sqrt{\dot{3}46\dot{3}3\dot{2}1}$ shows that there will be *four* figures, and also that the first will be 1, because the nearest Square Root of 3 is 1. (Answer is 1861.)

**Ex. 236.**—Apply the above to $\sqrt{34609689}$. Here it will be seen that the Square Root will consist of *four* figures, of which the first will be 5. (Answer 5883.)

As to *Squares;* the squares of all numbers from 31·623 to 100, have *four* figures; and all from 100 to 316·22 both inclusive, will have *five* figures; and all above 316·22, as far as 1000, will have *six* figures. The square of 1000 is 1 million.

---

*Squares of Decimals.*

Multiply the given decimal by 10, or 100, or 1000, so as to make *the first digit an integer.* Then square this in the usual way, and divide by the square of the number used as a multiplier.

**Ex. 237.**—How many 0's will there be in $\cdot05568^2$? First multiply by 100, to bring it to 5·568. Then $5\cdot568^2$ (as the line C will show) is 31. Now divide this by the square of the multiplier, *i.e.* by $100^2$, or 10000, and we have $\frac{31}{10000} = \cdot0031$. (*Two 0's.*)

**Ex. 238.**—How many 0's will there be in $\cdot0176^2$? Multiply by 100, and bring it to 1·76. The $1\cdot76^2$ is found on C to 3·1. Divide by the square of the multiplier, and we have $\frac{3\cdot1}{10000} = \cdot00031$. (*Three 0's.*)

---

*Square Roots of Decimals.*

Multiply the given decimal either by $10^2$ (100), or by $100^2$ (10000), but *not* by 1000,* so as to make either the first, or the two first digits,

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* If we multiply by 1000, we should have afterwards to divide by $\sqrt{1000}$, or 31·623, an awkward number.

I

into integers. Find the Square Root of this on the line D, and divide it by 10, if $10^2$ has been used as a multiplier; or by 100, if $100^2$ has been used as a multiplier.

**Ex. 239.**—Solve $\sqrt{\cdot 00624}$. To multiply by $10^2$ will not be enough, so we multiply by $100^2$ (*i.e.* by 10000), and obtain 62·4. We find the Square Root of this, on the line D, to be 7·9. Divide by 100 (because $100^2$ was the multiplier), and we have the *answer* ·079.

**Ex. 240.**—Solve $\sqrt{\cdot 0234}$. Multiply by $10^2$ (*i.e.* by 100) and obtain 2·34. Find the Square Root of this on D, to be 1·53. Divide by 10 (because we multiplied by $10^2$), and we have the *answer* ·153.

N.B. Compare these two Examples 239 and 240, with the Rule in page 84.

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*Remarks on using the lines D and C together.*

In using the line D, with C only, observe that as the numbers on D increase or decrease by *tens*, the numbers over them on C, increase or decrease by *squares of tens* (*i.e.* by hundreds). Also, that as the numbers on C increase or decrease by *tens*, the numbers under them on D, increase or decrease by *Square Roots* of tens (or 3·1623).

Hence the Rule

$$\text{Mult.}\begin{cases}\text{C by 10,000, and D by 100.}\\ \text{C by 100, and D by 10.}\\ \text{C by 10, and D by }\sqrt{10}.\end{cases}\quad \text{Divide}\begin{cases}\text{C by 10,000, and D by 100.}\\ \text{C by 100, and D by 10.}\\ \text{C by 10, and D by }\sqrt{10}.\end{cases}$$

So, if we set the Slide with 4 of D, under 768 of C, the run of readings, *supposing the Slide C to be long enough*, would be

| C | 12 | 76·8 | 120 | $\boxed{768}$ | 1200 | 7680 | 12000 | 76800 |
|---|---|---|---|---|---|---|---|---|
| D | ·5 | 1·265 | 1·581 | $\boxed{4}$ | 5 | 12·65 | 15·81 | 40 |

Here, it will be seen, that if for 768 on C we read 76800, the 4 on D will become 40; but if for 768 we read 7680, the number under it

will be 12·65, that is, $4 \times \sqrt{10}$, or $4 \times 3 \cdot 1623$. So if for 768 on C, we read 76·8, the 4 on D will become 1·265 or $4 \div \sqrt{10}$. N.B. If the 768 of the *first* radius of C, is set over the 4 of D, the Slide C will not be long enough to see what is under 76·8, or under 7680. This difficulty is easily obviated, as shown in Examples 241 to 249.

So again in the following, where 6·67 is set over 1760.*

| C | ·1 | 1 | 6 | 6·67 | 60 | 100 | 600 |
|---|---|---|---|---|---|---|---|
| D | 215·5 | 682 | 1670 | 1760 | 5280 | 6820 | 16700 |

If the 6 on C, is read as 60, the 1670 under it becomes not 16700, but $1670 \times \sqrt{10}$, or 5280. Remember then, that if any number on D is multiplied by $\sqrt{10}$ or 3·1623, the corresponding number over it must be multiplied by 10.

---

*When the Slide seems too short.*

Since D is only a line of *single* radius, it sometimes happens that when worked with C, which is a line of double radius, the latter seems too short, as observed in the N.B. above. The difficulty may be obviated, either by multiplying or dividing the given numbers on C by 100, and those under them on D by 10,—as in Examples 241, 242,—or when the number 1 comes into the question, by attending to *what radius the given number on C should be set*,—as in Examples 242½, 243, 244, 245, 254 ; or lastly, by *shifting the Slide*, as in Examples 247, 248, 249.

**Ex. 241.**—If the Slide Rule is set $\frac{C}{D}\rule{5cm}{0.4pt}\frac{1273}{5}$, what on C is over ·5 on D ?

Having divided 5 by 10, to get ·5, we divide 1273 by 100 ; and the set of the Rule is the same as if it were $\frac{C}{D}\frac{12 \cdot 73}{\cdot 5}$.

**Ex. 242.**—If the Slide Rule is set $\frac{C}{D}\rule{5cm}{0.4pt}\frac{300}{79}$, what on D is under 3 on C ?

---

* The numbers on D represent the distances in yards from an observer, and the numbers over them on C, the depression of the horizon in inches, allowing for refraction. See under "Depression of the Horizon."

Here 3 being $\frac{1}{100}$th of 300, we divide 79 by 10, and thus the Slide is as if read with 7·9 under 3.

**Ex. 242½.**—If the Slide is set $\frac{\text{C} \qquad 1 \qquad\qquad}{\text{D} \qquad 7 \qquad\qquad}$, what on D, is under the 3 of the line C?

By the Rule in page 86, this is the same as $\frac{\text{C} \qquad 100 \qquad\qquad}{\text{D} \qquad 70 \qquad\qquad}$; so set the 1 on *the extreme right of the line C*, over 7 on D, and reading back to the left, along C, we find under the 3 of the first radius of C, the answer 12·12.

***Ex. 243.**—If the Slide is set $\frac{\text{C} \qquad 1 \qquad\qquad}{\text{D} \qquad 246{\cdot}6 \qquad\qquad}$ what on D is under 51 on C?

By the Rule in page 86, this is the same as $\frac{\text{C} \qquad 100 \qquad\qquad}{\text{D} \qquad 2466 \qquad\qquad}$; so if we set the 1 in *the middle of the line C* (calling it 100), over 2466 on D, we read back to 51 on C, and find under it, the *answer* 1760, on D.

**Ex. 244.**—If the Slide is set $\frac{\text{C} \qquad 3{\cdot}9 \qquad\qquad}{\text{D} \qquad 1 \qquad\qquad}$, what on C, is over the 6·4 of D?

By the Rule in page 86, this is the same as $\frac{\text{C} \qquad 390 \qquad\qquad}{\text{D} \qquad 10 \qquad\qquad}$; so if we set the 390 of the *second radius of C*, over the 10 at the extreme right of D, and read back along D, as far as 6·4, we see over it, on C, the *answer* 160.

**Ex. 245.**—If the Slide is set $\frac{\text{C} \qquad\qquad\qquad 49}{\text{D} \qquad\qquad\qquad 42}$, what on D, is under the 1 of C?

By the Rule in page 86, this is the same as $\frac{\text{C} \qquad {\cdot}49 \qquad\qquad}{\text{D} \qquad 4{\cdot}2 \qquad\qquad}$;

---

* In Ex. 243, the numbers on D represent circumferences of a Circle in yards, and the numbers over them on C, the areas of these Circles in Acres. (See under "Land Measuring.")

so set the ·49 of the *first* radius of C, over the 4·2 of D, and reading on along C till we come to 1·0, we find under it, on D, the *answer* 6.

**Ex. 246.**—If the Slide is set $\frac{C}{D}\text{———}\frac{43560}{33}$, what on C, is over the 1 of D?

By the Rule in page 86, this is the same as $\frac{C}{D}\text{———}\frac{435{\cdot}6}{3{\cdot}3}$; so set the 435·6 of the *second* radius of C, over the 3·3 of D, and reading back on D to the 1 on its extreme left, we see over it the *answer* on C, namely 40.

---

*Shifting the Slide.*

**Ex. 247.**—If the Slide is set $\frac{C}{D}\frac{9{\cdot}5}{12}\text{———}$, what on C, is over 81 on D?

It is no use to read it as $\frac{C}{D}\frac{950}{120}$, or $\frac{C}{D}\frac{{\cdot}095}{1{\cdot}2}$, as in Examples 241, 242, though both these are equivalent to $\frac{C}{D}\frac{9{\cdot}5}{12}$. The Slide will still not show anything over the 7 of D. The only way is to run the eye along the instrument, till two well-defined numbers are found on C and D, exactly over each other, such as $\frac{C}{D}\frac{32}{22}$. Then *shift the Slide*, and set it with the 32 of the *second* radius of C over the 22 of D. We then can read straight on till we come to 81 on D, over which, on C, is the answer 433.

**Ex. 248.**—If the Slide is set $\frac{C}{D}\text{—}\frac{43}{7{\cdot}4}\text{———}$, what on D, is under 92 of C?

It is no use to read it as $\frac{C}{D}\frac{4300}{74}$, or $\frac{C}{D}\frac{{\cdot}043}{{\cdot}74}$, as in Examples 241, 242, though both these are equivalent to $\frac{C}{D}\frac{43}{7{\cdot}4}$. The Slide will still not show anything under the 92 of C. The only way is to run the eye along, till we come to two well-defined numbers on C and

D, exactly over each other, such as $\frac{C}{D}\frac{78{\cdot}5}{10}$. Then *shift the Slide*, till 78·5 of the *first* radius of C is over the 1 (which read as 10) on the extreme left of D, and then under 92 on the first radius of C, we see the answer on D, to be 10·8.*

**Ex. 249.**—If the Slide is set $\frac{C}{D}\frac{7}{1{\cdot}128}$, what on C, is over 4·75 on D?

Look along, till we see two numbers on C and D exactly over each other; as $\frac{C}{D}\frac{60}{3{\cdot}3}$. Then shift the Slide, till 60 on the *first* radius of C, is over 3·3 on D. Now we can read on along D, till we come to 4·75, over which is the *answer* 124 on C.

---

## CHECK NUMBERS.

It has been shown in page 86, that if any number on D is multiplied by $\sqrt{10}$, the number over it, on C, must be multiplied by 10, to keep up the proportion. Thus

$$\frac{C}{D}\frac{9{\cdot}5}{12} = \frac{C}{D}\frac{95}{12 \times \sqrt{10}} = \frac{C}{D}\frac{95}{37{\cdot}95}.$$

Now in the above Examples 247, 248, 249, if we multiply the given numbers on D, 12, 7·4, and 1·128 each by $\sqrt{10}$, that is, by 3·1623, and at the same time multiply the numbers over them on C *by ten*, we shall have $\frac{C}{D}\frac{95}{37{\cdot}95}$, $\frac{C}{D}\frac{430}{23{\cdot}4}$, $\frac{C}{D}\frac{70}{3{\cdot}568}$; and then can solve all the three Examples without shifting the Slide.

But it would be so troublesome to be always multiplying by 3·1623 with pen and pencil, that the method shown above, in Examples 247,

---

* Or we may see $\frac{C}{D}\frac{3{\cdot}8}{2{\cdot}2} = \frac{C}{D}\frac{380}{22}$, and reading back, come to $\frac{C}{D}\frac{92}{10{\cdot}8}$.

248, 249, that is, shifting the Slide, is always resorted to *except* in Formulæ (such as we shall find under Mensuration) where the given number on D, is *invariable*. It is then worth while to multiply it by 3·1623, and use the number so found (with the number over it on D, multiplied by 10) as what is called a "*Check number.*" It so happens as will be seen under "Mensuration" of Parallelopipeds, Areas of Circles, Solidity of Cylinders, &c., that 12, 7·4, 1·128, &c., are constantly wanted on the line D, as "Gauge Points," and hence it *is* worth while in these cases to note their respective "Check numbers," 37·95, 23·4, and 3·568.

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## LINES C AND D USED TOGETHER FOR MULTIPLICATION AND DIVISION, etc.

It has been shown in page 83 how to use these two lines for Involution and Evolution: it now remains to explain to what other cases they are applicable. All questions that can be presented in the two first forms noted in the margin, can be solved by the lines C and D only.

The forms III., IV., V., require the use of the *four* lines A, B, C, D, as shown in page 94.

$$\text{(I.)}\ x = \frac{m^2 \times o}{r^2}$$

$$\text{(II.)}\ x = \frac{\sqrt{m} \times o}{\sqrt{r}}$$

$$\text{(III.)}\ x = \frac{l \times n^2}{s}$$

$$\text{(IV.)}\ x = \frac{l \times n}{s^2}$$

$$\text{(V.)}\ x = \frac{\sqrt{l} \times \sqrt{n}}{\sqrt{s}}$$

The following shows the setting of the Slide Rule for the lines D *used with C only.*

| | | |
|---|---|---|
| C | $a$ | $b$ |
| D | $c$ | $d$ |

The "Proportions" read as follows:

1. $c^2 : d^2 :: a : b$, or $c^2 : a :: d^2 : b$.
2. $\sqrt{a} : \sqrt{b} :: c : d$, or $\sqrt{a} : c :: \sqrt{b} : d$.

Whence we have

$$a = \frac{c^2 \times b}{d^2}\ ;\ b = \frac{d^2 \times a}{c^2}\ ;\ c = \frac{\sqrt{a} \times d}{\sqrt{b}}\ ;\ d = \frac{\sqrt{b} \times c}{\sqrt{a}}.$$

The *two last* may also be in the forms $c = \sqrt{\frac{a \times d^2}{b}}$, and $d = \sqrt{\frac{b \times c^2}{a}}$.

To make this more clear by an *example,* see the Plate, which is set as follows:

| | | |
|---|---|---|
| C | 3·2 | 26 |
| D | 20 | 35 |

$$\text{Here } 3{\cdot}2 = \frac{20^2 \times 26}{57^2} = \frac{400 \times 26}{3249} = 3{\cdot}2 = a.$$

$$26 = \frac{57^2 \times 3{\cdot}2}{20^2} = \frac{3249 \times 3{\cdot}2}{400} = 26 = b.$$

$$20 = \frac{\sqrt{3{\cdot}2} \times 57}{\sqrt{26}} = \frac{17{\cdot}9 \times 57}{5{\cdot}1} = 20 = c.$$

$$57 = \frac{\sqrt{26} \times 20}{3{\cdot}2} = \frac{5{\cdot}1 \times 20}{1{\cdot}79} = 57 = d.$$

**Ex. 250.**—Solve $\left(\frac{232}{354}\right)^2$, or $\frac{232^2}{354^2}$.

Here $354^2 : 232^2 :: 1 : x$; or $x = \frac{232^2 \times 1}{354^2}$. (*See the Plate.*)

| | | |
|---|---|---|
| C | 4·3 | 1 |
| D | 232 | 354 |

**Ex. 251.**—What is the Square Root of $6\frac{7}{8}$; or $\sqrt{\frac{55}{8}}$; or $\frac{\sqrt{55}}{\sqrt{8}}$?

Here $\sqrt{8} : \sqrt{55} :: 1 : x$; or $x = \frac{\sqrt{55} \times 1}{\sqrt{7}}$. (*See the Plate.*)

| | | |
|---|---|---|
| C | 8 | 55 |
| D | 1 | 2 622 |

***Ex. 252.**—Multiply $2{\cdot}5^2$ by 8 ; or $\dfrac{2{\cdot}5^2 \times 8}{1^2}$

Here 1 (or $1^2$) : 8 :: $2{\cdot}5^2 : x$. (*See the Plate*).

| | | |
|---|---|---|
| C | 8 | 50 |
| D | 1 | 2·5 |

†**Ex. 253.**—Multiply 11·2 by $\sqrt{16}$ ; or $\dfrac{\sqrt{16} \times 11{\cdot}2}{\sqrt{1}}$, Ex. 319.

Here $\sqrt{1} : 11{\cdot}2 :: \sqrt{16} : x$. (*See the Plate.*)

| | | |
|---|---|---|
| C | 1 | 16 |
| D | 11·2 | 4·48 |

‡**Ex. 254.**—Divide 45000 by $75^2$.

Here $75^2 : 45000 : 1^2 : x$; or $x = \dfrac{1^2 \times 45000}{75^2}$. (*See the Plate.*)

| | | |
|---|---|---|
| C | 8 | 45000 |
| D | 1 | 75 |

**Ex. 255.**—Divide 5 by $\sqrt{20}$; or $\dfrac{5}{\sqrt{20}}$; or $\sqrt{\dfrac{5^2}{20}}$.

Here $\sqrt{20} : \sqrt{1} :: 5 : x$; or $x = \dfrac{\sqrt{1} \times 5}{\sqrt{20}}$. (*See the Plate.*)

| | | |
|---|---|---|
| C | 1 | 20 |
| D | 1·118 | 5 |

---

* In Ex. 252, we use $1^2 : 8$, instead of $1 : 8$, because when the line D is used with C only, the first term must be either a Square, or a Square Root. (See I. and II., page 91.)

† In Ex. 253 we use $\sqrt{1} : 11{\cdot}2$, instead of $1 : 11{\cdot}2$ for the reason given in the preceding footnote.

‡ In Ex. 254, if we set the 45 of either radius of C, over 75 on D, the line D is not long enough to read back to 1 on the left, to see what stands over it. We therefore read forward, till we have $\dfrac{\text{C} \quad 80000}{\text{D} \quad 100}$. Then, as explained in page 16, we divide the 100 on D by 100, to get 1 on D, and at the same time divide the 80000 on C by 10000, and we have $\dfrac{\text{C} \quad 8}{\text{D} \quad 1}$ giving 8 as the answer.

**Ex. 256.**—Solve $x = \frac{72 \times 26^2}{30^2}$.

Here $30^2 : 72 :: 26^2 : x$. (*See the Plate.*)

| | | |
|---|---|---|
| C | 54 = answer | 72 |
| D | 26 | 30 |

**Ex. 257.**—Solve $x = \frac{57 \times \sqrt{3 \cdot 2}}{\sqrt{26}}$; or $57 \times \sqrt{\frac{3 \cdot 2}{26}}$. (*See the Plate.*)

| | | |
|---|---|---|
| C | 3·2 | 26 |
| D | 20 = answer | 57 |

(In line 9, page 92, for 25 read 57.)

***Ex. 258.**—What is the "Geometric Mean" of 125 and 180?

Here $x = \sqrt{125 \times 180}$; or $x = \sqrt{125} \times \sqrt{180}$. (See footnote.)

| | | | |
|---|---|---|---|
| Either | C | 125 | 180 |
| | D | 125 | 150 answer |

as in the *Plate;* or else it may be set as follows :

| | | |
|---|---|---|
| C | 125 | 180 |
| D | 150 answer | 180 |

---

## TO USE THE FOUR LINES A, B, C, D, TOGETHER.

As explained in page 91, there are *three* cases (see margin), which cannot be solved by the line D with C only, but which require all four lines A, B, C, D.

(III.) $x = \frac{l \times n^2}{s}$

(IV.) $x = \frac{l \times n}{s^2}$

(V.) $x = \frac{\sqrt{l} \times \sqrt{n}}{\sqrt{s}}$

---

* In Ex. 258, the Equation is $\frac{\sqrt{125} \times \sqrt{180}}{1}$, or $\frac{\sqrt{125} \times \sqrt{180}}{\sqrt{1}}$, and in *this* form it cannot be solved by the two lines C, D, as it does not come under either I. *or* II., of page 91. But since $\sqrt{a} = \frac{a}{\sqrt{a}}$, we change the Equation $x = \sqrt{a} \times \sqrt{b}$, into $x = \frac{a}{\sqrt{a}} \times \sqrt{b}$; or $x = \frac{a \times \sqrt{b}}{\sqrt{a}}$, and then it is solvable as in Ex. 257. To solve it by the *four* lines, see Ex. 262½.

The following is the set of the Rule :

| | | |
|---|---|---|
| A | $a$ | |
| B | $b$ | |
| C | $c$ | |
| D | $d$ | |

and the "Proportions" read as follows :

1. $c : d^2 :: b : a$, or $a = \dfrac{b \times d^2}{c}$.

2. $d^2 : c :: a : b$, or $b = \dfrac{a \times c}{d^2}$.

3. $a : b :: d^2 : c$, or $c = \dfrac{b \times d^2}{a}$.

4. $\sqrt{b} : \sqrt{a} :: \sqrt{c} : d$, or $d = \dfrac{\sqrt{a} \times \sqrt{c}}{\sqrt{b}}$, or $\sqrt{\dfrac{a \times c}{b}}$.

To make this more clear by an Example, *see the Plate.*

| | | |
|---|---|---|
| A | 20 | |
| B | 16 | |
| C | | 45 |
| D | | 7·5 |

or, the same may be set thus :

| | | |
|---|---|---|
| A | 45 | |
| B | 16 | |
| C | | 20 |
| D | | 7·5 |

$$20 = \frac{16 \times 7{\cdot}5^2}{45} = \frac{16 \times 56{\cdot}25}{45}.$$

$$16 = \frac{20 \times 45}{7{\cdot}5^2} = \frac{20 \times 45}{56{\cdot}25}.$$

$$45 = \frac{16 \times 7{\cdot}5^2}{20} = \frac{16 \times 56{\cdot}25}{20}.$$

$$7{\cdot}5^2 = \frac{\sqrt{20} \times \sqrt{45}}{\sqrt{16}} = \frac{4{\cdot}472 \times 6{\cdot}71}{4}, \text{ or } \sqrt{\frac{20 \times 45}{75}} = \sqrt{\frac{900}{75}}.$$

**Ex. 259.**—Solve $x = \dfrac{4 \times 5{\cdot}7^2}{5}$, or $\frac{4}{5}$ of $5{\cdot}7^2$. (Here $5 : 4 :: 5{\cdot}7^2 : x$.)

| | | | |
|---|---|---|---|
| A | 5 | | |
| B | 4 | | |
| C | | 26 | *See the Plate.* |
| D | | 5·7 | |

or it may be set *thus:*

| | | |
|---|---|---|
| A | 26 | |
| B | 4 | |
| C | | 5 |
| D | | 5·7 |

**Ex. 260.**—Solve $x = \dfrac{3{\cdot}8 \times 9{\cdot}5}{2{\cdot}18^2}$. (Here $2{\cdot}18^2 : 3{\cdot}8 :: 9{\cdot}5 : x$.)

| | | | |
|---|---|---|---|
| A | | 9·5 | |
| B | | 7·6 | *See the Plate.* |
| C | 3·8 | | |
| D | 2·18 | | |

or it may be set *thus:*

| | | |
|---|---|---|
| A | | 3·8 |
| B | | 7·6 |
| C | 9·5 | |
| D | 2·18 | |

**Ex. 261.**—Divide $320^2$ by 125; or $\dfrac{320^2}{125}$; or $\frac{1}{125}$ of $320^2$.

| | | | |
|---|---|---|---|
| A | 125 | | |
| B | 1 | | *See the Plate.* |
| C | | 820 | |
| D | | 320 | |

N.B. Questions like $\dfrac{320^2}{125}$ can be solved, and perhaps more accurately, by the lines A, B, in the form $\dfrac{320 \times 320}{125}$, or $\dfrac{\text{A}}{\text{B}}\ \dfrac{320}{125}\ \dfrac{820}{320}$. See Ex. 43.

**Ex. 262.**—What is the Square Root of $\frac{7}{56}$ of 162? (This is the same as $\sqrt{\frac{7 \times 162}{56}}$, or $\frac{\sqrt{7} \times \sqrt{162}}{\sqrt{56}}$, where

$$\sqrt{56} : \sqrt{57} :: \sqrt{162} : x.)$$

Either *as in the Plate:*

| | | |
|---|---|---|
| A | 7 | |
| B | 56 | |
| C | | 162 |
| D | | 4·5 Ans. |

Or

| | | |
|---|---|---|
| A | 162 | |
| B | 56 | |
| C | | 7 |
| D | | 4·5 Ans. |

**Ex. 262½.**—What is the "Geometric Mean" of 125 and 180?

Here $x = \frac{\sqrt{125} \times \sqrt{180}}{\sqrt{1}}$. (See footnote to Ex. 258.)

| | | |
|---|---|---|
| A | 125 | |
| B | 1 | |
| C | | 180 |
| D | | 150 Ans. |

*Index to the Examples of the use of the D line.*

With the two lines C, D.

| Form. | | Form. | |
|---|---|---|---|
| I. | $\frac{232^2}{354^2}$ Ex. 250. | II. | $\sqrt{16 \times 11{\cdot}2^2}$ Ex. 253. |
| I. | $\left(\frac{232}{354}\right)^2$ Ex. 250. | II. | $\frac{5}{\sqrt{20}}$ Ex. 255. |
| I. | $2{\cdot}5^2 \times 8$ Ex. 252. | II. | $\sqrt{\frac{5^2}{20}}$ Ex. 255. |
| I. | $\frac{45000}{75^2}$ Ex. 254. | II. | $\frac{\sqrt{3{\cdot}2 \times 57}}{\sqrt{26}}$ Ex. 257. |
| I. | $\frac{72 \times 26^2}{30^2}$ Ex. 256. | II. | $\sqrt{\frac{3{\cdot}2 \times 57^2}{26}}$ Ex. 257. |
| II. | $\frac{\sqrt{55}}{\sqrt{8}}$ Ex. 251. | II. | $57 \times \sqrt{\frac{3{\cdot}2}{26}}$ Ex. 257. |
| II. | $\sqrt{\frac{55}{8}}$ Ex. 251. | II. | $\sqrt{125 \times 180}$ Ex. 258. |
| II. | $\sqrt{6\frac{7}{8}}$ Ex. 251. | II. | $\sqrt{125} \times \sqrt{180}$ Ex. 258. |
| II. | $11{\cdot}2 \times \sqrt{16}$ Ex. 253. | | |

With the four lines A, B, C, D.

| Form. | | Form. | |
|---|---|---|---|
| III. | $\frac{4 \times 5{\cdot}7^2}{5}$ Ex. 259. | IV. | $\frac{3{\cdot}8 \times 9{\cdot}5}{2{\cdot}18^2}$ Ex. 260. |
| III. | $\frac{4}{5}$ of $5{\cdot}7^2$ Ex. 259. | V. | $\sqrt{\frac{7 \times 162}{56}}$ Ex. 262. |
| III. | $\frac{320^2}{125}$ Ex. 261. | V. | Sq. Rt. of $\frac{7}{56}$ of 162 Ex. 262. |
| III. | $\frac{1}{125}$ of $320^2$ Ex. 261. | V. | $\frac{\sqrt{7} \times \sqrt{162}}{\sqrt{56}}$ Ex. 262. |

Such cases as $x = \frac{\sqrt{n}}{s}$ are not solvable with the Slide Rule, unless the Square Root of $s$ is known, and then it would come under Ex. 251.

* *Extra Examples* for the line D with C only.

$$(\text{I.})\ x = \frac{m^2 \times o}{r^2},\ \text{or (II.)}\ x = \frac{\sqrt{m} \times o}{\sqrt{r}}\ \text{(page 91).}$$

**Ex. 263.**—The Land-measuring chain used in some parts of India, is 33 feet long. How many Square chains make one Acre?

(Here $x = \frac{43560}{33^2}$, as in Ex. 254.)

| | | |
|---|---|---|
| C | 40 = Answer | 43560 |
| D | 1 | 33 |

**Ex. 264.**—The diameter of a Standard Bushel, is 18·79 inches, and its depth 8 inches. Required the diameter of a Bushel whose

* The extra Examples for line D, with A, B, C, will be found after Ex. 278.

depth is 1 foot? (Here, as diameters of cylinders, of equal capacity, vary as the Square Roots of their depths, we have $\sqrt{12} : \sqrt{18} :: x$, or $x = \dfrac{\sqrt{8} \times 18{\cdot}79}{\sqrt{12}}$, as in Ex. 257.)

| | | |
|---|---|---|
| C | 8 | 12 |
| D | 15·3 inches | 18·79 |

**Ex. 265.**—What must be the length, in feet, of one of the *sides* of a square tent, so as to cover an area equal to that of a rectangular tent measuring $20\frac{3}{4}$ feet by $14\frac{1}{4}$ feet? (Answer = Geometrical mean of $20\frac{3}{4}$ and $14\frac{1}{4}$, as in Ex. 258.)

| | | |
|---|---|---|
| C | 14·25 | 20·75 |
| D | 14·25 | 16·47 ft. |

**Ex. 265½.**—When the pressure of the wind is 1lb. per Square foot, the velocity is 15 miles an hour. What will be the velocity when the pressure is 2, 3, or 4 lbs. per Square foot, remembering that the velocity increases as the Square Roots of the pressure?

| | | | | |
|---|---|---|---|---|
| C | 1 lb. | 2 lbs. | 3 lbs. | 4 lbs. |
| D | 15 miles | 21·2 miles | 26 miles | 30 miles |

**Ex. 266.**—What is the "Mean Square" of a bar of iron which is $2\frac{3}{4}$ inches broad, and $\frac{3}{8}$ inch thick? ($x = \sqrt{2{\cdot}75 \times {\cdot}375}$.)

| | | | |
|---|---|---|---|
| C | ·375 | 2·75 | (Ex. 258.) |
| D | 1·015 inches | 2·75 | |

**Ex. 267.**—If 6 Square *mètres* = 7·2 Square yards, what is the length of the mètre in inches? (Here $\dfrac{\sqrt{7{\cdot}2}}{\sqrt{6}}$ = length in *yards*, but $36 \times \sqrt{\dfrac{7{\cdot}2}{6}}$ = length in *inches*; as in Ex. 257.)

| | | |
|---|---|---|
| C | 6 | 7·2 |
| D | 36 | 39·4 inches |

**Ex. 268.**—One day in the 18th century, a person observed that his age was the Square Root of the year. In what year was he born? (The Slide Rule shows instantly what is the only whole number that is a Square Root, between 1700 and 1800.)

**Ex. 269.**—A malt-kiln is $16\frac{1}{2}$ feet square. Required the *side* of another square kiln, capable of drying three times as much. (Since areas of Squares vary as the squares of their sides, $1 : 3 :: 16{\cdot}5^2 : x^2$ or $x = \sqrt{16{\cdot}5^2 \times 3}$, or $x = 16{\cdot}5 \times \sqrt{3}$, as in Ex. 253.)

| | | |
|---|---|---|
| C | 1 | 3 |
| D | 16·5 | 28·6 ft. |

**Ex. 270.**—In "Timber measuring" (as will be shown under that head), the *customary* method of obtaining the content in cubic feet is to multiply the length in feet, by the square of the quarter-girt in inches, and divide the product by $12^2$. What, by this method, is the content of a log 24 feet long, with a quarter-girt of $9\frac{3}{4}$ inches? (Here $x = \dfrac{24 \times 9{\cdot}75^2}{12^2}$, as in Ex. 256.)*

| | | |
|---|---|---|
| C | 24 | 15·84 cubic feet |
| D | 12 | 9·75 |

**Ex. 271.**—I have a cylindrical vessel 13 inches in diameter, and I want another of the *same* depth, but to contain twice as much. What must be its diameter? (As contents of Cylinders of the same depth vary as the squares of their diameters, we have $1 : 2 : 13^2 :: x^2$, or $x = \sqrt{13^2 \times 2}$, or $x = 13 \times \sqrt{2}$, as in Ex. 253.)

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* In Ex. 270 the Slide seems *too short* (see page 87), but if we read on, we see $\dfrac{C \quad 600}{D \quad 60}$, which is the same as $\dfrac{C \quad 6}{D \quad 6}$. Then set 6 of the *first* radius of C over 6 on D, and we see the answer on C over 9·75 of D. Or if we use the "Check number" of 12 (page 91, line 10), namely 37·95, we may at once set this on D under 10 *times* the number on C; or $\dfrac{C \quad 240}{D \quad 37{\cdot}95} = \dfrac{C \quad 2{\cdot}4}{D \quad 3{\cdot}795}$, 2·4 being on the *first* radius of C.

| | | |
|---|---|---|
| C | 1 | 2 |
| D | 13 | 18·4 inches |

**Ex. 272.**—I have a cylindrical vessel 28 inches deep, and 46 inches diameter, and I want another, 36 inches deep, to hold twice as much. What must be its diameter? (Here 36 : 56 : $46^2$ : $x^2$, or $x = \dfrac{46 \times \sqrt{56}}{\sqrt{36}}$, as in Ex. 257.)

| | | |
|---|---|---|
| C | 36 | 56 |
| D | 46 | 57·4 inches |

**Ex. 273.**—The French kilomètre is 1093·63 yards, and the population in France is on an average 70 to a square kilomètre. How many is this to a square mile? (Here $1093{\cdot}63^2 : 1760^2 :: 70 : x$, or $x = \dfrac{70 \times 1760^2}{1093{\cdot}63^2}$, as in Ex. 256.)

| | | |
|---|---|---|
| C | 70 | 181 |
| D | 1093·63 | 1760 |

***Ex. 274.**—What must be the diameter, in inches, of the mouth of a circular Rain-gauge, so that its area may be 17⅓ square inches? (Here $x = \sqrt{\dfrac{17{\cdot}33}{{\cdot}7854}}$, according to a known formula.)

| | | | |
|---|---|---|---|
| C | ·7854 | 17·33 | (Ex. 251.) |
| D | 1 | 4·7 | |

**Ex. 275.**—Suppose a field measured with a Gunter's chain is recorded as 16·6 acres; but it is subsequently found that the chain used was 9 inches too short; what is the *true* area of the field? (Here $33^2 : 32{\cdot}75^2 :: 16{\cdot}6 : x$, as in Ex. 256.)

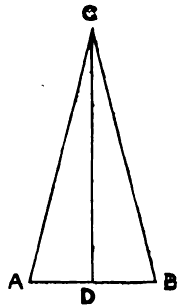
---

* In Ex. 274 the Slide seems too short: but we may use the expedient referred to in Ex. 248. Here $\dfrac{C \quad {\cdot}7854}{D \quad 1} = \dfrac{C \quad 78{\cdot}54}{D \quad 10}$. So if we place the 78·54 of the *second* radius of C over the 10 at the extreme right of D, and read back, we find under 17·33 of C, the answer on D.

| | | |
|---|---|---|
| C | 16·35 acres | 16·6 |
| D | 32·75 | 33 |

**Ex. 276.**—Let CD be the wall of a house, against which a ladder (AC) is placed, with its foot 6 feet (AD) from the wall. How long must the ladder be, to reach a window at C, which is 24·3 feet (DC) from the ground?

(Here A C = $\sqrt{24 \cdot 3^2 + 6^2}$.)




| | | | |
|---|---|---|---|
| C | 36 | 589 | 625 (= 36 + 589) |
| D | 6 = AD | 24·3 = DC | 25 = A C |

Here, the Slide Rule shows the squares of 6 and 24·3, and the sum of these being read on C = 625, we see under it the answer, *without moving the Slide* at all in the whole process.

**Ex. 277.**—In the preceding figure, let ABC be a Cone, of which the slant height AC is 21 inches, and the base AB 12 inches. Required the perpendicular height CD. (Here CD = $\sqrt{AC^2 - AD^2}$, or $\sqrt{(AC + AD) \times (AC - AD)}$ which last is better adapted to the Slide Rule: so that we have CD = $\sqrt{(21 + 6) \times (21 - 6)} = \sqrt{27 \times 15}$, as in Ex. 258.)

| | | |
|---|---|---|
| C | 15 | 27 |
| D | 15 | 20·12 |

**Ex. 277½.**—Given the hypothenuse AC = 23·77, and one of the sides CD = 18·6. Required the length of the third side DA (Here DA = $\sqrt{(23 \cdot 77 + 18 \cdot 6) \times (23 \cdot 77 - 18 \cdot 6)}$, or $\sqrt{42 \cdot 37 \times 5 \cdot 17}$, as in Ex. 258.)

| | | |
|---|---|---|
| C | 42·37 | 5·17 |
| D | 14·8 = DA | 5·17 |

In *Right-angled* Triangles, when two sides are *equal*, use the following :—

| | | | |
|---|---|---|---|
| A | 1 | 29 | Either equal side |
| B | 1·4142 (= $\sqrt{2}$) | 41 | Hypothenuse |

**Ex. 278.**—The circumference of a circle is equal to the Square Root of the area, divided by ·0796. What is the circumference of a circle whose area is 23 square feet? (Here $x = \sqrt{\frac{23}{\cdot 0796}}$, as in Ex. 251.)*

| | | |
|---|---|---|
| C | ·0796 | 23 |
| D | 1 | 17 |

---

*Extra Examples* **for the four lines A, B, C, D.**

See p. 94. (III.) $x = \frac{l \times n}{s}$. (IV.) $x = \frac{l \times n}{s^2}$. (V.) $x = \frac{\sqrt{l} \times \sqrt{n}}{\sqrt{s}}$

†**Ex. 279.**—A map drawn to a scale of $4\frac{5}{8}$ inches to a mile, is 3 feet $9\frac{1}{2}$ inches, by 3 feet $0\frac{1}{2}$ inches. Required the number of Square miles in it. (Here $x = \frac{45{\cdot}5 \times 36{\cdot}5}{4{\cdot}672^2}$, as in Ex. 260.)

| | | |
|---|---|---|
| A | | 36·5 |
| B | | 76·1 Square miles |
| C | 45·5 | |
| D | 4·67 | |

†**Ex. 280.**—How many Square *yards* are there in a garden bed 35 feet in diameter? (Here $x = \frac{35^2 \times \cdot 7854}{9}$, as in Ex. 259.)

---

* In Example 278, we read the Slide as $\frac{C\ \ 7{\cdot}96}{D\ \ 10}$ ——, and then it will not appear *too short*. (See pages 87, 88.)

† There are *two* ways of setting the Slide, when the four lines A, B, C, D are used, as shown in page 95 and in Ex. 283. One is as good as the other, except in the case of a "Series," as explained in page 107.

| | | |
|---|---|---|
| A | 9 | |
| B | ·7854 | |
| C | | 10·7 Square yards |
| D | | 35 |

**Ex. 281.**—The difference of level *in feet*, on the surface of the earth, is $\frac{5}{9}$ of the square of the distance in Statute *miles*. What is the distance of the sea-horizon, when the eye is 125 feet above the ground? (Here $x = \sqrt{\frac{9 \times 125}{5}}$, as in Ex. 262, or $x$ = Square Root of $\frac{9}{5}$ of 125.)

| | | |
|---|---|---|
| A | | 9 |
| B | | 5 |
| C | 125 | |
| D | 15 stat. miles | |

**Ex. 282.**—3·847 Cubic inches of cast iron weigh 1lb. What is the weight of a *cube* of cast iron which measures 12 inches each side? (Here $x = \frac{12^3}{3 \cdot 847}$, or $x = \frac{12^2 \times 12}{3 \cdot 847}$, as in Ex. 259.)

| | | |
|---|---|---|
| A | | 3·847 |
| B | | 12 |
| C | 449 lbs. | |
| D | 12 | |

**Ex. 283.**—If $d$ = diameter of a cylinder, and $l$ = length, the content is $\frac{d^2 \times l}{1 \cdot 2732}$. Required the content in Cubic inches, of a cylinder whose length is $6\frac{1}{2}$ inches, and diameter 4·6 inches. (Here $x = \frac{4 \cdot 6^2 \times 6 \cdot 5}{1 \cdot 273}$, as in Ex. 259.)

Either

| | | |
|---|---|---|
| A | 1·273 | |
| B | 6·5 | |
| C | | 108 cub. in. |
| D | | 4·6 = diam. |

Or*

| | | |
|---|---|---|
| A | 108 cub. in. | |
| B | 6·5 | |
| C | | 1·273 |
| D | | 4·6 = diam. |

**Ex. 284.**—Using ·1606 as a "Divisor," what is the content in Gallons, of a rectangular tank $9\frac{1}{2}$ feet long, and $6\frac{3}{4}$ feet in width, and the same in depth? (Here $x = \dfrac{9{\cdot}5 = 6{\cdot}75^2}{{\cdot}1606}$, as in Ex. 259.)

| | | |
|---|---|---|
| A | ·1606 | |
| B | 9·5 | |
| C | | 2698 gallons |
| D | | 6·75 = Mean square |

---

## To solve a "Series," with the lines C, D.

See page 91. (I.) $x = \dfrac{m^2 \times o}{r^2}$. (II.) $x = \dfrac{\sqrt{m} \times o}{\sqrt{r}}$.

N.B. 1. If $m^2$ and $o$, or $\sqrt{m}$ and $o$ are the two constants, the Slide must be *inverted*, as shown in Examples 298, &c.

N.B. 2. If $o$ and $r^2$, or $o$ and $\sqrt{r}$ are the two constants, the lines C and D are to be used as below; remembering that the two constants are to be set one over the other. In (I.) $r^2$ on the line D, and in (II.) $\sqrt{r}$ on the line C. (See N.B. to Ex. 20.)

**Ex. 285.**—Multiply $3^2$, $4^2$, $5^2$, each by 2·3. (Here $\dfrac{3^2 \times 2{\cdot}3}{1^2}$, $\dfrac{4^2 \times 2{\cdot}3}{1^2}$, $\dfrac{5^2 \times 2{\cdot}3}{1^2}$, where $o$ and $r^2$ are constant.)

| | | | | |
|---|---|---|---|---|
| C | 2·3 | 20·7 | 36·8 | 57·5 |
| D | 1 | 3 | 4 | 5 |

---

* See footnote to Ex. 279.

***Ex. 286.**—Solve at one setting $\frac{2{\cdot}5^2 \times 18}{1{\cdot}5^2}$, $\frac{3{\cdot}5^2 \times 18}{1{\cdot}5^2}$, $\frac{4{\cdot}6^2 \times 18}{1{\cdot}5^2}$. (Here $o$ and $r^2$ are constant.) *See the Plate.*

| | | | | |
|---|---|---|---|---|
| C | 18 | 50 | 98 | 169 |
| D | 1·5 | 2·5 | 3·5 | 4·6 |

**Ex. 287.**—Multiply $\sqrt{9}$, $\sqrt{14}$, $\sqrt{16}$, $\sqrt{25}$, each by 12. (Here $\frac{\sqrt{9} \times 12}{\sqrt{1}}$, $\frac{\sqrt{14} \times 12}{\sqrt{1}}$, $\frac{\sqrt{16} \times 12}{\sqrt{1}}$, $\frac{\sqrt{25} \times 12}{\sqrt{1}}$, where $o$ and $\sqrt{r}$ are constant.)

| | | | | | |
|---|---|---|---|---|---|
| C | 1 | 9 | 14 | 16 | 25 |
| D | 12 | 36 | 44·8 | 48 | 60 |

**Ex. 288.**—Divide $\sqrt{4096}$, $\sqrt{5184}$, $\sqrt{6400}$, each by $\sqrt{16}$ (Here $\frac{\sqrt{4096} \times 1}{\sqrt{16}}$, $\frac{\sqrt{5184} \times 1}{\sqrt{16}}$, $\frac{\sqrt{6400} \times 1}{\sqrt{16}}$, where $o$ and $\sqrt{r}$ are constant.)

| | | | | |
|---|---|---|---|---|
| C | 16 (1600) | 4096 | 5184 | 6400 |
| D | 1 (10) | 16 | 18 | 20 |

**Ex. 289.**—Solve, at one setting, $\frac{68 \times \sqrt{5{\cdot}4}}{\sqrt{37}}$, $\frac{68 \times \sqrt{9{\cdot}8}}{\sqrt{37}}$, $\frac{68 \times \sqrt{26}}{\sqrt{37}}$. (Here $o$ and $\sqrt{r}$ are constant.) *See the Plate.*

| | | | | |
|---|---|---|---|---|
| C | 5·4 | 9·8 | 26 | 37 |
| D | 26 | 35 | 57 | 68 |

**Ex. 290.**—If $d$ = diameter of a cylinder, and $l$ its depth or length, its Cubic content is $\frac{d^2 \times l}{1{\cdot}128^2}$. What are the contents in Cubic inches of three cylinders, whose *lengths are the same,* namely 7·2

* Ex. 286. Set as above, the Slide seems too short to see what is over the 4·6 on D. We must therefore shift the Slide as shown in page 89, or use the "Check number" 3·568 as shown in page 90.

inches, but whose diameters vary, being respectively 3·2, 4·2, and 4·6 inches? (Here $\frac{3 \cdot 2^2 \times 7 \cdot 2}{1 \cdot 128^2}$, $\frac{4 \cdot 2^2 \times 7 \cdot 2}{1 \cdot 128^2}$, $\frac{4 \cdot 6^2 \times 7 \cdot 2}{1 \cdot 128^2}$, where $o$ and $r^2$ are constant, as in Ex. 286.)

| | | | | |
|---|---|---|---|---|
| C | 7·2 | 58 cub. in. | 100 cub. in. | 120 cub. in. |
| D | 1·128 | 3·2 | 4·2 | 4·6 diams. |

N.B. If $m^2$ and $r^2$ are constant, there is no method of solving a "series" with one setting of the Slide, either with C, D inverted or not, or with A, B, C, D inverted or not. The only way is to find the square of $r$, and use it as a constant Divisor, as in Ex. 294. This is explained in the N.B. after Ex. 297.

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## To solve a "Series," with the lines A, B, C D.

See p. 94. (III.) $x = \frac{l \times n^2}{s}$. (IV.) $x = \frac{l \times n}{s^2}$. (V.) $x = \frac{\sqrt{l} \times \sqrt{n}}{\sqrt{s}}$

N.B. 1. Where the *two multipliers* are constant, the Slide must be *inverted*, as shown a few pages on.

In other cases (remembering always that the Square number, whether in the question or in the answer, must be on line D) the following Notes may be useful.

(IV.) Where $s^2$ and either Multiplier are constant.

| | |
|---|---|
| A | varying Multiplier |
| B | answers |
| C | constant Multiplier |
| D | $s$ constant |

(III.) *a*. Where $l$ and $s$ are constant

| | |
|---|---|
| A | $s$ constant |
| B | $l$ constant |
| C | answers |
| D | $n$ varying |

(III.) *b*. Where $s$ and $n^2$ are constant

| | |
|---|---|
| A | answers |
| B | $l$ varying |
| C | $s$ constant |
| D | $n$ constant |

(V.) Where $\sqrt{s}$ and either Multiplier are constant.

| | |
|---|---|
| A | constant Multiplier |
| B | *s* constant |
| C | varying Multiplier |
| D | answers |

**Ex. 291.**—Divide 48, 64, and 80, each by $4^2$. (Here $\frac{48 \times 1}{4^2}$, $\frac{64 \times 1}{4^2}$, $\frac{80 \times 1}{4^2}$, and $s^2$ and one Multiplier are constant.)

(IV.)

| | | | | |
|---|---|---|---|---|
| A | | 48 | 64 | 80 |
| B | | 3 | 4 | 5 |
| C | 1 | | | |
| D | 4 | | | |

***Ex. 292.**—Divide $49^2$, $32^2$, and $29^2$, each by 20. (Here $\frac{49^2 \times 1}{20}$, $\frac{32^2 \times 1}{20}$, $\frac{29^2 \times 1}{20}$, and *l* and *s* are constant.)

(III*a*.)

| | | | | |
|---|---|---|---|---|
| A | 20 | | | |
| B | 1 | | | |
| C | | 42 | 51·2 | 120 |
| D | | 29 | 32 | 49 |

**Ex. 293.**—Solve $\frac{3^2 \times 7}{1 \cdot 3}$, $\frac{4^2 \times 7}{1 \cdot 3}$, $\frac{5^2 \times 7}{1 \cdot 3}$. (Here *l* and *s* are constant.)

(III*a*.)

| | | | | |
|---|---|---|---|---|
| A | 1·3 | | | |
| B | 7 | | | |
| C | | 48·46 | 86·15 | 134·6 |
| D | | 3 | 4 | 5 |

---

* Since $\frac{a^2}{b} = \frac{a^2 \times b}{b^2}$, Examples such as 292 can be solved by C, D only. Thus $\frac{49^2 \times 20}{20^2}$, $\frac{32^2 \times 20}{20^2}$, $\frac{29^2 \times 20}{20^2}$, as in Ex. 286. $\frac{C\ 20\ \ 42\ \ 51 \cdot 2\ \ 120}{D\ 20\ \ 29\ \ 32\ \ 49}$.

**Ex. 294.**—Solve $\frac{7{\cdot}5^2 \times 16}{45}$, $\frac{7{\cdot}5^2 \times 24}{45}$, $\frac{7{\cdot}5^2 \times 32}{45}$. (Here $s$ and $n^2$ are constant.) *See the Plate.*

(III*b*.)

| | | | | |
|---|---|---|---|---|
| A | | 20 | 30 | 40 |
| B | | 16 | 24 | 32 |
| C | 45 | | | |
| D | 7·5 | | | |

**Ex. 295.**—Solve $\frac{\sqrt{21} \times \sqrt{162}}{\sqrt{168}}$, $\frac{\sqrt{21} \times \sqrt{260}}{\sqrt{168}}$, $\frac{\sqrt{21} \times \sqrt{620}}{\sqrt{168}}$. (Here $\sqrt{s}$ and one Multiplier are constant.) *See the Plate.*

(V.)

| | | | | |
|---|---|---|---|---|
| A | 21 | | | |
| B | 168 | | | |
| C | | 162 | 260 | 620 |
| D | | 45 | 57 | 88 |

**Ex. 296.**—There are three maps, all drawn to the scale of $4\frac{2}{3}$ inches to a mile. They are each 45 inches wide, but their lengths vary; being respectively 34, $36\frac{1}{2}$, and 40 inches. How many *square miles* are represented in each map? (Here $\frac{45{\cdot}5 \times 34}{4{\cdot}67^2}$, $\frac{45{\cdot}5 \times 36{\cdot}5}{4{\cdot}67^2}$, $\frac{45{\cdot}5 \times 40}{4{\cdot}67^2}$, where $s^2$ and one Multiplier are constant.) See Ex. 279.

(IV.)

| | | | | |
|---|---|---|---|---|
| A | | 34 | 36·5 | 40 |
| B | | 70·9 | 76·1 | 83·2 |
| C | 45·5 | | | |
| D | 4·67 | | | |

**Ex. 297.**—Taking the data in Ex. 281, what distance in statute miles, can lights be seen from the sea when they are at the respectiv elevations of 22 feet, 80 feet, and 125 feet? (Here $\sqrt{\frac{9 \times 22}{5}}$, $\sqrt{\frac{9 \times 80}{5}}$, $\sqrt{\frac{9 \times 125}{5}}$; where $\sqrt{s}$ and one Multiplier are constant.)

(V.)

| | | | | |
|---|---|---|---|---|
| A | 9 | | | |
| B | 5 | | | |
| C | | 22 | 80 | 125 |
| D | | 6·3 | 12 | 15 |

N.B. 1. If we have $x = \frac{m^2 \times o}{r^2}$, it can be solved with the lines C, D, as in Ex. 256, but if we have a "*series*" where $m^2$ and $r^2$ are constant, while $o$ varies, the only way is to find the square of $r$ which call $w$, and use the four lines A, B, C, D. Thus, suppose we have $\frac{9^2 \times 8}{6^2}$, $\frac{9^2 \times 12}{6^2}$, $\frac{9^2 \times 14}{6^2}$, where $9^2$ and $6^2$ are constant. We know that $6^2 = 36$. Then we get the series $\frac{9^2 \times 8}{36}$, $\frac{9^2 \times 12}{36}$, $\frac{9^2 \times 14}{36}$, as in Ex. 293.

| | | | | |
|---|---|---|---|---|
| A | | 18 | 27 | 31·5 |
| B | | 8 | 12 | 14 |
| C | 36 | | | |
| D | 9 | | | |

When the square Divisor occurs very often in practice, it is worth while to make a note of it in case of a "series." Thus in Mensuration of content of cylinders, suppose we have three all of the same diameter 4·6 inches, but whose *lengths vary*, namely 6·5, 7·2, and 8·0 inches, the contents in cubic inches (compare Ex. 290) would be $\frac{4{\cdot}6^2 \times 6{\cdot}5}{1{\cdot}128^2}$, $\frac{4{\cdot}6^2 \times 7{\cdot}2}{1{\cdot}128^2}$, $\frac{4{\cdot}6^2 \times 8{\cdot}0}{1{\cdot}128^2}$. This "series" is not solvable in any way by the Slide Rule (see N.B. to Ex. 290), but if we keep a note of the Square of 1·128 which is 1·2732, we have $\frac{4{\cdot}6^2 \times 6{\cdot}5}{1{\cdot}2732}$, $\frac{4{\cdot}6^2 \times 7{\cdot}2}{1{\cdot}2732}$, $\frac{4{\cdot}6^2 \times 8{\cdot}0}{1{\cdot}2732}$, easily solvable as in Ex. 294. On this head more will be found under "*Mensuration of Cylinders.*"

## Use of the lines $\frac{⅁}{D}$ INVERTED.

If in the Formulæ (I.) $x = \frac{m^2 \times o}{r^2}$, (II.) $x = \frac{\sqrt{m} \times o}{\sqrt{r}}$, we see that in a "series" of Examples the *two Multipliers are constant*, the Slide must be inverted; and the multipliers set one over the other; $m^2$ is always on D, and $\sqrt{r}$ on ⅁. In other cases of a "series," see p. 105.

| | | | | |
|---|---|---|---|---|
| ⅁ | $b$ | (8) | $a$ | (2) |
| D | $c$ | (3) | $d$ | (6) |

The "Proportion" is $c^2 : d^2 :: a : b$, or $\sqrt{a} : \sqrt{b} :: c : d$. Thus, in the above setting, $8 \times 3^2 = 2 \times 6^2$, and $3 \times \sqrt{8} = 6 \times \sqrt{2}$.

N.B. If a number on D is multiplied by 10, the number over it on ⅁ must be divided by 100. Thus $\frac{⅁ \quad 800}{D \quad 2{\cdot}7} = \frac{⅁ \quad 8{\cdot}0}{D \quad 27}$; and if a number on ⅁ is divided by 100, the number under it on D must be multiplied by 10. Thus $\frac{⅁ \quad 92{\cdot}16}{D \quad 10} = \frac{⅁ \quad {\cdot}9216}{D \quad 1}$.

**Ex. 298.**—Divide 2800 by $14^2$, by $18^2$, and by $19^2$. (Here $\frac{2800 \times 1^2}{14}, \frac{2800 \times 1^2}{18}, \frac{2800 \times 1^2}{19}$.) Compare Ex. 254.

| | | | | | |
|---|---|---|---|---|---|
| ⅁ | 2800 | (28) | 14·3 | 8·64 | 7·75 |
| D | 1 | (10) | 14 | 18 | 19 |

Observe how the N.B. above applies in this case.

**Ex. 299.**—Divide 42 by $\sqrt{36}$, by $\sqrt{39}$, and by $\sqrt{196}$. (Here $\frac{\sqrt{1} \times 42}{\sqrt{36}}, \frac{\sqrt{1} \times 42}{\sqrt{49}}, \frac{\sqrt{1} \times 42}{\sqrt{196}}$.) Compare Ex. 255.

| | | | | | |
|---|---|---|---|---|---|
| ⅁ | 1 | (100) | 196 | 49 | 36 |
| D | 42 | (4·2) | 3 | 6 | 7 |

Observe how the N.B. above applies, and enables us to see at once what is over 3, 6, 7.

**Ex. 300.**—Solve $\frac{5^2 \times 108}{2^2}$, $\frac{5^2 \times 108}{2{\cdot}4^2}$, $\frac{5{\cdot}2 \times 108}{3^2}$.

| | | | | |
|---|---|---|---|---|
| B | 675 | 469 | 300 | 108 |
| D | 2 | 2·4 | 3 | 5 |

**Ex. 301.**—Divide $\sqrt{9216}$ by $\sqrt{4}$, by $\sqrt{16}$, and by $\sqrt{64}$, or Square Roots of $\frac{9216}{4}$, $\frac{9216}{16}$, $\frac{9216}{64}$. $\left(\text{Here } \frac{\sqrt{9216} \times 1}{\sqrt{4}}, \frac{\sqrt{9216} \times 1}{\sqrt{16}}, \frac{\sqrt{9216} \times 1}{\sqrt{64}}.\right)$

| | | | | | |
|---|---|---|---|---|---|
| B | 9216 | (92·16) | 64 | 16 | 4 |
| D | 1 | (10) | 12 | 24 | 48 |

**Ex. 302.**—Solve $\frac{\sqrt{144} \times 8}{\sqrt{36}}$, $\frac{\sqrt{144} \times 8}{\sqrt{64}}$, $\frac{\sqrt{144} \times 8}{\sqrt{256}}$.

| | | | | |
|---|---|---|---|---|
| B | 64 | 36 | 256 | 144 |
| D | 12 | 16 | 6 | 8 |

**Ex. 303.**—If Surveying chains of 33 feet, 50 feet, and 66 feet, are used, how many *square chains* of each go to one Acre? $\left(\text{Here } \frac{43560 \times 1^2}{33^2}, \frac{43560 \times 1^2}{50^2}, \frac{43560 \times 1^2}{66^2}.\right)$ Ex. 254.

| | | | | |
|---|---|---|---|---|
| B | 43560 | 40 | 17·42 | 10 |
| D | 1 | 33 | 50 | 66 |

**Ex. 304.**—A seconds' pendulum vibrating 60 times in a minute is 39·14 inches long. Since the lengths of pendulums vary *inversely as the squares* of their number of vibrations in a given time, what will be the length of two pendulums to vibrate respectively 120 and 240 times in one minute? $\left(\text{Here } \frac{60^2 \times 39{\cdot}14}{120^2}, \frac{60^2 \times 39{\cdot}14}{240^2}.\right)$

| | | | |
|---|---|---|---|
| B | 39·14 | 9·78 inches | 2·45 inches |
| D | 60 | 120 | 240 |

See also Examples 328, 329.

## Use of the four lines $\frac{\frac{A}{C}}{\frac{B}{D}}$ INVERTED.

If in the Formulæ (III.) $\frac{l \times n^2}{s}$, (IV.) $\frac{l \times n}{s^2}$, (V.) $\frac{\sqrt{l} \times \sqrt{n}}{\sqrt{s}}$, we see that in a "series" of Examples, the *two multipliers are constant* whilst the Divisor varies, the Slide must be inverted. (In other cases of a "series," it need not be inverted. See page 107.)

| | | | | |
|---|---|---|---|---|
| A | $c$ | (25) | | |
| C | $a$ | (23) | | |
| B | | | $b$ | (16) |
| D | | | $d$ | (6) |

The "Proportion" is $d^2 : c :: a : b$. Or $\sqrt{b} : \sqrt{a} :: \sqrt{c} : d$.

N.B. The two Multipliers must be set one over the other; the Square number whether $n^2$, or $s^2$, is always on D, and the Square Root *Divisor* always on B.

**Ex. 305.**—Solve $\frac{324}{7^2}$, $\frac{324}{8^2}$, $\frac{324}{9^2}$. Here we have $\frac{324 \times 1}{7^2}$, $\frac{324 \times 1}{8^2}$, $\frac{324 \times 1}{9^2}$ : where 234 and 1 are the two constant Multipliers. The divisor being a Square number, will be on D.

| | | | | |
|---|---|---|---|---|
| A | 324 | | | |
| C | 1 | | | |
| B | | 6·61 | 5·16 | 4 |
| D | | 7 | 8 | 9 |

**Ex. 306.**—Solve $\frac{2200 \times 1{\cdot}3}{14^2}$, $\frac{2200 \times 1{\cdot}3}{18^2}$, $\frac{2200 \times 1{\cdot}3}{20^2}$.

| | | | | |
|---|---|---|---|---|
| A | 2200 | | | |
| C | 1·3 | | | |
| B | | 14·6 | 8·86 | 7·15 |
| D | | 14 | 18 | 20 |

**Ex. 307.**—Solve $\frac{7·5^2 \times 16}{45}$, $\frac{7·5^2 \times 16}{55}$, $\frac{7·5^2 \times 16}{65}$.

| | | | | |
|---|---|---|---|---|
| A | 13·85 | 16·36 | 20 | |
| C | 65 | 55 | 45 | |
| B | | | | 16 |
| | | | | 7·5 |

**Ex. 308.**—Divide $24^2$ by 8, by 9, and by 16. (Here $\frac{1 \times 24^2}{8}$, $\frac{1 \times 24^2}{9}$, $\frac{1 \times 24^2}{16}$.)

| | | | | |
|---|---|---|---|---|
| A | 8 | 9 | 16 | |
| C | 72 | 64 | 36 | |
| B | | | | 1 |
| D | | | | 24 |

N.B. This may also be solved by the line A and C *inverted*, as follows:

| | | | | |
|---|---|---|---|---|
| A | 24 | 36 | 64 | 72 * |
| C | 24 | 16 | 9 | 8 |

**Ex. 309.**—Solve $\sqrt{\frac{6·4 \times 20}{5·8}}$, $\sqrt{\frac{6·4 \times 20}{8}}$, $\sqrt{\frac{6·4 \times 20}{14·2}}$; (or $\frac{\sqrt{6·4} \times \sqrt{20}}{\sqrt{5·8}}$, $\frac{\sqrt{6·4} \times \sqrt{20}}{\sqrt{8}}$, $\frac{\sqrt{6·4} \times \sqrt{20}}{\sqrt{14·2}}$.)

| | | | | |
|---|---|---|---|---|
| A | 6·4 | | | |
| C | 20 | | | |
| B | | 14·2 | 8 | 5·8 |
| D | | 3 | 4 | 4·7 |

**Ex. 310.**—Solve $\sqrt{6\frac{7}{8}}$, $\sqrt{\frac{55}{8·5}}$, $\frac{\sqrt{55}}{\sqrt{9}}$. (Here $\frac{\sqrt{1} \times \sqrt{55}}{\sqrt{8}}$,

* Ex. 307 is $\frac{24 \times 24}{8}$, $\frac{24 \times 24}{9}$, $\frac{24 \times 24}{16}$, as in Ex. 75.

$\frac{\sqrt{1} \times \sqrt{55}}{\sqrt{8\cdot5}}$, $\frac{\sqrt{1} \times \sqrt{55}}{9}$. Or divide the Square Root of 55, by 8, 8·5, and 9.)

| | | | | |
|---|---|---|---|---|
| A | 55 | | | |
| C | 1 | | | |
| B | | 9 | 8·5 | 8 |
| D | | 2·47 | 2·54 | 2·62 |

---

## LINE E, AND "CUBES."

[As some Slide Rules have not the line E, it will be shown in Examples 321, 322, how to find Cubes and Cube Roots *with the line D.* It is also to be observed that there are *some* questions involving Cubes and Cube Roots which the line E will *not* solve, but which the line D *will;* as Examples 318, 319, 320.]

When the Slide is taken out, and put in again with the other side of it, (having the E line on it) upwards, and *shut in even*, so that the 1 at the extreme left of E coincides with the 1 at the extreme left of D, the numbers on E will be the *Cubes* of the numbers under them on D; and the numbers on D will be the *Cube Roots* of the numbers over them on E.

The 1 at the extreme left of E will represent either 1, or ·001, or 1000, or 1 million; as under :—

| First 1 | Second 1 | Third 1 | Last 1 |
|---|---|---|---|
| 1 million | 10 millions | 100 millions | 1000 millions |
| 1,000 | 10,000 | 100,000 | 1 million |
| 1 | 10 | 100 | 1,000 |
| ·001 | ·01 | ·1 | 1 |

Thus, if we want $\sqrt[3]{250}$, we look under the *Third* radius, and find 6·3. If we want $\sqrt[3]{\cdot0229}$, we look under the *Second* radius, and find ·284. If we want $\sqrt[3]{2\cdot34}$, we look under the *First* radius, and find 1·35. If we want $\sqrt[3]{62,100000}$, we look under the *Second* radius, and find 396.

The following would be the readings if the Slide is *shut in even*, and the third 1 on E is considered as 100:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| E | 100 | 1000 | 10000 | 100000 | 1 mill. | 10 mill. | 100 mill |
| D | 4·64 | 10 | 21·54 | 46·4 | 100 | 215·4 | 464 |

So with the second 1 on E, as ·01; *shut in even.*

| | | | | | | |
|---|---|---|---|---|---|---|
| E | ·000234 | ·00234 | ·0234 | ·01 | ·234 | 2·34 |
| D | ·0616 | ·133 | ·286 | ·2154 | ·616 | 1·33 |

The Cubes of all numbers from 10 inclusive, to 46·41 inclusive, have five figures. Beyond this to 100 inclusive, they have six figures.

As the numbers on D increase or decrease by *tens*, those above them on E increase or decrease by *thousands*; and as the numbers on E increase or decrease by *tens*, those under them on D increase or decrease by *Cube Roots of tens*. (The Cube Root of 10 is 2·1544, and $\sqrt[3]{100} = 4·6416$, and $\sqrt[3]{1000} = 10$.)

Hence

Mult. $\begin{cases} \text{E by 1000, and D by } \sqrt[3]{1000}, \text{ or } 10. \\ \text{E by 100, and D by } \sqrt[3]{100}, \text{ or } 4·64. \\ \text{E by 10, and D by } \sqrt[3]{10}, \text{ or } 2·15. \end{cases}$ Div. $\begin{cases} \text{E by 1000, and D by } \sqrt[3]{1000}. \\ \text{E by 100, and D by } \sqrt[3]{100}. \\ \text{E by 10, and D by } \sqrt[3]{10}. \end{cases}$

Thus if we set 8 on E over 3 on D,

| | | | | | | |
|---|---|---|---|---|---|---|
| E | ·8 | 8 | 80 | 800 | 8000 | 8 mill. |
| D | 1·392 | 3 | 6·47 | 13·92 | 33 | 300 |

$$\text{Here } \frac{·8}{1·392^3} = \frac{8}{3^3} = \frac{80}{6·47^3} = \frac{800}{13·92^3} = \frac{8000}{30^3} = \frac{8 \text{ millions}}{300^3}.$$

(See also, the two last Examples under "Solidity of Spheres.")

If we have $\frac{\text{E}}{\text{D}} \frac{50 \qquad\qquad 500}{\qquad\qquad\quad 60}$, and wish to know what on D is under the 50 of E, it will not be $\frac{60}{10}$, or 6; but it will be $\frac{60}{\sqrt[3]{10}}$, or $\frac{60}{2·154} = 27·85$, as will be seen if tried with the Slide.

If we have $\dfrac{\text{E} \qquad \cdot5236 \qquad\qquad}{\text{D} \qquad\;\; 1 \qquad\qquad\quad}$, and wish to know what is under the 221 of E, the Slide apparently not being long enough,—we multiply D by 10, and E by 1000, as shown in the Rule, page 116, and read it $\dfrac{\text{E} \qquad\qquad\qquad\qquad\quad 523\cdot6}{\text{D} \qquad\qquad\qquad\qquad\qquad\; 10}$, and then we can easily read back to 221 of E, and find under it 7·5, the answer, on D.

The "Proportions" on the lines E and D, are as follows:

$$\frac{\text{E} \quad a \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad b}{\text{D} \quad c \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad d}$$

$$\begin{cases} d^3 : c^3 :: b : a, \text{—or } a = \dfrac{b \times c^3}{d^3} \\ c^3 : d^3 :: a : b, \text{—or } b = \dfrac{a \times d^3}{c^3} \end{cases}$$

$$\begin{cases} \sqrt[3]{b} : \sqrt[3]{a} :: d : c, \text{—or } c = \dfrac{d \times \sqrt[3]{a}}{\sqrt[3]{a}}, \text{ or } \sqrt{\dfrac{d^3 \times a}{b}} \\ \sqrt[3]{a} : \sqrt[3]{b} :: c : d, \text{—or } d = \dfrac{c \times \sqrt[3]{b}}{\sqrt[3]{a}}, \text{ or } \sqrt{\dfrac{c^3 \times b}{a}} \end{cases}$$

It will be seen from the above, that the line E with D, solves *two* forms of Equation, viz.: $x = \dfrac{m \times n^3}{s^3}$, and $x = \dfrac{m \times \sqrt[3]{n}}{s}$. The first term must always be a Cube or Cube Root, but 1 may always be made $1^3$ or $\sqrt[3]{1}$.

N.B. The line E is of no use in solving $x = \dfrac{n^3}{s}$, nor in finding the Square Root of a Cube, or the Cube Root of a Square: but the first of these may be solved by the lines A, B, C, D, as in Ex. 318, and the two others by the lines C, D, or g, D, as in Examples 319, 320.

**Ex. 311.**—Solve $x = 4{\cdot}2^3 \times 8$, or $x = \dfrac{4{\cdot}2^3 \times 8}{1}$.

$$\frac{\text{E} \quad 8 \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad 593}{\text{D} \quad 1 \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad 4{\cdot}2}$$

**Ex. 312.**—Solve $x = \frac{750}{5^3}$, or $\frac{1^3 \times 750}{5^3}$.

| | | |
|---|---|---|
| E | 6 | 750 |
| D | 1 | 5 |

**Ex. 313.**—Solve $x = \left(\frac{9}{12}\right)^3$, or $\frac{1 \times 9^3}{12^3}$.

| | | |
|---|---|---|
| E | 1 | ·421 |
| D | 12 | 9 |

**Ex. 314.**—Solve $x = \sqrt[3]{\frac{112}{14}}$, or $\frac{1 \times \sqrt[3]{112}}{\sqrt[3]{14}}$.

| | | |
|---|---|---|
| E | 14 | 112 |
| D | 1 | 2 |

**Ex. 315.**—Solve $x = \frac{4^3 \times 6}{5^3}$.

| | | |
|---|---|---|
| E | 3·07 | 6 |
| D | 4 | 5 |

**Ex. 316.**—Solve $x = \frac{\sqrt[3]{27} \times 8}{\sqrt[3]{64}}$.

| | | |
|---|---|---|
| E | 27 | 64 |
| D | 6 | 8 |

**Ex. 317.**—A shaft, 4 inches in diameter, will, at a given rate of revolution, transmit a 12-horse power. What diameter of shaft, at the same revolution, will suffice for a 38-horse power? (Here $12 : \sqrt[3]{38} :: 4 : x$.

| | | |
|---|---|---|
| E | 12 | 38 |
| D | 4 | 5·87 H. P |

N.B. Other Examples will be found under the heads of Solid Mensuration of Spheres and Cones, and at the end of APPENDIX E.

## Cases of "Cube" not solvable with the line E, but with A, B, C, D, or else with C, D.

**Ex. 318.**—Divide $8^3$ by 6. Solved as in Ex. 259 $\frac{8^2 \times 8}{6}$.

| | | |
|---|---|---|
| A | 6 | |
| B | 8 | |
| C | | 85·3 |
| D | | 8 |

**Ex. 319.**—Solve $\sqrt{5^3}$, in the form of $\sqrt{5 \times 5^2}$, as in Ex. 253.

| | | |
|---|---|---|
| C | 1 | 5 |
| D | ·5 | 11·2 |

**Ex. 320.**—Solve $\sqrt[3]{27^2}$. *Invert* the line ꓭ, and keep the Square number 27 on the D line.

| | | | |
|---|---|---|---|
| ꓭ | 9 | 1 | Ans. = 9. |
| D | 9 | 27 | |

This is rather an inconvenient way, but there is no other direct one. See N.B. after Ex. 321.

### Involution and Evolution of the Cube *by the line D.*

**Ex. 321.**—Find the Cube of 12. (Here $x = 12^2 \times 12$, as in Ex. 252.)

| | | |
|---|---|---|
| C | 12 | 1728 |
| D | 1 | 12 |

**Ex. 322.**—Find the Cube Root of 800. Invert the Slide, and see where two *similar numbers* are over and under each other, on the lines ꓭ and D.

| | | | |
|---|---|---|---|
| ꓭ | 800 | 9·28 | Ans. = 9·283. |
| D | 1 | 9·28 | |

N.B. In this case the uncertainty lies as to which of the 1's on the line D the given number on ꓭ is to be set over. If the 800 is set over

the 1 on the extreme *left*, the answer may seem to be either 20, or 4·31; the former being $\sqrt[3]{8000}$, and the latter $\sqrt[3]{80}$. It is only when the 800 is set over the *right* 1 (which in this instance s at the extreme *right* of D), that we get the answer at once.

**Ex. 323.**—Solve $4{\cdot}2^3 \times 8$ by the lines A, B, C, D. (See Ex. 259.)

This is easy with line E (Ex. 311), but cannot be done with line D, unless we happen to know the reciprocal of 8. In this case we have $x = \dfrac{4{\cdot}2^2 \times 4{\cdot}2}{{\cdot}125}$, as in Ex. 259.

| | | |
|---|---|---|
| A | ·125 | |
| B | 4·2 | |
| C | | 593 |
| D | | 4·2 |

---

## ACCELERATED MOTION.

A heavy body near the Earth's surface descends about 16·1 inches in the *first* second of time; that is *in vacuo:* and the spaces descended by falling bodies are proportional to the Squares of the times of the descents.

**Ex. 324.**—Through what space *in vacuo* will a body fall, in 3, 4, and 7 seconds respectively?

(Here $3^2 \times 16{\cdot}1$; $4^2 \times 16{\cdot}1$; $7^2 \times 16{\cdot}1$). (See Ex. 293.)

| | | | | |
|---|---|---|---|---|
| C | 16·1 | 145 ft. | 257·5 ft. | 789 ft. |
| D | 1 | 3 sec. | 4 sec. | 7 sec. |

**Ex. 325.**—How many seconds will a body be in falling 272 feet, and 360 feet *in vacuo?* (Here $\sqrt{\dfrac{272}{16{\cdot}1}}$ and $\sqrt{\dfrac{360}{16{\cdot}1}}$ both solvable at one setting of the Rule.)

| | | | |
|---|---|---|---|
| C | 16·1 | 272 ft. | 360 ft. |
| D | 1 | 4·11 sec. | 4·72 sec. |

N.B. The above Rule only applies to descent *in vacuo.* The resistance of the air increases greatly after the first 2 seconds : thus in Ex. 325 a bullet let fall 272 feet, would be 4¾ seconds, instead of 4·11 seconds in reaching the ground.

**Ex. 326.**—What *velocity* in feet per second, will a body have acquired after falling *in vacuo* through 360 feet ?

(Here $x = \sqrt{64{\cdot}4 \times 360}$, as in Ex. 258.)

| | | |
|---|---|---|
| C | 64·4 | 360 |
| D | 64·4 | 152·2 ft. |

**Ex. 327.**—What *velocity* in feet per second, will a body have acquired after falling *in vacuo* for 4·72 seconds ?

(Here $x = 4{\cdot}72 \times 32{\cdot}2$.)

| | | |
|---|---|---|
| A | 32·2 | 152·2 ft. |
| B | 1 | 47·2 |

## PENDULUMS.

The *times* of vibrations are as the Square Roots of lengths. The *number* of vibrations in a given time is inversely as the Square Roots of the lengths ; and the *lengths* are inversely as the Squares of the number of vibrations in a given time.

In the latitude of London, a "seconds' pendulum is 39·14 inches long. In Lat. 75° N. it is 39·2. On the Equator 39·01.

The following Formula (*Slide inverted,* p. 111) will be useful.

| | | | |
|---|---|---|---|
| B | 11 | 39·14 | Inches length |
| D | 35·8 | 60 | No. of vibr. in 1 minute |

Here *Inches length* $= \frac{39{\cdot}14 \times 60^2}{35{\cdot}8^2}$, and *No. of vibr. in* 1 *minute* $= \sqrt{\frac{39{\cdot}14 \times 60^2}{\text{Inches length}}}$.

**Ex. 328.**—How many vibrations in 1 minute will a 12-inch pendulum make? and how many inches long must a pendulum be to make *half-second* beats?

| | | | | |
|---|---|---|---|---|
| B | 11 | 39·14 | 12 | 9·78 inches |
| D | 35·8 | 60 | 108·4 vibr. | 120 |

**Ex. 329.**—Three military pendulums are required for "Slow" and "Quick" step, and "Double:" being respectively 75, 108, and 150 beats (a step or pace for a beat) per minute. Required their respective lengths.

| | | | | | |
|---|---|---|---|---|---|
| B | 11 | 39·14 | 25 | 12 | 6·26 inches |
| D | 35·8 | 60 | 75 | 108 | 150 vibr. |

## MENSURATION FORMULÆ.

(LINEAR.)

(For Examples, see Examples 330, &c.)

(I.)*

| | | | | | |
|---|---|---|---|---|---|
| A | 1 | 3·1416 | 22 | Circumference | Circ. $= d \times 3{\cdot}1416$. |
| B | .3183 | 1 | 7 | Diam. of Circle | Diam. $= c \times {\cdot}3183$. |

* In Formula I. the more exact multiplier is 3·141592636; but the ratio 7 : 22 is so near, that if we use it for a diameter of 791, it would make the circumference 2486; whereas with the more elaborate multiplier it would be 2484·99976076. So that with the proportion 7 : 22, if we add $\frac{1}{2486}$, or ·4024 per 1,000, to the circumference, it is exact.

(II.)

| | | | | | |
|---|---|---|---|---|---|
| A | ·1 | 1·1284 | 35 | Diam. of Circle | Side $= d \times$ ·8862. |
| B | ·8862 | 1 | 31 | Side of Square equal | Diam. $= s \times$ 1·1284. |

(III.)

| | | | | | |
|---|---|---|---|---|---|
| A | 1 | 1·414 | 99 | Diam. of Circle | Side $= d \times$ ·7071. |
| B | ·7071 | 1 | 70 | Side of inscr. Sq. | Diam. $= s \times$ 1·414. |

(IV.)

| | | | | | |
|---|---|---|---|---|---|
| A | 1 | 1·414 | *99 | Diag. of Square | Side $= d \times$ ·7071 |
| B | ·7071 | 1 | 70 | Side of Square | Diag. $= s \times$ 1·414. |

(V.)

| | | | | | |
|---|---|---|---|---|---|
| A | 1 | 3·545 | 39 | Circumf. of Circle | Side $= c \times$ ·2821. |
| B | ·2821 | 1 | 11 | Side of Square equal | Circ. $= s \times$ 3·545. |

(VI.)

| | | | | | |
|---|---|---|---|---|---|
| A | 1 | 4·442 | 40 | Circumf. of Circle | Side $= c \times$ ·2251. |
| B | ·2251 | 1 | 9 | Side of inscr. Square | Circ. $= s \times$ 4·442. |

(VII.)

| | | | | | |
|---|---|---|---|---|---|
| C | ·5 | 1 | 2 | *Area* of Squar | Area = Diag.$^2 \div 2$. |
| D | 1 | 1·414 | 2 | Diag. of Square | Diag. = $\sqrt{Area \times 2}$. |

---

*For sides of Triangles*, see Examples 276, 277, 277½.

(VIII.)

| | | | | | |
|---|---|---|---|---|---|
| A | ·5774 | 1 | 30 | Radius of Circle | Side of Tr. = Rad. $\times$ 1·732. |
| B | 1 | 1·732 | 52 | Side of inscr. equil. Tr. | Rad. $= s \times$ ·5774. |

---

* This $\frac{99}{70}$ is the same value as the $\frac{49}{21}$ of page 103.

(IX.)

| | | | | | |
|---|---|---|---|---|---|
| A | 3·464 | 1 | 52 | Side of Equilat. Tr. | Rad. = Side of Tr. × ·2887. |
| B | 1 | ·2887 | 15 | Rad. of inscr. Circle | Side = Rad. × 3·464. |

(X.)

| | | | | | |
|---|---|---|---|---|---|
| A | 1 | 1·52 | 76 | Side of Equilat. Tr. | Side of Sq. = Side of Tr. × ·658. |
| B | ·658 | 1 | 50 | Side of Square equal | Side Tr. = Side Sq. × 1·52. |

---

*The following Polygons are Equilateral.*

(XI.)

| | | | | | |
|---|---|---|---|---|---|
| A | 1 | 1·453 | 61 | Side of Pentagon | Rad. = Side × ·6882. |
| B | ·6882 | 1 | 42 | Rad. of inscr. Circle | Side = *r* × 1·453. |

(XII.)

| | | | | | |
|---|---|---|---|---|---|
| A | 1 | 1·1547 | 52 | Side of Hexagon | Rad. = Side × ·866. |
| B | ·866 | 1 | 45 | Rad. of inscr. Circle | Side = *r* × 1·1547. |

(XIII.)

| | | | | | |
|---|---|---|---|---|---|
| A | ·9631 | 1 | 80 | Side of Heptagon | Rad. = Side × 1·038. |
| B | 1 | 1·038 | 83 | Rad. of inscr. Circle | Side = *r* × ·9361. |

(XIV.)

| | | | | | |
|---|---|---|---|---|---|
| A | ·8284 | 1 | 53 | Side of Octagon | Rad. = Side × 1·207. |
| B | 1 | 1·2071 | 64 | Rad. of inscr. Circle | Side = *r* × ·8284. |

For 10 sides $\frac{\text{A} \quad 1 \quad \text{Side}}{\text{B} \quad 1{\cdot}539 \quad \text{Rad.}}$; for 12 sides $\frac{\text{A} \quad 1 \quad \text{Side}}{\text{B} \quad 1{\cdot}866 \quad \text{Rad.}}$.

N.B. 1st. In the above Formulæ for Equilateral Polygons, remember that the "radius of an inscribed circle" is the perpendicular from the centre of a polygon to the middle of any one side, as DC in Ex. 337.

(XV.)

| | | | | | |
|---|---|---|---|---|---|
| A | 1·236 | 1 | 58 | Rad. of circumscr. Pent. | Rad. of Pent. $= r \times 1{\cdot}236$. |
| B | 1 | ·8093 | 47 | Radius of Circle | Rad. of Circ. $= r$ of P. $\times {\cdot}8093$. |

(XVI.)

| | | | | | |
|---|---|---|---|---|---|
| A | 1 | 1·1755 | 40 | Side of Pentagon | Side = Rad. $\times 1{\cdot}1755$. |
| B | ·8506 | 1 | 34 | Rad. of circumscr. Circle | Rad. = Side $\times {\cdot}8506$. |

(XVII.)

| | | | |
|---|---|---|---|
| A | 1 | Side of Hexagon | Same. |
| B | 1 | Rad. of circumscr. Circle | |

(XVIII.)

| | | | | | |
|---|---|---|---|---|---|
| A | 1 | ·8678 | 46 | Side of Heptagon | Side = Rad. $\times {\cdot}868$. |
| B | 1·1524 | 1 | 23 | Rad. of circumscr. Circle | Rad. = Side $\times 1{\cdot}152$. |

(XIX.)

| | | | | | |
|---|---|---|---|---|---|
| A | 1 | ·7654 | 13 | Side of Octagon | Side = Rad. $\times {\cdot}7654$. |
| B | 1·307 | 1 | 17 | Rad. of circumscr. Circle | Rad. = Side $\times 1{\cdot}307$. |

For 10 sides $\dfrac{A \quad {\cdot}618 \quad \text{Side}}{B \quad 1 \quad \text{Rad.}}$; for 12 sides $\dfrac{A \quad {\cdot}518 \quad \text{Side}}{B \quad 1 \quad \text{Rad.}}$.

N.B. 2d. In the above seven Formulæ, the "radius of a circumscribing circle" is the distance from the centre of a polygon to any one of its corners, as DB in Ex. 337.

N.B. 3d. In the case of a *Rectangle,* where the sides $a$ and $b$ are given, the *diagonal* $= \sqrt{a^2 + b^2}$ as in Ex. 276.

**Ex. 330.**—What will be the circumference in inches of four circles whose diameters are 29·6, 42·7, 113, and 159?

(I.)

| | | | | | |
|---|---|---|---|---|---|
| A | 22 | 93 | 132 | 355 | 500 |
| B | 7 | 29·6 | 42·7 | 113 | 159 |

**Ex. 331.**—What length in inches must each side of a square pipe be, to make its area equal to that of a round pipe of one foot diameter?

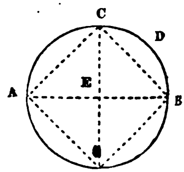
| | | | |
|---|---|---|---|
| (II.) | A | 12 | 35 |
| | B | 10·634 inches | 31 |

**Ex. 332.**—What is the distance in feet from the centre to the corner of a tent 16 feet square?

| | | | |
|---|---|---|---|
| (IV.) | A | 22·6 feet = diagonal | 99 |
| | B | 16 | 70 |

whence from the *centre* the answer is 11·3 feet.

**Ex. 333.**—In the figure, if EB the *radius* of the circle be $3\frac{1}{2}$ inches, what will be the length of AC?




| | | | |
|---|---|---|---|
| (III.) | A | 7 = diameter | 99 |
| | B | 4·95 inches = AC | 70 |

**Ex. 334.**—In the preceding figure, let the circumference be that of a log of timber, measuring 39 inches round. What is the difference between its "Quarter-girt" CDB (9·75 inches) and one of its sides AC, when *squared*?*

| | | | |
|---|---|---|---|
| (VI.) | A | 39 | 40 |
| | B | 8·78 = AC | 9 |

Hence *the difference* is ·97 inches.

**Ex. 335.**—If the diagonal of a square piece of ground is 37·4 feet, how many square feet will there be in it?

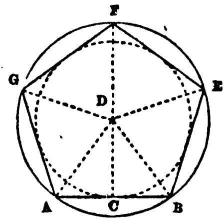
| | | | |
|---|---|---|---|
| (VII.) | C | 2 | 700 Square feet |
| | D | 2 | 37·4 |

* The "Squared side" may always be taken as $\frac{9}{10}$ of the "Quarter-girt."

**Ex. 336.**—If a circle has a radius of 45, what will be the length of each side of an *Equilateral* Triangle inscribed within it ?

| | | | |
|---|---|---|---|
| (VIII.) | A | 30 | 45 |
| | B | 52 | 78 = side |

**Ex. 337.**—If the side of a Pentagon, as AB in the figure, is 40, what will be the distance from D to C ? (*i.e.* the radius of an inscribed circle.)




| | | | | |
|---|---|---|---|---|
| (XI.) | A | 40 = AB | 61 | or Side × ·6882 |
| | B | 27·5 = DC | 42 | |

**Ex. 338.**—If the side of a Pentagon, as AB in the above figure, is 25, with what length AD, BD, must arcs cutting each other at D, be described from centres A and B, so as to find the centre of a circle within which the Pentagon is to be set out ?

| | | | |
|---|---|---|---|
| (XVI.) | A | 25 = AB | 40 |
| | B | 21·26 = BD | 34 |

**Ex. 338½.**—If a given circle have a radius DB of 51, what will be the length of each side of a regular Pentagon to be drawn within it ?

| | | | |
|---|---|---|---|
| (XVI.) | A | 40 | 60 = AB |
| | B | 34 | 51 = DB |

**Ex. 339.**—If a given circle, such as the dotted one in the preceding figure, have a radius DC of 30, and it is required to draw a regular Pentagon *outside* it, what must be the radius DB of an outside circle, which the angles of the outside Pentagon shall touch at five points in its circumference ?

$$\text{(XV.)}\ \frac{\text{A} \qquad 37{\cdot}1 = \text{DB} \qquad\qquad\qquad\qquad 58}{\text{B} \qquad 30 = \text{DC} \qquad\qquad\qquad\qquad 47}$$

N.B. In Ex. 339, when drawing a Pentagon outside a given circle, having found DB = 37·1, we describe a circle with this radius, and then find the lengths of AB, BE, &c. from Formula (XVI.) to be 43·6 each, as follows :— $\frac{\text{A} \quad 40 \qquad\qquad 43{\cdot}6}{\text{B} \quad 34 \qquad\qquad 37{\cdot}1}$, and we then step off the sides on the circumference.

**Ex. 340.**—Suppose a circular piece of ground 1360 yards in diameter, has its circumference marked out by 8 flags at equal distances, how many yards apart will the flags be ?

$$\frac{\text{A} \qquad 13 \qquad\qquad\qquad\qquad 520 \text{ yards} = \text{Ans.}}{\text{B} \qquad 17 \qquad\qquad\qquad\qquad 680 = \text{semi-diam.}}$$

*Notes.*

In drawing Regular Polygons the above Formulæ are often useful (Ex. 339 and N.B.).

To find the central angle ADB in Ex. 337 of *any* Equilateral Polygon, divide 360° by the number of sides. Thus in a Pentagon ADB $= \frac{360}{5} = 72°$, and CDB = 36°. In a Heptagon the central angle $= \frac{360}{7} = 51{\cdot}43°$. In a Pentagon (Ex. 337), if AB = 1, then DC = ·6882 and DB = ·8506. If DB = 1, then AB = 1·1755, and DC = ·8090. If DC = 1, then AB = 1·453, and DB = 1·236.

*The following may also be found useful.*

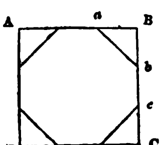



FIG. 1.

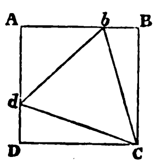



FIG. 2.

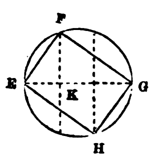



FIG. 3.

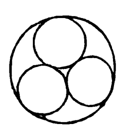


FIG. 4.

**Ex. 341.**—Let ABCD, fig. 1, be a square piece of board of 87 inches to a side. Required the length in inches, of the side ($bc$) of the largest Octagon that can be cut from it. (Here $x = 87 \times \cdot 4143$.)*

| | | |
|---|---|---|
| A | ·4143 | 36 = $bc$ |
| B | 1 | 87 |

**Ex. 342.**—Let fig. 2 be a square, each side of which is 27. What is the length $db$ of each side of an Equilateral Triangle † inscribed within it? ($x = 27 \times 1 \cdot 0353$.)

| | | |
|---|---|---|
| A | 1·0353 | 28 |
| B | 1 | 27 |

**Ex. 343.**—In fig. 3, let EFGH be a Rectangle, one of whose sides is 10·4, and the other 14·7. Required its diagonal EG. (This is, to find from the Hypothenuse EG of a right angled triangle, the sides EF, GH, as in Ex. 276 and N.B. 3, page 125) $x = \sqrt{10 \cdot 4^2 + 14 \cdot 7^2}$.

| | | | | |
|---|---|---|---|---|
| C | 1 | 108 | 216 | 324 = 108 + 216 |
| D | 1 | 10·4 | 14·7 | 18 = Ans. |

**Ex. 344.**—Let fig. 3 represent the surface end of a Cylinder whose diameter EG is 18 inches. Required the two sides EF, GF, of the strongest rectangular beam that can be cut from it. ($EF = \frac{18}{1 \cdot 732}$ and $GF = \frac{18}{1 \cdot 732} \times 18$.)‡

| | | | |
|---|---|---|---|
| A | 10·4 = EF | 14·7 = GF | 18 |
| B | 1 | 1·414 | 1·732 |

or, we may take $EF = \frac{d}{\sqrt{3}}$, and $GF = \frac{d \times \sqrt{2}}{\sqrt{3}}$.

---

* In fig. 1, if the side BC = 1, B$b$ = ·29285, and $bc$ = ·4143.

† In fig. 2, if the side BC = 1, C$b$ = 1·0353, and B$b$ = ·268, and the angle B$cb$ = 15°; as also angle DC$d$.

‡ To *construct* the sides required in Ex. 344, divide EG into three parts, and from K raise the perpendicular KF. Join FE and FG, which will be the two required sides.

| C | 1 | 2 | 3 |
|---|---|---|---|
| D | 10·4 = EF | 14·7 = GF | 18 |

**Ex. 345.**—Within a circle having a diameter of 25 inches, it is required to have either three or four circles described (fig. 4). Required the diameter of each circle if there are three, or if there are four, $x = 25 \times {\cdot}416$, and $y = 25 \times {\cdot}464$.

| A | 10.4 four | 11·6 three | |
|---|---|---|---|
| B | ·416 | ·464 | 1 |

**Ex. 346.**—A circular Safety valve 7 inches diameter, is to be replaced by two smaller ones, which *together* are to have the same area of aperture as the 7-inch one, and both to be the same size. Required the diameter of each. (Here $x = \frac{7}{\sqrt{2}}$, as in Ex. 255.)

| C | 1 | 2 |
|---|---|---|
| D | 4·95 | 7 |

Each will then have an *area* of $19\frac{1}{4}$ square inches; the original 7-inch aperture having an area of $38\frac{1}{2}$.

**Ex. 347.**—A circular Safety valve of 7 inches diameter (as in Ex. 346), is to be replaced by two smaller ones which *together* are to have the same area of aperture as the 7-inch one; but the *area* of one is to be $\frac{2}{3}$ the area of the other. Required the diameter of each. (Here $x = \sqrt{\frac{7^2 \times 2}{2 + 3}}$; and $y = \sqrt{\frac{7^2 \times 3}{2 + 3}}$; as in Ex. 257; and both solved at *one setting* of the Slide.)

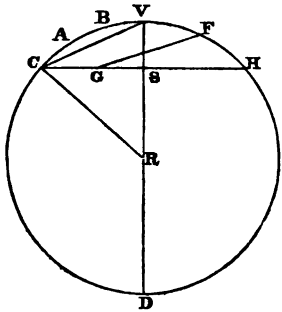
| C | 2 | 3 | 5 (= 2 + 3) |
|---|---|---|---|
| D | 4·43 = $x$. | 5·42 = $y$. | 7 |

Their *Areas* will be respectively 15·4 and 23·1 square inches, or a total of $38\frac{1}{2}$, the Area of the 7-inch aperture. Here 15·4 : 23·1 :: 2 : 3.

## CHORDS, ARCS, VERSED SINES, RADII.

(Linear Measure.)

| | | |
|---|---|---|
| V D. | Diameter | = 100. |
| C H. | Chord | = 80. |
| C S. | $\frac{1}{2}$ Chord | = 40. |
| V S. | Ht. of Arc | = 20. |
| C V. | Chord of $\frac{1}{2}$ Arc | = 44·72. |
| V R. | Radius | = 50. |
| C R. | Radius | = 50. |
| DS. = VD − VS | | = 80. |




$$\text{I. } CV = \sqrt{VS \times VD} = \sqrt{20 \times 100} = \sqrt{2000} = 44{\cdot}72.$$

$$\text{II. } CV = \sqrt{CS^2 + VS^2} = \sqrt{40^2 + 20^2} = \sqrt{1600 + 400} = \sqrt{2000} = 44{\cdot}72.$$

$$\text{III. } DS = \frac{CS^2}{VS} = \frac{40^2}{20} = \frac{1600}{20} = 80.$$

$$\text{IV. } VD = DS + VS = 80 + 20 = 100. \quad \text{Or } \frac{CH^2 \times {\cdot}25}{VS} + VS.$$

$$\text{V. } VD = \frac{CV^2}{VS} = \frac{44{\cdot}72^2}{20} = \frac{2000}{100} = 100. \quad \text{Or Rad.} = \frac{CV^2 \times {\cdot}5}{VS}.$$

$$\text{VI. } VS = \frac{CV^2}{VD} = \frac{44{\cdot}72^2}{100} = \frac{2000}{20} = 20. \quad \text{Or } CR - \sqrt{CR^2 - CS^2}.$$

$$\text{VII. } VS = \sqrt{(CV + CS) \times (CV - CS)} = \sqrt{84{\cdot}72 \times 4{\cdot}72} = \sqrt{400} = 20.$$

$$\text{VIII. } CS = \sqrt{(CV + VS) \times (CV - VS)} = \sqrt{64{\cdot}72 \times 24{\cdot}72} = \sqrt{1600} = 40.$$

$$\text{IX. } CH = 2 \times \sqrt{(VD - VS) - VS} = 2\sqrt{80 \times 20} = 2\sqrt{1600} = 80. \quad \text{Or } 2 \times \sqrt{(CV \div VS) \times (CV - VS)}.$$

X. $RS = \sqrt{(CR + CS) \times (CR - CS)} = \sqrt{90 \times 10} = \sqrt{900} = 30$.

XI. CABVFH. *Length of Arc* $= \dfrac{\text{Radius} \times \text{Arc or Angle}}{57{\cdot}3}$, or

$$\frac{(8 \times CV) - CH}{3} = \frac{(8 \times 44{\cdot}72) - 80}{3} = 92{\cdot}6.$$

XII. CRV. *Degrees of Arc* $= \dfrac{57{\cdot}3 \times \text{length of Arc}}{\text{Radius}} = 106°$.

**Ex. 348.**—If in a bridge, the Chord CH = 240 feet, and the rise of the crown or Versed Sine = 34 feet, what is the length of the curve CABVFH? First find CV the Chord of $\frac{1}{2}$ the Arc, which is $\sqrt{120^2 + 34^2} = 125$ feet, as in Ex. 276. Then use Formula XI.: $x = \dfrac{(8 \times CV) - CH}{3} = \dfrac{1000 - 240}{3} = \dfrac{760}{3} = 253\frac{1}{3}$ feet for the Answer. The correct answer is $252\frac{7}{12}$.

N.B. The same may be obtained very nearly, by *construction.* Take CB = $\frac{1}{2}$ CH, and measure this distance as a Chord from H *to* G. Join GB, and its length is equal to *half* the length of the curve CABVFH.

**Ex. 349.**—On a plan, the Chord CH of a railway curve measured 37 inches, and its Versed Sine or VS was 5 inches. With what *radius* in inches was the curve set out? Here since CV $= \dfrac{37}{2}$ or 18·5, we have by Formula III., $DS = \dfrac{18{\cdot}5^2}{5} = 68{\cdot}45$, to which if we add VS = 5, we have 73·45 for the diameter, and 36.725 for the *Radius* or answer.

(*The following require a "Table of Natural Sines."*)

When the length of the Radius is given, let A = any angle, and C = its Chord (in terms of the Radius):

Then C = (Natural Sine $\frac{1}{2}$ A) × 2; and Natural Sine $\frac{1}{2}$ A = $\dfrac{\text{Chord}}{2}$.

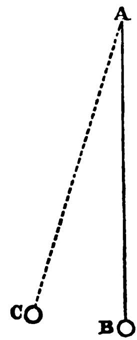
**Ex. 350.**—Required the length of a Chord to an arc or angle of 30°.

The Natural Sine of $\frac{1}{2}$ A, *i.e.* Natural Sine of 15° is ·259. Double this, and we have ·518. The required Chord, then, is ·518 times the Radius, whatever that may be.

**Ex. 351.**—Radius being 1·0, the Chord given is ·766. What is the angle that it subtends?

Here $\frac{\cdot 766}{2} = \cdot 383$, which is the Natural Sine of $22\frac{1}{2}°$; whence the angle subtended is $2 \times 22\frac{1}{2}$ or 45°.

**Ex. 352.**—If a Pendulum AB 8 inches long swings to C, and BC is measured 4·8 inches, what is the angle BAC?




Here $\frac{4 \cdot 8}{4} = \cdot 6$, *i.e.* the Chord is ·6 of the Radius. Half this is ·3, which is the Natural Sine of $17\frac{1}{2}°$. Double this is 35°, the angle required. Or we may obtain the ·3 sooner, by dividing 4·8 by double the Radius or $\frac{4 \cdot 8}{16} = \cdot 3$.

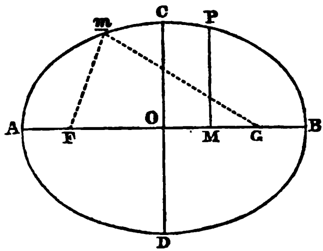
**Ex. 353.**—Given in the fig., page 131, the Chord CH = 326; and the Versed Sine VS = 97. Required the "length of the Arc" CABVTH.

Find DS (III. page 131) to be $\frac{163^2}{97} = 274$; whence we get the Diameter DV = 274 + 97 = 371; and the Radius RC = 185·5. But CS is given $\frac{326}{2}$ or 123. Then if RC = 1, CS = ·879, and is the Natural Sine of $61\frac{1}{2}°$, or CRV. Double this, or 123°, is the *Degrees of Arc* or angle CRH; and then by Formula XI. page 131, $\frac{185 \cdot 5 \times 123°}{57 \cdot 3} = 398$, the *length of the Arc* CABVFH.

N

## ELLIPSES. (Linear Measure.)

To describe an Ellipse, take a thread F$m$G of the length of the Axis major; and to such Foci F, G, as may be selected, fasten the ends of the thread. Stretch the thread as far as it will go to $m$ with the point of a pencil, and keeping it at the stretch, keep on tracing out the circumference with the pencil-point. The nearer the Foci, the nearer the Ellipse will be to a Circle.




In the figure, let the Axis major AB = 12. Axis minor CD = 9. Semi-Axis major AO = 6. Focal distance FO = 4 (nearly). One Ordinate PM = 4·24 Longer Absciss AM = 8. Shorter Absciss MB = 4.

FC is the "mean" distance of either Focus from the Circumference, and is = AO the Semi-Axis major.

The following Formulæ (as can be shown from the above figure) may be useful. They may all be readily worked out by the Slide Rule.

I. *Ellipticity* $= \dfrac{\text{Axis major} - \text{Axis minor}}{\text{Axis major}}$, or $\dfrac{AB - CD}{AB}$, or $\dfrac{12 - 9}{12} = \dfrac{3}{12} = \dfrac{1}{4} = {\cdot}25.$

II. *Eccentricity* $= \dfrac{\text{Focal distance}}{\text{Semi-Axis major}} = \dfrac{FO}{AO}$, or $\frac{4}{6} = \frac{2}{3} = {\cdot}666$ nearly: but more exactly as follows: $\sqrt{1 - \dfrac{\text{Axis major}^2}{\text{Axis minor}^2}}$, or $\sqrt{1 - \dfrac{AB^2}{CD^2}}$, or $\sqrt{1 - \dfrac{81}{144}} = \sqrt{1 - 5625} = \sqrt{{\cdot}4375} = {\cdot}6614.$

II. *Focal distance* FO $= \sqrt{\text{Axis major}^2 - \text{Axis minor}^2}$ $= \sqrt{144 - 81}$, or $\sqrt{21 \times 3} = 3{\cdot}969$, as in Ex. 258.

IV. *Circumference* = (Axis major + Axis minor) × 1·58 or

$(12 + 9) \times 1{\cdot}58 = 21 \times 1{\cdot}58 = 33{\cdot}18$; but more exactly as follows:
$\left(\frac{AB + CD}{2} + \sqrt{\frac{AB^2 + CD^2}{2}}\right) \times 1{\cdot}5708$, or $\left(\frac{21}{2} + \sqrt{\frac{144+81}{2}}\right)$
$\times 1{\cdot}5708 = (10{\cdot}5 + \sqrt{112{\cdot}5}) \times 1{\cdot}5708 = (10{\cdot}5 + 21{\cdot}11) \times 1{\cdot}5708$
$= 21{\cdot}11 \times 1{\cdot}5708 = 33{\cdot}1596 =$ Circumference.

**Ex. 354.**—In the figure, given the two Axes, AB and CD = 12 and 9 respectively; as also the two Abscisses AM and MB 8 and 4 respectively. Required their Ordinate PM.

Here *Ordinate* $= \frac{\text{Axis minor} \times \sqrt{\text{absc.} \times \text{absc.}}}{\text{Axis major}} = \frac{CD \times \sqrt{AM \times MB}}{AB}$

1st. $\sqrt{8 \times 4}$, as in Ex. 258.

| | | |
|---|---|---|
| C | 4 | 8 |
| D | 4 | 5·657 |

2d. $\frac{9 \times 5{\cdot}657}{12}$, as in Ex. 37.

| | | |
|---|---|---|
| A | 9 | 4·243 |
| B | 12 | 5·657 |

**Ex. 355.**—Given, the two Axes, 12 and 9, also the Ordinate PM = 4·243. Required the Abscisses, AM, MB. (Here OM the distance from the centre must first be found, and added to AO.)

$$OM = \frac{AB \times \sqrt{(CO + PM) \times (CO - PM)}}{CD} = \frac{12 \times \sqrt{8{\cdot}743 \times {\cdot}257}}{9}.$$

1st. $\sqrt{8{\cdot}743 \times {\cdot}257}$, as in Ex. 258.

| | | |
|---|---|---|
| C | ·257 | 8·743 |
| D | 1·5 | 8·743 |

2d. $\frac{12 \times 1{\cdot}5}{9}$, as in Ex. 37.

| | | |
|---|---|---|
| A | ·0 = OM | 12 |
| B | 1·5 | 9 |

Then $AO + OM = 6 + 2 = 8,$ = *Longer Absciss.*

$BO - OM = 6 - 2 = 6,$ = *Shorter Absciss.*

**Ex. 356.**—Given the Ordinate $PM = 4·243$, and the two Abscisses $AM$, $MB$, = 8 and 4. Required $CD$ the Axis minor. (The Axis major of course $= 8 + 4$.)

$$CD = \frac{AB \times PM}{\sqrt{AM \times MB}} = \frac{12 \times 4·243}{\sqrt{8 \times 4}}.$$

Find $\sqrt{8 \times 4}$ as in Ex. 258, to be 5.657. Then $\frac{12 \times 4·243}{5·657} = 9$ = CD the *Axis minor.*

**Ex. 357.**—Given, the Ordinate $PM = 4·243$; its lesser Absciss $MB = 4$; and the minor Axis $CD = 9$. Required $AB$ the major Axis.

$$AB = \frac{(CO + \sqrt{(CO + PM) \times (CO - PM)}) \times (CD \times MB)}{PM^2}$$

$= \frac{(4·5 + \sqrt{8·743 \times = ·257}) \times (9 \times 4)}{4·243^2}$. First find $\sqrt{8·743 \times ·257}$ as in Ex. 258 to be 1·5. Then $\frac{6 \times 36}{4·243^2}$, as in Ex. 260.

$$\left\{\begin{array}{lll} A & 6 & \\ \hline B & 12 = AB & \\ C & & 36 \\ \hline D & & 4·243 \end{array}\right.$$

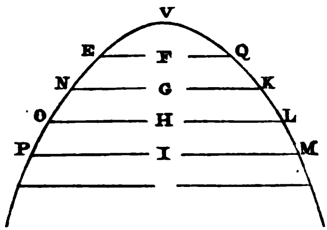
**Ex. 358.**—If the "mean distance" $CF$ of the Earth from the Sun is 25 millions of miles, and the *Eccentricity* of its orbit = ·0168; what will be the shortest distance $AF$ that the Earth is from the Sun, and what its farthest distance $FB$?

$95 - (95 \times ·0168) = 95 - 1·6 = 93·4$ millions $= AF.$

$95 + (95 \times ·0168) = 95 + 1·6 = 96·6$ millions $= FB.$

## PARABOLAS. (Linear Measure.)

EFQ is the Parameter, which always passes *through the focus* F; and in the figure, is four times VF. The Abscisses vary as the Squares of the Ordinates; and the Ordinates vary as the Square Roots of the Abscisses. VF, VG, VH, &c. are the Abscisses.




**Ex. 359.**—In the figure, VF = 3·5 and FQ = 7. If VG = 7, VH = 10·5, and VI = 14, what will be the length of the respective Ordinates GK, HL, IM?

| | | | | |
|---|---|---|---|---|
| C | 3·5 = VF | 7 = VG | 10·5 = VH | 14 = VI Absc. |
| D | 7 = FQ | 9·9 = GK | 12·1 = HL | 14 = IM Ord. |

$$\text{Absciss, as VH} = \frac{\text{Ord. HL}^2}{\text{Param. EFQ}}.$$

$$\text{Ordinate, as HL} = \sqrt{\text{Absc. VH} \times \text{Param. EFQ}}.$$

$$\text{Parameter EFQ} = \frac{\text{FQ}^2}{\text{VF}}.$$

The length of an arc $= 2 \times \sqrt{\text{Ord.}^2 + \frac{\text{Absc.}^2 \times 4}{3}}$. Thus if HL = 12·1 and HV = 10·5, the length of the arc OVL $= 2 \times \sqrt{12{\cdot}1^2 + \frac{10{\cdot}5^2 \times 4}{3}} = 17{\cdot}16$.

## SQUARE. (AREA-MEASURE.)

*See also under* "LAND-MEASURING."

(I.)

| | | | |
|---|---|---|---|
| C | 1 | Area | Area = Side². |
| D | 1 | Side of Square | Side = $\sqrt{\text{Area}}$. |

(II.)

| | | | |
|---|---|---|---|
| C | 1 | Area in Sq. yds. | Area in Sq. yds. $\frac{\text{Side}^2}{3^2}$. |
| D | 3 | Side in feet | Side in ft. = $\sqrt{\text{Area in yds.} \times 3^2}$. |

(III.)

| | | | |
|---|---|---|---|
| C | 1 | Area in Sq. ft. | Area in Sq. ft. = $\frac{\text{Side}^2}{12^2}$. |
| D | 12 | Side in inches | Side in in. = $\sqrt{\text{Area in ft.} \times 12^2}$. |

(IV.)

| | | | | | |
|---|---|---|---|---|---|
| C | ·5 | 1 | 2 | Area | Area = $\frac{\text{Diag.}^2}{2}$. |
| D | 1 | 1·414 | 2 | Diagonal | Diag. = $\sqrt[2]{\frac{\text{Area} \times 2^2}{2}}$. |

**Ex. 360.**—I have a square piece of board whose surface is 2 Square feet. Required the length of each side *in inches* ($x = \sqrt{2 \times 12^2}$, as in Ex. 253).

| (III.) | | |
|---|---|---|
| C | 1 | 2 |
| D | 12 | 17 inches |

**Ex. 361.**—What is the area in Square inches of a Square whose diagonal is 25·46 inches? $\left(x = \frac{25{\cdot}46^2}{2}.\right)$

| (IV.) | | |
|---|---|---|
| C | 2 | 324 Sq. in. |
| D | 2 | 25·46 |

N.B. Here $\frac{25{\cdot}46^2}{2}$ is adapted to the lines C, D in the form of $\frac{25{\cdot}46^2 \times 2}{2^2}$; otherwise, if we do not care to adhere to Formula IV., we may solve it thus $\frac{\text{A} \quad 25{\cdot}46 \qquad\qquad 324}{\text{B} \quad 2 \qquad\qquad 25{\cdot}46}$, as in N.B. to Ex. 261. But the lines C, D are more useful when the area is given, and the diagonal required.

---

## RECTANGLE—RHOMBUS—RHOMBOID. (Area-measure.)

*See also under* "Land-measuring."

(V.)

$$\frac{\text{A} \quad \text{Length} \qquad\qquad \text{Area}}{\text{D} \quad 1 \qquad\qquad \text{Perpendicular}}, \quad \text{Area} = \text{length} \times \text{height}.$$

(VI.)

$$\frac{\text{A} \quad \text{Yards long} \qquad\qquad \text{Area in Sq. ft.}}{\text{B} \quad 4 \qquad\qquad \text{Inches wide}} \quad \text{Area in Sq. ft.} = \frac{\text{Yards} \times \text{inches}}{4}.$$

(VII.)

$$\frac{\text{A} \quad \text{Feet long} \qquad\qquad \text{Area in Sq. yds.}}{\text{B} \quad 9 \qquad\qquad \text{Feet perpendicular}} \quad \text{Area in Sq. yds.} = \frac{\text{Feet} \times \text{Feet}}{9}.$$

(VIII.)

$$\frac{\text{A} \quad \text{Inches long} \qquad\qquad \text{Area in Sq. ft.}}{\text{B} \quad 144 \qquad\qquad \text{Inches perpendicular}} \quad \text{Area in Sq. ft.} = \frac{\text{Inches} \times \text{Inches}}{144}.$$

N.B. In a *Rhombus* the Area $= \frac{1}{2}$ the product of the Diagonals.

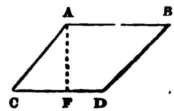
**Ex. 362.**—If in a Rectangular figure, having an area of 25 Square feet, the length is known to be 6 ft. 8 in., what will the breadth be in feet?

| | | | |
|---|---|---|---|
| (V.) | A | 6·67 | 25 |
| | B | 1 | 3·75 ft. |

**Ex. 363.**—How many *Square feet* are there in a piece of papering 12 yards long and 21 inches wide ?

| | | | |
|---|---|---|---|
| (VI.) | A | 12 | 68 Sq. ft. |
| | B | 4 | 21 |

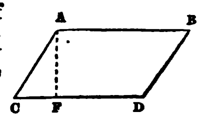
**Ex. 364.**—In the figure (a *Rhombus*), if each side is 26 feet, and the perpendicular A F 18 feet, what is the area in Square feet ?



| | | | |
|---|---|---|---|
| (V.) | A | 26 | 468 Sq. ft. |
| | B | 1 | 18 = perp. |

N.B. If in the above figure the Diagonals 48 and 19½ feet had been given, the area would be $\frac{48 \times 19{\cdot}5}{2} = 468$ Square feet. (See N.B. after Formula VIII.)

**Ex. 365.**—In the figure (a *Rhomboid* *), if the parallel sides AB, CD are each 456 feet, and the perpendicular AF = 237·7 feet, what is the area in Square feet ?



| | | | |
|---|---|---|---|
| (V.) | A | 456 | 108390 Sq. ft. |
| | B | 1 | 237·7 = perp. |

**Ex. 366.**—In the above *Rhomboid,** if the area is known to be 1·6 Square feet, and the length 21 inches, what is the perpendicular height AF in inches ?

| | | | |
|---|---|---|---|
| (VIII.) | A | 1·6 | 21 |
| | B | 11 inches | 144 |

* By some the Rhomboid is called a Parallelogram.

## TRAPEZOID. (AREA-MEASURE.)

*See also under* "LAND-MEASURING."*

(IX.)

| | | |
|---|---|---|
| A | Sum of Parallels | Area |
| B | 2 | Perpendicular height |

$$\text{Area} = \frac{\text{Sum paral.} \times \text{perp.}}{2}.$$

(X.)

| | | |
|---|---|---|
| A | Sum of Paral. in ft. | Area in Sq. yds. |
| B | 18 | Feet perp. |

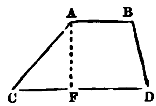
$$\text{Area in Sq. yds.} = \frac{\text{Sum paral.} \times \text{perp.}}{18}.$$

(XI.)

| | | |
|---|---|---|
| A | Sum of Paral. in in. | Area in Sq. ft. |
| B | 288 | Inches perp. |

$$\text{Area in Sq. ft.} = \frac{\text{Sum paral.} \times \text{perp.}}{288}.$$

**Ex. 367.**—In the figure (a *Trapezoid*) let AB, CD be 320 feet, and 720 feet, respectively; and the perpendicular height AF = 360 feet. Required the area in Square yards.




| | | | |
|---|---|---|---|
| (X.) | A | 1040 = sum | 20800 Sq. yards |
| | B | 18 | 360 = Area |

---

## TRAPEZIUMS. (AREA-MEASURE.)

*See also under* "LAND-MEASURING."

(XII.)

| | | |
|---|---|---|
| A | Sum of perpendiculars | Area |
| B | 2 | Diagonal |

$$\text{Area} = \text{Diag.} \times \text{mean of perps.}$$

---

*In "Land-measuring" (qu. *v.*) the *diagonal* is measured, both in the case of Trapezoids and Trapeziums.

(XIII.)

| | | |
|---|---|---|
| A | Sum of Perps. in ft. | Area in Sq. yds. |
| B | 18 | Diagonal in ft. |

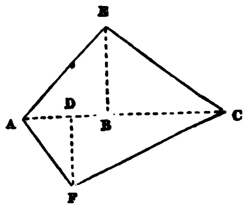
$$\text{Area in Sq. yds.} = \frac{\text{Sum perps.} \times \text{diag.}}{18}.$$

(XIV.)

| | | |
|---|---|---|
| A | Sum of Perps. in in. | Area in Sq. ft. |
| B | 288 | Diag. in in. |

$$\text{Area in Sq. ft.} = \frac{\text{Sum perps.} \times \text{diag.}}{288}.$$

**Ex. 368.**—In the figure (a *Trapezium*) let the diagonal AC = 42 feet, and the perpendiculars DF, BE = 16 and 18 feet respectively. Required the area in Square feet.




| | | |
|---|---|---|
| A | 34 = sum of perps. | 714 Sq. ft. |
| B | 2 | 42 |

**Ex. 369.**—In the above Trapezium let AC measure 486 feet, DF 144 feet, and BE 162 feet. Required the area in Square yards.

| | | |
|---|---|---|
| A | 306 = sum of perps. | 8262 Sq. yds. |
| B | 18 | 486 |

*More Examples will be given under* "LAND-MEASURING."

---

## TRIANGLES. (AREA-MEASURE.)

*See also under* "LAND-MEASURING."

(XV.)

| | | |
|---|---|---|
| A | Base | Area of *any* Triangle |
| B | 2 | Perpendicular |

$$\text{Area} = \frac{\text{Base} \times \text{Perp.}}{2}.$$

(XVI.)

| | | |
|---|---|---|
| A | Base in feet | Area in Sq. yds. |
| B | 18 | Perp. in ft. |

$$\text{Area in sq. yds.} = \frac{\text{Base in ft.} \times \text{Perp. in ft.}}{18}.$$

(XVII.)

| | | |
|---|---|---|
| A | Base in inches | Area in Sq. ft. |
| B | 288 | Perp. in inches |

Area in Sq. yds. $= \dfrac{\text{Base in in.} \times \text{Perp. in in.}}{288}$

(XVIII.)

| | | | | |
|---|---|---|---|---|
| A | 1 | 1·52 | 76 | Side of Equilat. Tr. |
| B | ·658 | 1 | 50 | Side of Sq. equal |

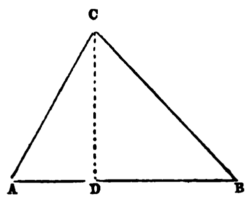
Side of Sq. = Side of Tr. × ·658.

(XIX.)

If $s = \frac{1}{2}$ the sum of the *three Sides*, $a$, $b$, $c$,—then in *any* Triangle the area $= \sqrt{(s - a) \times (s - b) \times (s - c) \times s}$.

N.B. When three Sides are given, two of which are equal, the Perpendicular is first found, as in Ex. 276, from the Hypothenuse and $\frac{1}{2}$ Base; and thence the area, as in Formula XV. above.

**Ex. 370.**—In the figure (*a Triangle*) let AB = 42 feet, and the Perpendicular DC = 33 feet. Required the area in Square feet.




| (XV.) | | | |
|---|---|---|---|
| | A | 42 | 693 Sq. f |
| | B | 2 | 33 |

**Ex. 371.**—In a Triangle whose area is 117·5 Square yards, the Base is 49 feet, 6 inches. What will be the length of the Perpendicular *in feet?*

| (XVI.) | | | |
|---|---|---|---|
| | A | 49·5 | 117·5 |
| | B | 18 | 42·75 = Perp |

**Ex. 372.**—Given three *Equilateral* Triangles whose sides are respectively 5·1, 6·0, and 7·2 inches. Required their areas in Square inches.

| (XIX.) | | | | | |
|---|---|---|---|---|---|
| | C | 3·9 | 11·26 | 15·6 | 22·45 Sq. in. |
| | D | 3 | 5·1 | 6 | 7·2 |

**Ex. 373.**—In the fig. let AC = 39, CB = 42, and AB = 4 feet. Required the area in Square feet (Formula XIX.)

(Here $s$ or $\frac{1}{2}$ sum of $a + b + c = 63$ ; and $s - a = 18, s - b = 21$ $s - c = 24$.)

| | | | |
|---|---|---|---|
| 1st. $18 \times 21$ | A | 18 | 37 |
| | B | 1 | 2 |

| | | | |
|---|---|---|---|
| 2d. $378 \times 24$ | A | 24 | 907 |
| | B | 1 | 37 |

| | | | |
|---|---|---|---|
| 3d. $9070 \times 63$ | A | 63 | 571000 |
| | B | 1 | 9070 |

| | | | |
|---|---|---|---|
| 4th. $\sqrt{571000}$ | C | 1 | 571000 |
| | D | 1 | 756 = Answer |

The answer read off is *exact*, although the Square number is strictly 571536. By dotting off the alternate figures of the Square, beginning with the last figure, we at once see how many figures there are in the Square Root. (See page 84.)

N.B. To find the area from three sides, the Logarithmic line *Log* (or Num.) may be used, as will be shown under Appendix B.

**Ex. 374.**—What will be each side of an Equilateral Triangle whose area equals that of a Square, each side of which is 7 inches

| | | | | |
|---|---|---|---|---|
| (XVIII.) | A | 1·52 | 10·6 Sq. in. | 76 |
| | B | 1 | 7 | 50 |

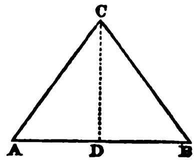
**Ex. 375.**—If in a Right-angled Triangle two sides are equal and the Hypothenuse = 11·3 inches, what is the area in Square inches?

(Here area = $\sqrt{\frac{11 \cdot 3^2}{2}}$ as in Ex. 254.)

| | | |
|---|---|---|
| C | 2 | 1 |
| D | 11·3 | 8 Sq. in. |

**Ex. 376.**—If in a Triangle the Base is 19 feet, and the two sides 15 feet each, what is the area in Square feet? (See N.B. after Formula XIX.)

Here we first find the Perpendicular CD as in Ex. 277; and thence the area as in Ex. 370.



1st. $CD = \sqrt{(15 - 9) \times (15 + 9)}$

| | | |
|---|---|---|
| C | 24 | 6 |
| D | 12 = CD | 6 |

2d. $\text{Area} = \dfrac{12 \times 19}{2}$

| | | |
|---|---|---|
| A | 12 | 144 Sq. ft. |
| B | 2 | 19 |

**Ex. 377.**—In the figure to Ex. 376 let AB = 45 feet, AC = 39 feet, BC = 42 feet. Required the length of the Perpendicular CD. [First find the *difference* of the Segments AD, DB, and thence the Segments themselves. Having thus obtained BC and BD, or AC and AB, we find the Perpendicular CD as in Ex. 277.]

$$\text{Difference of Segm.} = \frac{(BC + AC) \times (BC - AC)}{AB} = \frac{81 \times 3}{45} = 5{\cdot}4.$$

$$\text{Then } \frac{AB + 5{\cdot}4}{2} = 25{\cdot}2 = DB\text{; and } \frac{AB - 5{\cdot}4}{2} = 19{\cdot}8 = DA.$$

Then $CD = \sqrt{(BC + BD) \times (BC - BD)} = \sqrt{67{\cdot}2 \times 16{\cdot}8}$ as in Ex. 277.

| | | | |
|---|---|---|---|
| C | 1 | 16·8 | 67·2 |
| D | 1 | 33·6 = CD | 67·2 |

N.B. If the *three sides* of any Triangle are given, the area may be found by another method than that of Ex. 376; for we may find the Perpendicular as in the preceding Example, and thence the area as in Ex. 370.

**Ex. 378.**—In the figure to Ex. 376 let AB = 45 feet, AC = 39 feet, BC = 42 feet. Required the area.

First find the Perpendicular CD to be 33·6 feet, as in the preceding Example. Then area $= \dfrac{45 \times 33{\cdot}6}{2} = 756$ Square feet. (Ex. 370.)

N.B. If we know the Radius of a circle that circumscribes a triangle of which the three sides are known, we can find the *area* of that triangle: for area $= \dfrac{\text{Product of three sides}}{\text{Radius} \times 4}$. Thus if the three sides are 45, 42, 39 feet, and the Radius 24·375 feet, the *area* will be $\dfrac{45 \times 42 \times 39}{24{\cdot}375 \times 4} = \dfrac{73710}{97{\cdot}5} = 756$ Square feet.

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## EQUILATERAL POLYGONS. (AREA-MEASURE.)

(XX.)

| | | | |
|---|---|---|---|
| C | 1·7205 | 1200 | Area |
| D | 1 | 26·4 | Side of Pentagon |

Area = Side$^2$ × 1·7205.
Side = $\sqrt{\text{Area} \div 1{\cdot}7205}$.

(XXI.)

| | | | |
|---|---|---|---|
| C | 2·598 | 3000 | Area |
| D | 1 | 34 | Side of Hexagon |

Area = Side$^2$ × 2·598.
Side = $\sqrt{\text{Area} \div 2{\cdot}598}$.

(XXII.)

| | | | |
|---|---|---|---|
| | 3·634 | 4200 | Area |
| D | 1 | 34 | Side of Heptagon |

Area = Side$^2$ × 3·634.
Side = $\sqrt{\text{Area} \div 3{\cdot}634}$.

(XXIII.)

| | | | |
|---|---|---|---|
| C | 4·828 | 2600 | Area |
| D | 1 | 23·2 | Side of Octagon |

Area = Side$^2$ × 4·828.
Side = $\sqrt{\text{Area} \div 4{\cdot}828}$.

(XXIV.)

$$\frac{\text{C} \quad \cdot 8284 \quad 29 \quad \text{Area of largest incl. Oct.}}{\text{D} \quad 1 \quad 35 \quad \text{Area of Square}} \left\{ \begin{array}{l} \text{Area} = \text{Area of Sq.} \times \cdot 8284. \\ \text{Side} = \sqrt{\text{Area} \div \cdot 8284}. \end{array} \right.$$

For 10 sides $\frac{\text{C} \quad 7{\cdot}694 \quad \text{Area}}{\text{D} \quad 1 \quad \text{Side}}$; for 12 sides $\frac{\text{C} \quad 11{\cdot}2 \quad \text{Area}}{\text{D} \quad 1 \quad \text{Side}}$.

N.B. 1st. Any side = $\sqrt{\text{Area} \div \text{multiplier}}$, solved as in Ex. 251.

N.B. 2d. If the Slide is given in feet, and the area required in *Square yards*, for 1 on D read 3 (or its "check number" 9·49 with 10 times the number * on C). If the side is in inches, and area in Square feet, for 1 on D, read 12 (or its "check number" * 37·95).

**Ex. 379.**—What is the area, in Square feet, of a Pentagonal garden bed, each side of which is 8½ feet ?

$$\text{(XX.)} \frac{\text{C} \quad 1200 \quad (12) \quad 124{\cdot}3 \text{ Sq. ft.}}{\text{D} \quad 26{\cdot}4 \quad (2{\cdot}64) \quad 8{\cdot}5 = \text{side}}$$

In the above Example we see what is over 8·5 on D sooner, if we read 2·64 under 12, instead of 26·4 under 1200. See page 87.

**Ex. 380.**—Required the length in inches, of each side of a Hexagon, which contains 1040 Square inches.

$$\text{(XXI.)} \frac{\text{C} \quad 2{\cdot}598 \quad 3000 \quad 1040 = \text{area}}{\text{D} \quad 1 \quad 34 \quad 20 \text{ ft.}}$$

**Ex. 381.**—Required the areas in Square *feet* of two Octagons whose sides are respectively 8·28 and 20 *inches*.

$$\text{(XXIII.)} \frac{\text{C} \quad 2{\cdot}29 \quad 4{\cdot}83 \quad 13{\cdot}41 \quad 21 \text{ Sq. feet area}}{\text{D} \quad 8{\cdot}28 \quad 37{\cdot}95 \quad 20 \quad 25 \text{ inches side}}$$

* See explanation of "Check number" in page 90.

**Ex. 382.**—If there are two square pieces of board whose sides are respectively 7 and 20 inches, what are the areas in Square inches of the largest Octagons that can be cut from them ?

| | | | |
|---|---|---|---|
| C | 1 | 40·6 | 334·1 areas |
| D | 1·1 | 7 | 20 sides |

---

## CIRCLES. (AREA-MEASURE.)

See also under "LAND-MEASURING."

(I.)*

| | | | | | |
|---|---|---|---|---|---|
| C | ·7854 | 1 | 43 | 380 | Area |
| D | 1 | 1·1284 | 7·4 | 22 | Diam. |

Area = Diam.$^2$ × ·7854.
Diam. = $\sqrt{\frac{\text{area}}{·7854}}$ ; or $\sqrt{\text{area}}$ × 1·1284.

(II.)*

| | | | | |
|---|---|---|---|---|
| C | 3·1416 | 1 | 43 | Area |
| D | 1 | ·5642 | 3·7 | Radius |

Area = Radius$^2$ × 3·1416.
Radius = $\sqrt{\frac{\text{area}}{3·1416}}$ ; or $\sqrt{\text{area}}$ × ·5642.

(III.)*

| | | | | |
|---|---|---|---|---|
| C | ·07958 | 1 | 23 | Area |
| D | 1 | 3·542 | 17 | Circumf. |

Area = Circumf.$^2$ × ·07958.
Circumf. = $\sqrt{\frac{\text{area}}{·0796}}$ ; or $\sqrt{\text{area}}$ × 3·54.

(IV.)

| | | | | |
|---|---|---|---|---|
| A | 1 | 1·2732 | 14 | Circular inches |
| B | ·7854 | 1 | 11 | Square inches |

1 Sq. foot = 183·35 circ. inches.

---

* In Formulæ I., II., III., if Diameter, Radius, or Circumference is given in inches and the Area required in Square feet, for 1 on D, read 12. (See Formulæ X. and XI.) If line D is given in feet, and area required in Square yards, for 1 on D, read 9.

(V.)

| | | | | | |
|---|---|---|---|---|---|
| A | 1 | 1·2732 | 14 | Area of Square | Area of Circ. = area Sq. × ·7854. |
| B | ·7854 | 1 | 11 | Area of inscr. Circle | Area of Sq. = area Circ. × 1·2732. |

(VI.)

| | | | | | |
|---|---|---|---|---|---|
| A | 1 | 1·571 | 22 | Area of Circle | Area of Sq. = area Circ. × ·6366. |
| B | ·6366 | 1 | 14 | Area of inscr. Sq. | Area of Circ. = area Sq. × 1·571. |

(VII.)

| | | | | | |
|---|---|---|---|---|---|
| A | 1 | 1·1284 | 35 | Diam. of Circle | Side = Diam. × ·8862. |
| B | ·8862 | 1 | 31 | Side of Square equal | Diam. = Side × 1·1284. |

(VIII.)

| | | | | | |
|---|---|---|---|---|---|
| C | 1 | 3·545 | 39 | Circumf. of Circle | Side = Circumf. × ·2821. |
| D | ·2821 | 1 | 11 | Side of Square equal | Circumf. = Side × 3·545. |

(IX.)

| | | | | |
|---|---|---|---|---|
| C | 1·571 | 10 | Area of Circle | Area = Side² × 1·571. |
| D | 1 | 2·52 | Side of inscr. Sq. | Side = $\sqrt{\text{Area} \div 1{\cdot}571}$. |

(X.)

| | | | | |
|---|---|---|---|---|
| C | ·7854 | 29 | Area in Sq. ft. | Area = $\frac{\text{Diam.}^2 \times {\cdot}7854}{12^2}$. |
| D | 12 | 73 | Diam. in inches | Diam. = $\sqrt{\frac{\text{Sq. ft.} \times 12^2}{{\cdot}7854}}$. |

(XI.)

| | | | | |
|---|---|---|---|---|
| C | ·0796 | ·7 | Area in Sq. ft. | Area = $\frac{\text{Circ.}^2 \times {\cdot}0796}{12^2}$. |
| D | 12 | 35·6 | Circumf. in inches | Circ. = $\sqrt{\frac{\text{Sq. ft.} \times 12^2}{{\cdot}0796}}$. |

**Ex. 383.**—If the area of a circle is 1 Square foot, (144 Square inches) what is its diameter in *inches?*

| | | | |
|---|---|---|---|
| (I.) | C | 144 Square inches | 43 |
| | D | 13·54 inches | 7·4 |

| | | | |
|---|---|---|---|
| or (X.) | C | 1 | 29 |
| | D | 13·54 inches | 73 |

**Ex. 384.**—What must be the diameter, in inches, of a circular rain gauge, so that the area may be 17⅓ Square inches?

| | | | |
|---|---|---|---|
| (I.) | C | 17·33 | 4·3 |
| | D | 4·7 | 7·4 |

**Ex. 385.**—What are the areas of three pipes, whose diameters are respectively 2·6, 4·1, and 9·7 inches?

| | | | | | |
|---|---|---|---|---|---|
| (I.) | C | 5·31 Sq. in. | 13·2 Sq. in. | 43 | 73·9 Sq. in. |
| | D | 2·6 | 4·1 | 7·4 | 9·7 |

**Ex. 386.**—If the circumferences of four circles are respectively 31, 34, 44, and 51 inches, what are their areas in Square inches?

| | | | | | | |
|---|---|---|---|---|---|---|
| (III.) | C | 23 | 76·5 | 92 | 154 | 207 Sq. in. |
| | D | 17 | 31 | 34 | 44 | 51 |

**Ex. 387.**—The atmospheric pressure being 14·7 lbs. on every *square* inch, how many lbs. is it on a *circular* inch?

| | | | |
|---|---|---|---|
| (IV.) | A | 14 | 14·7 |
| | B | 11 | |

**Ex. 388.**—If the area of a circle is 121 Square inches, what is the area of a Square inscribed within it?

| | | | |
|---|---|---|---|
| (VI.) | A | 22 | 121 |
| | B | 14 | 77·1 Sq. in. |

**Ex. 389.**—A cylindrical log (Ex. 384), after having been carefully squared (as in Ex. 388) measured 8·78 inches each side. What was the *area* of the section before it was squared?

| | | | |
|---|---|---|---|
| (IX.) | C | 10 | 121 Sq. in. |
| | D | 2·52 | 8·78 |

**Ex. 390.**—Required the number of Square *feet* of a piston-head, whose diameter is 85 *inches*—and of another whose diameter is 59 inches.

| | | | | |
|---|---|---|---|---|
| (X.) | C | 29 | 19 Sq. ft. | 39·4 Sq. ft. |
| | D | 73 | 59 | 85 |

**Ex. 391.**—Required the Square *feet* in three circles whose circumferences are respectively 84, 105, and 115 *inches*.

| | | | | | |
|---|---|---|---|---|---|
| (XI.) | C | ·7 | 3·9 | 6·1 | 7·3 Sq. ft. |
| | D | 35·6 | 84 | 105 | 115 |

**Ex. 392.**—Required the pressure in lbs. on the piston of the cylinder of a hydraulic pump, the diameter of the cylinder being 8 inches, and that of the smaller pipe being $\frac{3}{4}$ inch, whilst the pressure on the latter is $\frac{1}{2}$ cwt.

| | | | | |
|---|---|---|---|---|
| 1st. (I.) | C | ·442 area | 43 | 50·2 area |
| | D | ·75 | 7·4 | 8 |

2d. $\dfrac{50{\cdot}2 \times 56}{{\cdot}442}$

| | | | |
|---|---|---|---|
| A | 50·2 | 6372 lbs. = Ans. |
| B | ·442 | 56 |

N.B. The above Example may be solved without finding the two areas, as the pressures increase as the squares of the diameters, or $\cdot75^2 : 8^2 :: 56 : x$. (Ex. 256.)

| | | |
|---|---|---|
| C | 56 | 6372 lbs. |
| D | ·75 | 8 |

---

The following will show how in figures all having the *same perimeter*, the greatest area is included by the figure nearest approaching to a *Circle*.

All eight have the same Perimeter, viz. : 36 inches.

| | Area in Sq. inches. |
|---|---|
| Right-angled Triangle, with sides 9, 12, 15 . . . | 54·0 |
| Triangle, with sides 10, 12, 14 . . . . . . . . . | 58·8 |
| Equilateral Triangle, 12 to a side . . . . . . . | 62·3 |
| SQUARE, of 9 to a side . . . . . . . . . . . | 81·0 |
| Equilateral Pentagon, 7·2 to a side . . . . . | 89·2 |
| Equilateral Octagon, 4·5 to a side . . . . . . . | 97·8 |
| Equilateral Duodecagon, 3·0 to a side . . . . . | 101·0 |
| CIRCLE, with diameter of 11·459 . . . . . . . | 103·13 |

Hence it will be seen that with the *same perimeter*, the area of a Circle exceeds that of a Square, as 1 exceeds ·7854; or as 127·3 exceeds 100; or as 14 exceeds 11 (Formula V.) (See Ex. 123.)

## ELLIPSES. (AREA-MEASURE.)

| | | |
|---|---|---|
| A | One axis | Area |
| B | 1·2732 | Other axis |

$$\text{Area} = \frac{\text{Product of axes}}{1{\cdot}2732}.$$

N.B. If the axes are given in *feet*, and the area required in Square *yards*, use as a Divisor, 11·46 (= 1·2732 × 9).

**Ex. 393.**—If the two axes of an Elliptical piece of water are 300 and 200 *feet*, what is the area in Square yards?

| | | |
|---|---|---|
| A | 200 = One axis | 5236 Sq. yds. |
| B | 11·46 | 300 = Other axis |

## PARABOLAS. (AREA-MEASURE.)

| | | |
|---|---|---|
| A | Vertical axis | Area |
| B | 1·5 | Base |

$$\text{Area} = \frac{\text{Vert. axis} \times \text{Base}}{1{\cdot}5}.$$

## SECTORS. (AREA-MEASURE.)

Either the "length of arc" (XI. page 132) or "degrees of arc" (XII. page 132) must first be found, *and then*

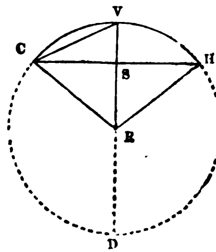
(I.)

| | | | |
|---|---|---|---|
| A | Length of arc | Area of Sector | Area $= \frac{\text{Rad.} \times \text{length of arc}}{2}$. |
| B | 2 | Radius | |

(II.)*

| | | | |
|---|---|---|---|
| C | Degrees of arc | Area of Sector | Area $= \frac{\text{Rad.}^2 \times \text{Degrees}}{10{\cdot}71}$. |
| D | 10·71 (check 33·9) | Radius | |

**Ex. 394.**—In the figure, let the Chord CH be given = 16 inches; and the Radius CR or VR be given = 10 inches ($\frac{1}{2}$ the Chord or CS will of course be 8). Required the area of the Sector CRHV.




Here we first find RS to be 6, by (X.) page 132, and then deduct this from VR, leaving VS = 4. We then find CV to be 8·9 by (II.) page 131, and the "length of arc" to be 18·518 $\left(\frac{8 \times CV - CH}{3}\right)$ as in (XI.) page 132.

Then by Formula I. above

| | | |
|---|---|---|
| A | 18·518 = *l. of arc* | 92·59 Sq. inches = Area |
| B | 2 | 10 = *radius* |

**Ex. 395.**—Given VD = 32 and VS = 6 in the above figure. Find the area of the Sector, from the "Degrees of arc."

Here CV = 13·85 (by I. page 131), and CH = 25 (by IX. page 131). Then if we have a *Table of Sines* (for if we have not, we must first find the *length* of arc to get the degrees, as in the next Example), we find ·7806 ($\frac{CS}{VR}$ as shown in Ex. 350) to be the Nat. sine of 51·3° which is the angle CRV. Double this, or 102·6° is the whole angle CRH, or "Degrees of arc." Then by Formula II.† above,

* Or area of Sector $= \frac{\text{Degrees of arc} \times \text{Area of circle}}{360}$.

| | | | |
|---|---|---|---|
| (II.) | C | 102·6 | 229 Sq. inches = Area |
| | D | 10·71 | 16 |

**Ex. 396.**—Solve the preceding Example *without* a Table of Nat. sines.

Find CH = 25 as in the preceding Example. Then "length of arc" $= \frac{(8 \times 13{\cdot}85) - 25}{3}$ = 28·6 (as in XI. page 132). Then by Formula I. page 153,

| | | | |
|---|---|---|---|
| (I.) | A | 28·6 | 229 Sq. inches = Area |
| | B | 2 | 16 = VR |

**Ex. 397.**—In the figure, page 153, let the "Degrees of arc," or angle CRH, *be given* as 27°, and the diameter = 194 inches. What is the area of the Sector?

| | | | |
|---|---|---|---|
| (II.) | C | 27° | 2217 Sq. inches = Area |
| | D | 10·71 | 97 = radius |

In this Example, we must use the check number (page 90) 33·9, on D, with 10 times 27 or 270 on C.

N.B. Instead of using Formula II. page 153, we may find the area of the whole circle by Formula I. page 148, and then use the proportion 360° : to Degrees given :: area of whole circle : area of Sector. Thus in Ex. 397, the area of the whole Circle would be (with a diameter of 388) 29560 Square inches. Then

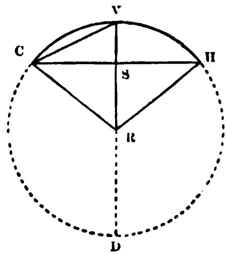
| | | |
|---|---|---|
| A | 27° | 2217 = area of Sector |
| B | 360° | 29560 area of Circle |

---

## SEGMENTS. (Area-measure.)

First find the area of a "Sector" having the same Arc and Radius as the required Segment : and subtract from it the area of the Triangular part.

**Ex. 398.**—Required the area of the Segment CHV in the figure, where VD = 32 inches, and VS = 6 inches.




The area of the whole *Sector* CRHV is found, as in Ex. 395, to be 229 Square inches. The perpendicular of the triangle, or SR is evidently $\frac{1}{2}$ VD − VS, or 10; and the base CH = 25, is found during the process of finding the area of the Sector.

Then the area of the Triangle CHR, as in Ex. 370, $\frac{25 \times 10}{2} = 125$ Square inches. Subtract this from the 229 Square inches of the whole Sector, and we have 104 Square inches = ***the area of the Segment.***

**Ex. 399.**—Required the arêa of the Segment CHV in the preceding figure, where the Chord CH = 12, and Radius CR = 10 inches.

Here we find the area of the whole *Sector* as in Ex. 398. The "length of arc" will be $\frac{(8 \times 6{\cdot}3245) - 12}{3} = 12{\cdot}87$, and the area of the whole Sector by Formula I. page 153, will be 64·35 Square inches. In this process, SR the perpendicular of the Triangle will have been found = 8, whence Triangular area = $\frac{12 \times 8}{2}$ = 48 Square inches. This deducted from 64·35, leaves 16·35 Square inches = ***the area of the Segment.***

---

## CIRCULAR RINGS. (AREA-MEASURE.)

(See also under "LAND-MEASURING.")

| | | |
|---|---|---|
| A | Sum of radii | Area |
| B | ·3183 | Diff. of radii |

Area = $\frac{(R + r) \times (R -}{{\cdot}3183}$

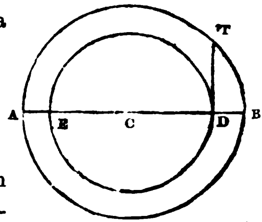
| A | Sum of diams. | Area |
|---|---|---|
| B | 1·2732 | Diff. of diams. |

$$\text{Area} = \frac{(D + d) \times (D - d)}{1{\cdot}2732}.$$

| A | Sum of Circumfs. | Area |
|---|---|---|
| B | 12·566 | Diff. of Circumfs. |

$$\text{Area} = \frac{(C + c) \times (C - c)}{12{\cdot}566}.$$

**Ex. 400.**—In the figure, let CB = 50, and CD = 40 feet. Required the area of the outer ring * in Square feet.



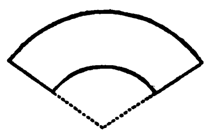

| A | 90 | 2827·5 Sq. ft. |
|---|---|---|
| B | ·3183 | 10 |

**Ex. 401.**—The circular fences on each side of a gravel walk, round a shrubbery, are 800 and 714 feet in length. Required the area of the walk in Square feet.

| A | 1514 | 359 Sq. ft. |
|---|---|---|
| B | 12·566 | 86 |

N.B. To find the diameter of a circle having the same area as a circular ring, let D = outer, and $d$ = inner diameter of the ring. Then $x = \sqrt{D^2 - d^2}$, or $\sqrt{(D + d) \times (D - d)}$. Ex. 258.

The area of *part* of a Ring (as in the figure), is found by multiplying half the sum of the two arcs (Formula XI. page 132) by the distance between them.



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* The area of the outer ring is also the area of a circle whose radius is DT. In Ex. 400, DT would be 30 feet, and the area, by (II.) page 148, = 2827 Square feet.

## *PRISMS AND CYLINDERS. (SURFACE MEASURE.)

I. Surface = (Perimeter of one end × length) *plus* the two end areas.

II. The Surface of a *Cube*, is any side$^2$ × 6.

III. The Surface of a Tetrahedron = linear edge$^2$ × 1·732; of a Octahedron, linear edge$^2$ × 3·464. Ex. 252.

**Ex. 402.**—What is the surface in Square feet, of a chest, whose length is 7 ft. 8 in.; breadth 4 ft. 7 in.; and depth 2 ft. 9 in.?

(Reduce inches to decimals of a foot, Appendix G.)

1st. $(4{\cdot}58 + 2{\cdot}75) \times 2 \times 7{\cdot}67 = \dfrac{7{\cdot}33 \times 7{\cdot}67}{2}$.

| | | |
|---|---|---|
| A | 7·33 | 112·4 Sq. ft. |
| B | ·5 | 7·67 |

2d. $(4{\cdot}58 \times 2{\cdot}75) \times 2 = \dfrac{4{\cdot}58 \times 2{\cdot}75}{{\cdot}5}$.

| | | |
|---|---|---|
| A | 4·58 | 25·2 Sq. ft. |
| B | ·5 | 2·75 |

Then 112·4 + 25·2 = 137·6 Square feet, the *Answer*.

**Ex. 403.**—Required the Surface of a cube having each side 37 inches. (Here $x = 37^2 \times 6$, as in Ex. 252.)

| | | |
|---|---|---|
| C | 6 | 8214 Sq. in. |
| D | 1 | 37 |

Or if the answer is required in Square *feet*, we have $\dfrac{37^2 \times 6}{12^2}$.

*The difference between a "triangular prism" and a "wedge" is, that in the former, the two ends are parallel and perpendicular. For "Wedges," see page 161.

P

| C | 6 | 57 Sq. ft. | |
|---|---|---|---|
| D | 12 | 37 | Ex. 256. |

**Ex. 404.**—Required the Surface in Square feet, of a Triangular prism, of which the length is 13 *feet*, and the sides of the base 23, 34, and 19 *inches?*

1st. Perim. of end × length ; or $\frac{76}{12} \times 13$.

| A | 76 | 82·3 Sq. ft. |
|---|---|---|
| B | 12 | 13 |

2d. Area of end* × 2, or $\frac{208 \times 2}{144}$.

| A | 2 | 2·9 Sq. ft. |
|---|---|---|
| B | 144 | 208 |

Then 82·3 + 2·9 = 85·2 Square feet, the *answer.*

**Ex. 405.**—Required the whole Surface of a Cylinder whose length is 15 feet, and diameter $5\frac{1}{3}$ feet.

1st. Perim. of end × *l*, or $\frac{5·33}{·3183} \times 15$ (Formula (I.) page 122).

| A | 5·33 | 251·3 Sq. ft. |
|---|---|---|
| B | ·3183 | 15 |

2d. Area of 2 ends, or $\frac{5·33^2 \times ·7854}{·5}$ as in Ex. 259.

| | | |
|---|---|---|
| A | ·5 | |
| B | ·7854 | |
| C | | 44·7 Sq. ft. |
| D | | 5·33 |

Then 251·3 + 44·7 = 296 Square feet, the *answer.*

* The area of the triangular end is found as in Ex. 373. $\sqrt{43320} = 208$.

## PYRAMIDS. (SURFACE MEASURE.)

Add the area of the triangles which constitute its sides, to the area of the base.

**Ex. 406.**—Required the Surface in Square feet of a Triangular pyramid, each side of the base of which is 32 *inches*, and the slant height $11\frac{1}{2}$ *feet*.

1st. $\dfrac{11 \cdot 5 \times 2 \cdot 67}{2} \times 3$, or $\dfrac{11 \cdot 5 \times 2 \cdot 67}{\cdot 667}$.

| | | |
|---|---|---|
| A | 11·5 | 46 = area of sides |
| B | ·667 | 2·67 |

2d. $s^2 \times \cdot 433$, as in Ex. 372.

| | | |
|---|---|---|
| C | ·433 | 3·08 = area of base |
| D | 1 | 2·667 |

Then 46 + 3·08 = 49·08 Square feet, the *answer*.

---

## CONES. (SURFACE MEASURE.)

$$* \text{Surface} = \begin{cases} \text{diam.} \times (\text{rad.} + \text{slant side}) \div \cdot 6366. \\ \text{circumf.} \times (\text{rad.} + \text{slant side}) \div 2. \end{cases}$$

**Ex. 407.**—The slant side of a Cone is 10 ft. and the diameter of its base 32 inches. Required the whole Surface in Square feet.

$$\left(\text{Here } x = \frac{2 \cdot 67 \times 11 \cdot 33}{\cdot 637}.\right)$$

| | | |
|---|---|---|
| A | 2·67 | 47·47 Sq. ft. |
| B | ·637 | 11·33 |

---

* If we only require the curved surface and not the base,

$$x = \frac{\text{Circumf.} \times \text{slant}}{2} \text{ or } \frac{\text{Diam.} \times \text{slant}}{\cdot 6366}.$$

## FRUSTRUMS OF PYRAMIDS AND CONES.

(SURFACE MEASURE.)

Let $p'$ = perimeter at top, and $p$ at bottom. And $s$ = slant height.

Then Surface $= \dfrac{(p' + p) \times s}{2} +$ Areas of the ends.

N.B. This supposes the Pyramid to be regular. If not so, the lateral planes are trapezoids, and their areas must be found separately as in page 141.

**Ex. 408.**—Required the whole Surface of a frustrum of a square pyramid, whose slant height is 20 feet, and its two ends having sides of 4 and 6 feet respectively. The Slide Rule will assist in finding the answer, or

$$\frac{(16 + 24) \times 20}{2} + 4^2 + 6^2 = 400 + 16 + 36 = 652 \text{ Square feet.}$$

**Ex. 409.**—Required the entire surface of the frustrum of a Cone, whose slant height is $10\frac{1}{2}$ feet, and the diameters of the ends 1·695 and ·715 feet respectively.

1st. $(1{\cdot}695 + {\cdot}715) \times 3{\cdot}1416$ = sum of perimeters.

| | | |
|---|---|---|
| A | 2·41 | 7·58 |
| B | 1 | 3·1416 |

2d. $\dfrac{7{\cdot}58 \times 10{\cdot}5}{2}$ = Area without the ends.

| | | |
|---|---|---|
| A | 7·58 | 39·9 Sq. ft. |
| B | 2 | 10·5 |

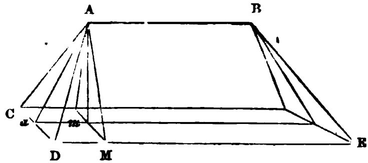
3d. Areas of two ends, by Formula (I.), page 148.

| | | | |
|---|---|---|---|
| C | 4·03 | 22·63 | 43 |
| D | ·715 | 1·695 | 7·4 |

Then the *Answer* is 39·9 + 4·03 + 22·63 = 66·56 Square feet.

## WEDGES. (Surface Measure.)

Add together the area of the Base, the areas of the two Ends, and the areas of the two Sides.




**Ex. 410.**—In the figure, CD = 9, DE = 36, A$a$ = 22, AB = 24 inches. Required the superficies in Square inches.

| | | | |
|---|---|---|---|
| 1st. | $36 \times 9$ | = | 324 Base. |
| 2d. | $22 \times 9$ | = | 198 *two* ends. |
| 3d. | $(36 + 24) \times 22$ | = | 1320 *two* sides. |
| | | | 1842 Square inches. |

## SPHERES. (Surface Measure.)

| | | | | | |
|---|---|---|---|---|---|
| (I.) | C / D | 3·1416 / 1 | 172 / 7·4 | Surface / Diameter | Surf. $= d^2 \times 3·1416$. |
| (II.) | C / D | 12·56 / 1 | 172 / 3·7 | Surface / Radius | Surf. $= r^2 \times 12·5664$. |
| (III.) | C / D | ·3183 / 1 | 92 / 17 | Surface / Circumf. | Surf. $= c^2 \times ·3183$. |
| (IV.) | A / B | Circumference / 1 | | Surface / Diameter | Surf. $= c \times d$. |

(V.) 

| | | | |
|---|---|---|---|
| C | 3·1416 | Surf. in Sq. yds. | Surf. $= \frac{d^2 \times 3{\cdot}1416}{3^2}$. |
| D | 3 | Diam. in feet | |

N.B.—The surface of a Sphere is 4 times the area of a section of its centre.

**Ex. 411.**—Required the convex surface of a sphere $10\frac{1}{2}$ inches diameter.

(I.)

| | | |
|---|---|---|
| C | 172 | 346·3 Sq. in. |
| D | 7·4 | 10·5 |

**Ex. 412.**—How many Square feet of varnishing are required for a globe 4·72 feet in circumference?

(III.)

| | | |
|---|---|---|
| C | 92 | 7·09 |
| D | 17 | 4·72 |

---

## CUBES. (Solid Content.)

*Solidity* = Cube of length of any side.

*Length of any side* = Cube Root of solid content.

**Ex. 413.**—What is the length of each side of a Cubic block of stone, containing 15·62 cubic feet? See also Ex. 322.

| | | |
|---|---|---|
| E | 1 | 15·62 |
| D | 1 | 2·5 |

**Ex. 414.**—Required the Cubic *feet* in a cube of timber, each of whose sides is $28\frac{1}{4}$ *inches*. (Here Solidity $= \frac{28{\cdot}25^3}{1728}$, solved as in Ex. 318.

| | | |
|---|---|---|
| A | | 1728 |
| B | | 28·25 |
| C | 18 | |
| D | 28·25 | |

Or we may take $\sqrt{1728} = 41{\cdot}57$, and then use the *two* lines C, D, only, in the form $\frac{28{\cdot}25^2 \times 28{\cdot}25}{41{\cdot}57^2}$, as in Ex. 256.

N.B. If the measurement is given in *inches* and the Solidity required in Cubic *yards* it is $\frac{\text{side}^3}{27}$, or $\frac{\text{side} \times \text{side}^2}{27}$, solved as in Ex. 318.

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## PARALLELOPIPEDS. (Solid Content.)

For content in *Bushels, Gallons*, &c., see farther on, under "Gauging."

Let S = Solidity ; $b$ = breadth ; $d$ = depth ; $l$ = length.

Then $S = l \times b \times d$ ; $d = \frac{S}{l \times b}$ ; $b = \frac{S}{l \times d}$ ; $l = \frac{S}{b \times d}$.

| I. Measurement given. | II. Solidity required. | III. Formula for A, B, C, D. | IV. Formula for C, D, when two sides are alike, or a "Mean Square" known. |
|---|---|---|---|
| All in yds., ft., or in. | Cub. yds., Cub. ft., Cub. in. | $b \times d \times l$ | Sq. $\text{side}^2 \times l$ |
| All in feet. | Cubic yards | $\frac{b \times d \times l}{27}$ | $\frac{\text{Sq. side}^2 \times l}{5{\cdot}196^2}$ *check* 16 43 |
| All in inches. | Cubic feet | $\frac{b \times d \times l}{1728}$ | $\frac{\text{Sq. side}^2 \times l}{41{\cdot}57^2}$ *check* 131·5 |
| One in ft. two in in. | Cubic feet | $\frac{b \times d \times l}{144}$ | $\frac{\text{Sq. side}^2 \times l}{12^2}$ *check* 37·95 |
| One in ft. two in in. | Cubic inches | $\frac{b \times d \times l}{{\cdot}0833}$ | $\frac{\text{Sq. side}^2 \times l}{{\cdot}289^2}$ *check* ·913 |

N.B. When two of the three measurements are the same (column IV.), the computation is much facilitated. If the "Gauge Point" (5·196, 4·157, &c.) in column IV. is not convenient, use the *check* number on D, with ten times the original number on C, as explained in page 90.

**Ex. 415.**—Block of stone. Length, 66 feet. Breadth, 14 feet. Depth, 12 feet. } Required content in Cubic feet.

Here we must first find 12 × 14 to be 108 ; and then

| 168 × 66 | A | 168 | 11100 Cub. ft. |
|---|---|---|---|
| | B | 1 | 66 |

The *exact* content is 11088 Cubic feet.

**Ex. 416.**—Excavation. Length, 640 feet. Breadth, 30 feet. Depth, 26 feet. } Required content in Cubic *yards*.

Here first find 26 × 30 = 780 ; and then

| $\frac{780 \times 640}{27}$ | A | 780 | 18500 Cub. yds. |
|---|---|---|---|
| | B | 27 Divisor | 640 |

**Ex. 417.**—Case of goods. Length, 3 feet. Breadth, 18 inches. Depth, 14 inches. } Required content in Cubic *feet.*

Here $\frac{3 \times 18 \times 14}{144}$, or $\frac{54 \times 14}{144}$, as follows :

| A | 5·25 Cub. ft. | 54 |
|---|---|---|
| B | 14 | 144 Divisor |

**Ex. 418.**—Squared timber. Length, 9·5 feet. Breadth, 3·5 feet. Depth, 3·5 feet. } Required content in Cubic feet.

Here we have *two sides equal* (see N.B. page 163) or $3{\cdot}5^2 \times 9{\cdot}5$, as in Ex. 252.

| C | 9·5 | 116 Cub. ft. |
|---|---|---|
| D | 1 | 3·5 = Sq. side |

**Ex. 419.**—Water-tank. Length, 49 inches. Breadth, 49 inches. Depth, 73 inches. } Required content in Cubic *feet.*

| Here $\frac{49^2 \times 73}{41{\cdot}57^2}$ | C | 73 | 101 Cub. ft. |
|---|---|---|---|
| | D | 41·57 G. P. | 49 |

The *exact* answer is 101·41 Cubic feet.

**Ex. 420.**—Bar Iron. Length, 2 feet. Breadth, $1\frac{1}{2}$ inches. Thickness, $\frac{5}{8}$ inch. } Required content in Cubic inches.

Here $\dfrac{2 \times 1{\cdot}5 \times {\cdot}625}{{\cdot}0833}$, or $\dfrac{3 \times {\cdot}625}{{\cdot}0833}$, as follows :

| | | |
|---|---|---|
| A | ·625 | 22·5 Cub. in. |
| B | ·0833 | 3 |

N.B. In Examples like this, where no two measurements are the same, the content may also be obtained by finding a "Mean Square," and using the Divisor in column IV. of the Table in page 163. Thus,

as in Ex. 258,

| | | |
|---|---|---|
| C | ·625 | 1·5 |
| D | ·625 | ·968 "Mean Square" |

and then $\dfrac{{\cdot}968^2 \times 2}{2892}$

as in Ex. 256.

| | | |
|---|---|---|
| C | 2 | 22·5 Cub. in. |
| D | ·289 | ·968 |

; but it is just as short to multiply two sides with the lines AB, and find the Content by a third multiplication with lines AB.

**Ex. 421.**—A rectangular cistern is to be made, to hold 1920 Cubic feet of water. Its length is to be 32 feet and its breadth 8. What must be its depth ?

Here Depth $= \dfrac{1920}{32 \times 8} = \dfrac{1920 \times {\cdot}125}{32}$.*

| | | |
|---|---|---|
| A | 7·6 ft. deep | 1920 |
| B | ·125 | 32 |

**Ex. 422.**—If a squared log of timber be 22 inches by 15, at each end, what length in inches will make a Cubic foot ?

Here $x = \dfrac{1728}{\text{Area of end}}$, or $\dfrac{1728}{22 \times 15}$, or $\dfrac{1728 \times {\cdot}0666}{22}$.*

| | | |
|---|---|---|
| A | 5·23 in. | 1728 |
| B | ·0666 | 22 |

---

* We see in Examples 421 and 422, the advantage of having in the memory the *reciprocals* of a few frequently occurring numbers. (See N.B. 3, page 14.)

**Ex. 423.**—A Parallelopiped, with one end square, and 15 feet long, contains 45·9 Cubic feet. Required the length of the "Square side" in inches. (Here $x = \frac{\sqrt{144} \times \sqrt{45{\cdot}9}}{\sqrt{15}}$, or $\sqrt{\frac{144 \times 45{\cdot}9}{15}}$, as in Ex. 262.

| | | |
|---|---|---|
| A | 144 Divisor | |
| B | 15 Length | |
| C | | 45·9 Cub. ft. |
| D | | 21 in. |

N.B. Since 144 is a number in constant use, it is easy to substitute 12 for $\sqrt{144}$, and then the *two* lines C, D, are available, for $x = \frac{12 \times \sqrt{45{\cdot}9}}{\sqrt{15}}$, as in Ex. 257.

| | | |
|---|---|---|
| C | 15 Length | 45·9 Cub. ft. |
| D | 12 G. P. | 21 in. |

---

## CYLINDERS. (Solid Measure.)

For content in Bushels, Gallons, &c., see farther on under "Gauging."

Let S = solid content; $l$ = length or height; $d$ = diameter; $c$ = circumference.

For the *four* lines ABCD.

$$S = \frac{d^2 \times l}{1{\cdot}2732}, \text{ or } \frac{c^2 \times l}{12{\cdot}566} \text{ as in Ex. 259.}$$

$$l = \frac{S \times 1{\cdot}2732}{d^2}, \text{ or } \frac{S \times 12{\cdot}566}{c^2} \text{ as in Ex. 260.}$$

$$d = \frac{\sqrt{S} \times \sqrt{1{\cdot}273}}{\sqrt{l}}, \text{ or } c = \frac{\sqrt{S} \times \sqrt{12{\cdot}566}}{\sqrt{l}}$$

as in Ex. 262.

For the *two* lines C D. The *check number* (see page 90) for 1·128 is 3·568, and for 3·545 it is 11·21.

$$S = \frac{d^2 \times l}{1{\cdot}128^2}, \text{ or } \frac{c^2 \times l}{3{\cdot}545^2} \text{ as in Ex. 256.}$$

$$l = \frac{S \times 1{\cdot}128^2}{d^2}, \text{ or } \frac{S \times 3{\cdot}545^2}{c^2} \text{ as in Ex. 256.}$$

$$d = \frac{\sqrt{S} \times 1{\cdot}128}{\sqrt{l}}, \text{ or } c = \frac{\sqrt{S} \times 3{\cdot}545}{\sqrt{l}} \text{ as in Ex. 257.}$$

(See Appendices I and K.)

N.B. Where there is a "*Series*" of Cylinders of the same Cubic content, and the same diameter or same circumference, but the *lengths varying*, the solution must be made with the *four* lines A B C D, as in Ex. 427; but in all other cases, whether for single Cylinders or a "series" of them, the lines C, D, are sufficient. As it is generally more convenient to use the two lines than the four, the following sets of the Rule are for the *two lines only*. By reference, however, to the preceding Formulæ, and the Examples annexed, the learner can practise the operation with the four lines, as in N.B. to Ex. 426.

(I.) Where the measurements and the required content are of the same name, i.e. *all in inches, all in feet*, &c.*

| | | | | | If 4 lines are used, the "Divisor" is |
|---|---|---|---|---|---|
| (*a*) | C | Length | Length × 10 | Solid content | 1·273 |
| | D | 1·128 | 3·568 | Diameter | N.B. to Ex. 426. |
| (*b*) | Ɔ | Content | Content ÷ 10 | Length | |
| | D | 1·128 | 3·568 | Diameter | |
| (*c*) | C | Length | Length × 10 | Content | 12·566 |
| | D | 3·545 | 11·21 | Circumference | Ex. 431. |
| (*d*) | Ɔ | Content | Content ÷ 10 | Length | |
| | D | 3·545 | 11·21 | Circumference | |

* When $l$ is given in feet, and $d$ or $c$ in inches, see page 163.

**Ex. 424.**—Required the content in Cubic inches, of a cylinder 7 inches long, and $4\frac{3}{4}$ inches diameter.

| (a) | | | | |
|---|---|---|---|---|
| | C | 7 = length | (70) | 124 cub. in. |
| | D | 1·128 | (3·568 *check*) | 4·75 = diam. |

In this Example, we must use the *Check number* with 10 times 7 over it; for it will be seen that if 1·128 on D is under 7, and we read on to the right, there is nothing to be seen over 3·568. (See page 90.)

N.B. If the *inverted* formula (*b*) is used, the number over the *check* must be multiplied by 10.

| (b) | | | |
|---|---|---|---|
| | Ↄ | 12·4 or $\frac{1}{10}$ of 124 the answer | $7 = l$ |
| | D | 3·568 *check* | $4{\cdot}75 = d$ |

**Ex. 425.**—What is the circumference in feet, of a round log of timber 24 feet long, and containing 20·17 Cubic feet.

| (c) | | | |
|---|---|---|---|
| | C | 20·17 | 24 length |
| | D | 3·25 ft. circumf. | 3·545 |

**Ex. 426.**—There are three cylindrical vessels, all of the same length, 7·2 inches, but *varying diameters*, viz.: 3·2, 4·2, and 4·6 inches. Required the contents of each in Cubic inches.

| (a) | | | | | | |
|---|---|---|---|---|---|---|
| | C | 7·2 | (72) | 5·8 cub. in. | 100 cub. in. | 120 cub. in. |
| | D | 1·128 | (3·568) | 3·2 | 4·2 | 4·6 |

Here, as in Ex. 424, it is necessary to use the *check* number.

N.B. If in Ex. 426, we wished to use the Formula adapted for *four* lines given in page 166, we have

$$\frac{d^2 \times l}{1{\cdot}2732} = \left\{\begin{array}{lll} A & 1{\cdot}2732 \text{ (Divisor for 4 lines)} & \\ B & 7 \text{ length} & \\ C & & 124 \text{ content} \\ D & & 4{\cdot}75 \text{ diam.} \end{array}\right.$$

**Ex. 427.**—There are three cylindrical vessels, all of the same diameter, 4·6 inches, but *varying lengths*, viz.: 6·5, 7, and 8 inches. Required the contents of each in Cubic inches. The *four* lines must be used. See footnote.*

| $\frac{d^2 \times l}{1 \cdot 2732}$ | | | | | |
|---|---|---|---|---|---|
| | A | | 108 cub. in. | 120 cub. in. | 133 cub. in. |
| | B | | 6·5 | 7·2 | 8 lengths |
| | C | 1·2732 | | | |
| | D | 4·6 diam. | | | |

**Ex. 428.**—There are four rounded logs of timber, each 20 feet long, but their respective circumferences are 5, 5·2, $5\frac{1}{2}$, and 6 feet. Required the Cubic feet in each.

| | | | | | | |
|---|---|---|---|---|---|---|
| (c) | C | 20 | 39·8 | 43·0 | 48·1 | 57·3 cub. ft. |
| | D | 3·545 | 5 | 5·2 | 5·5 | 6 |

**Ex. 429.**—There are three cylinders, each containing 377 Cubic inches, but of varying diameters, *also* of varying lengths. If the lengths are 22, 25, and 30 inches respectively, what are the diameters?

| | | | | | | |
|---|---|---|---|---|---|---|
| (b) | B | 377 content | (37·7) | 30 | 25 | 22 lengths |
| | D | 1·128 | (3·568) | 4 | 4·28 | 4·67 diams. |

**Ex. 430.**—There are 69·318 Cubic inches in a Quart. What

---

* When, in a "Series," the *diameters* vary, the contents of all can be solved by one setting of the two lines C D, as shown in Ex. 290; for there the equation is $x = \frac{d^2 \times l}{1 \cdot 128^2}$, where $l$ and $1 \cdot 128^2$ are constant: but if the *lengths* vary, $d^2$ and $1 \cdot 128^2$ become constant, and as they cannot be set one over the other (see N.B. before Ex. 285), we must use the four-line Formula $x = \frac{d^2 \times l}{1 \cdot 2732}$, and $l$ and 1·2732 being constant, as in Ex. 293, we employ the form (III*a*.), page 107.

respective lengths and diameters will be suitable for four Cylindrical measures to be made each to hold a Quart? *

| | | | | | |
|---|---|---|---|---|---|
| (b) | Ɔ | 69·32 = 1 Quart | 7·3 | 6 | 5·4 lengths |
| | D | 1·128 | 3·48 | 3·84 | 4·05 diams. |

**Ex. 431.**—There are three cast-iron Cylinders, all having the same circumference, $3\frac{1}{4}$ feet, but with *varying lengths*, viz.: 19, 24, and 35 feet. Required their respective contents in Cubic feet. (See remarks on Ex. 427.)

| | | | | |
|---|---|---|---|---|
| A | | 16 cub. ft. | 20·2 cub. ft. | 29·5 cub. ft. |
| B | | 19 | 24 | 35 lengths |
| C | 12·566 | | | |
| D | 3·25 circumf. | | | |

(II.) *When the diameter or circumference is in Inches, and the length in Feet.*

N.B. To solve the following by the *four* lines A, B, C, D, see *Note* after Ex. 436, page 172.

| | | | | | Divisors for 4 lines. |
|---|---|---|---|---|---|
| (e) C | length in feet | $l \times 10$ | Cubic feet | | 183·35 |
| D | 13·54 | 42·82 | diam. in inches | | |
| (f) Ɔ | Cubic feet | Cubic feet ÷ 10 | length in feet | | |
| D | 13·54 | 42·82 | diam. in inches | | |
| (g) C | length in feet | $l \times 10$ | Cubic feet | | 1809·5 |
| D | 42·54 | 134·5 | circumf. in inches | | |
| (h) Ɔ | Cubic feet | Cubic feet ÷ 10 | length in feet | | |
| D | 42·54 | 134·5 | circumf. in inches | | |

* In Examples 429 and 430, $x = \frac{\sqrt{S} \times 1·128}{\sqrt{1}}$. Since the *two multipliers* are constant, the Slide must be inverted, as on page 111.

| | | | | | Divisors for 4 lines. |
|---|---|---|---|---|---|
| (i) | C | length in feet | $l \times 10$ | Cubic inches | ·1061 |
| | D | ·3257 | 1·030 | diam. in inches | |
| (k) | ꓭ | Cubic inches | Cubic inches ÷ 10 | length in feet | |
| | D | ·3257 | 1·03 | diam. in inches | |
| (l) | C | length in feet | $l \times 10$ | Cubic inches | 1·047 |
| | D | 1·023 | 3·235 | circumf. in inches | |
| (m) | ꓭ | Cubic inches | Cubic inches ÷ 10 | length in feet | |
| | ꓷ | 1·023 | 3·235 | circumf. in inches | |

**Ex. 432.**—How many Cubic *feet* are there in a cast-iron pillar, whose length is 8 feet, and diameter 5 *inches ?*

| | | | | | |
|---|---|---|---|---|---|
| (e) | C | 8 = $l$ | (80 | ·8) | 1·09 cub. in. |
| | D | 13·54 | (42·82 | 4·282) | 5 = diam. |

***Ex. 433.**—How many Cubic *feet* are there in a cylindrical log of timber, whose length is 24 feet, and circumference 39 *inches ?* (See under "Timber measuring.")

| | | | |
|---|---|---|---|
| (g) | C | 20·17 cub. ft. | 24 ft. long |
| | D | 39 = circumf. in inches | 42·54 |

**Ex. 434.**—From an iron bar $\frac{3}{4}$ inch in diameter, three pieces of 4, 5, and 7 *feet* are cut off. How many Cubic inches are there in each piece ? (See note to Ex. 427, to show that four lines are required.)

---

* In Ex. 432, if 8 on C is set over 13·54 on D, we cannot see what is over the 5 of D. Using the "check number" (page 90), we get $\frac{C}{D}\ \frac{80}{42{\cdot}82}$; but even this will not do, unless we reduce it as explained in page 87, and Ex. 246, by dividing C by 100, and D by 10 ; and then $\frac{C}{D}\ \frac{{\cdot}8}{4{\cdot}282}$.

(i)

| | | | |
|---|---|---|---|
| A | 21·2 cub. in. | 26·5 cub. in. | 37·1 cub. in. |
| B | 4 | 5 | 7 |
| C | | | ·1061 |
| D | | | ·75 = diam. |

**Ex. 435.**—What must be the diameter in inches of a Pump, capable of raising 26 Cubic feet of water per minute, the length of the stroke being $2\frac{1}{2}$ feet, and making 16 effective strokes in a minute. (Here the *length* is $16 \times 2\frac{1}{2} = 40$ feet.)

(e)

| | | |
|---|---|---|
| C | 26 Cubic feet | 40 = $l$ |
| B | 10·9 inches diam. | 13·54 |

**Ex. 436.**—$2\frac{1}{4}$ Cubic inches of copper were drawn out into a wire 149 feet long. Required the diameter of the wire in decimals of an inch.

(i)

| | | |
|---|---|---|
| C | 149 = $l$ | 2·25 cub. in. |
| D | ·3257 | ·04 inch |

*Note.* With reference to N.B. before Formula (*e*) page 170, the form, when the *our* lines are used, is

either

| | | |
|---|---|---|
| A | "Constant" | |
| B | length | |
| C | | Cubic content |
| D | | diam. or circumf. |

or

| | | |
|---|---|---|
| A | | Cubic content |
| B | | length |
| C | "Constant" | |
| D | diam. or circumf. | |

Thus Ex. 433 would be $x = \dfrac{24 \times 39^2}{1809{\cdot}5}$, or solved like Ex. 259.

| | |
|---|---|
| 1809·5 | |
| 24 | |
| | 20·17 |
| | 39 |

or

| | | |
|---|---|---|
| A | | 20·17 |
| B | | 24 |
| C | 1809·5 | |
| D | 39 | |

If a Cylinder is squared, its content will be reduced in the same proportion that the circular area of its section would be reduced if squared, as in Ex. 388, or in the proportion of 11 to 7.

For further particulars concerning "Cylinders," see Appendix E.

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## SPHERES. (Solid Content.)

[For contents in Gallons, Quarts, &c., see farther on, under Gauging.]

Let S = Solid content ; $d$ = diameter ; $c$ = circumference.

| Ex. 311. | Ex. 259. | Ex. 256. |
|---|---|---|
| $S = \begin{cases} d^3 \times \cdot 5236 \\ c^2 \times \cdot 01689 \end{cases}$ | $\frac{d^2 \times d}{1{\cdot}909}$ <br> $\frac{c^2 \times c}{59{\cdot}22}$ | $\frac{d^2 \times d}{1{\cdot}382^2}$ check 4·37 <br> $\frac{c^2 \times c}{7{\cdot}695^2}$ check 24·33 |
| Ex. 314. | Not solvable. | Ex. 433. |
| $d = \sqrt[3]{\frac{S}{\cdot 5236}}$ <br> $c = \sqrt[3]{\frac{S}{\cdot 01689}}$ | $\sqrt[3]{S \times 1{\cdot}909}$ <br> $\sqrt[3]{S \times 59{\cdot}22}$ | $\sqrt[3]{S \times 1{\cdot}382^2}$ <br> $\sqrt[3]{S \times 7{\cdot}695^2}$ |

*Sets of Rule for line E.*

| | | | | | |
|---|---|---|---|---|---|
| (I.) | E | ·5236 | 523·6 | 5·1 | Content |
| | D | 1 | 10 | 4·6 | Diameter |
| (II.) | E | ·0169 | 16·9 | 10 | Content |
| | D | 1 | 10 | 8·4 | Circumference |

| | | | | | |
|---|---|---|---|---|---|
| (III.) | E | 1 | 10 | 10000 | Content in Cubic feet |
| | D | 14·9 | 32·1 | 321 | Diameter in inches |

(IV.)

| | | | |
|---|---|---|---|
| E | 1 | 40 | Content in Cubic feet |
| D | 46·8 | 160 | Circumf. in inches |

**Ex. 437.**—A Sphere has a diameter of 7 inches, or a circumference of 22 inches. Required its contents in Cubic inches, by Formula (I.) and by Formula (II.)

(I.)

| | | |
|---|---|---|
| E | 5·1 | 179·6 cub. in. |
| D | 4·6 | 7 = diam. |

(II.)

| | | |
|---|---|---|
| E | 10 | 179·6 cub. in. |
| D | 8·4 | 22 = circumf. |

**Ex. 438.**—If a Sphere has a content of 221 Cubic feet, what is its diameter in feet?

(I.)

| | | |
|---|---|---|
| E | 5·1 | 221 Cubic feet |
| D | 4·6 | 7·5 feet diam. |

**Ex. 439.**—What is the circumference in inches, of a Sphere whose content is 98·4 Cubic inches?

(II.)

| | | |
|---|---|---|
| E | 16·9 | 98·4 |
| D | 10 | 18 in. circumf. |

**Ex. 440.**—Required the contents in Cubic inches, of four Spheres, whose circumferences are 3·9, 18·2, 39, and 84 inches.

(II.)

| | | | | | |
|---|---|---|---|---|---|
| E | 10 | 1 | 100 | 1000 | 10000 cub. in. |
| D | 8·4 | 3·9 | 18·2 | 39 | 84 circumfs. |

**Ex. 441.**—The diameter of a Sphere is 30 *inches*. What is its content in *Cubic feet?*

(III.)

| | | |
|---|---|---|
| E | 8·18 cub. ft. | 10 |
| D | 30 diam. in inches | 32·1 |

### Solve the following without the line E.

**Ex. 442.**—Take the dimensions of Ex. 437.

| | | | | |
|---|---|---|---|---|
| $\frac{7^2 \times 7}{1{\cdot}382^2}$ or $\frac{7^3}{1{\cdot}382^2}$ | C | 7 | (70) | 179·6 cub. in. |
| | D | 1·382 | (43·7 check) | 7 = diam. |

***Ex. 443.**—Take the dimensions of Ex. 439.

| | | | |
|---|---|---|---|
| $\sqrt[3]{98{\cdot}4 \times 7{\cdot}695^2}$ | ꓷ | 18 | 98·4 Cub. in. |
| | D | 18 | 7·695 |

*i.e.* the answer is 18 inches. In this Example we must either use the *Check* number, as $\frac{ꓷ}{D}\frac{9{\cdot}84}{24{\cdot}33}$ (N.B. page 111), or shift the Slide to

| | | |
|---|---|---|
| ꓷ | 800 | 8 |
| D | 27 | 2·7 |

as explained in Examples 247, 248.

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### Sphere and Inclosed Cube.

The distance from corner to corner of a Cube, *through the centre*, is the same as the diameter of a Sphere inclosing the Cube, and is = side of one edge of the Cube × $\sqrt{3}$, *i.e.* side × 1·73205. The Formula is

| | | | |
|---|---|---|---|
| A | 1·73205 | 7·1 | Diag. of Cube or diam. of Sphere |
| B | 1 | 4·1 | Side of Cube |

---

* The use of $\frac{ꓷ}{D}$ inverted is sometimes puzzling. (See N.B. to Ex. 322, and N.B. page 179); but Ex. 442 may *also* be solved by *inverting*, for Content of Sphere = $\frac{d^2 \times d}{1{\cdot}382^2}$. So if $d = 7$ we have

| | | |
|---|---|---|
| ꓷ | 1·796 Ans. | 7 |
| D | 1·382 (ch. 4·37) | 7 |

**Ex. 444.**—Three Cubes measure respectively, along their edges, 2·6, 3·0, and 7·5 inches. Required the lengths in inches of their diagonals or of the diameters of Spheres which would inclose them.

| | | | | |
|---|---|---|---|---|
| A | 7·1 | 4·5 | 5·2 | 13 diagonals |
| B | 4·1 | 2·6 | 3 | 7·5 sides |

**Ex. 445.**—The diameter of a Sphere is $35\frac{1}{2}$ inches. Required the side of the greatest Cube that can be cut from it.

| | | |
|---|---|---|
| A | 7·1 | 35·5 diam. of Sphere |
| B | 4·1 | 20·5 side of Cube |

---

## SPHERICAL SEGMENTS. (Solid Content.)

Let $r$ = radius of base, $h$ = height of Segmt. $\Big\}$ Solidity $= \dfrac{(3r^2 + h^2) \times h}{1{\cdot}91}$.

**Ex. 446.**—Required the Cubic inches in a basin whose depth is 5 inches, and diameter 16.

(Here $x = \dfrac{[(3 \times 8^2) + 5^2] \times 5}{1{\cdot}91} = \dfrac{217 \times 5}{1{\cdot}91} = 568$ Cubic inches.)

**Ex. 447.**—Required the Cubic inches in a small heap of grain, in the form of a Spherical segment, whose height is 1·75 inches, and diameter 5·5.

$$x = \frac{(3 \times 2{\cdot}75^2 + 1{\cdot}75^2) \times 1{\cdot}75}{1{\cdot}91} = \frac{24{\cdot}75 \times 1{\cdot}75}{1{\cdot}91} = 22{\cdot}68 \text{ Cub. in.}$$

---

## CYLINDRICAL RINGS. (Solid Content.)

Let $d$ = inner diameter, and $t$ = thickness of the ring: Then content $= \dfrac{(t + d) \times t^2}{{\cdot}405}$, as in Ex. 259; or content $= \dfrac{(t + d) \times t^2}{{\cdot}2013^2}$, as in Ex. 256.

**Ex. 448.**—Required the Cubic inches in an iron ring, whose inner diameter is 8 inches, and thickness 3 inches.

| | | |
|---|---|---|
| C | $11 = t + d$ | 244 cub. in. |
| D | ·2013 | 3 = thickness |

## PIPING. (Metal required.)

Let D = external diameter, and $d$ = internal diameter; *both in inches.* Then $\frac{(D + d) \times (D - d)}{\cdot 1061}$ is the content in Cubic inches of *every foot of length.*

**Ex. 449.**—How many Cubic inches of metal are there in *one foot length* of iron pipe, 8 inches bore, and $\frac{3}{4}$ inch thick?

$$\left(x = \frac{(9\cdot5 + 8) \times (9\cdot5 - 8)}{\cdot1061} = \frac{17\cdot5 \times 1\cdot5}{\cdot1061}.\right)$$

| | | |
|---|---|---|
| A | 17·5 | 247·4 cub. in. per foot |
| B | ·1061 | 1·5 |

## CONES. (Solid Content.)

Let S = solid content; $h$ = height; $d$ = diameter; $c$ = circumference.

For the four lines A, B, C, D.

$$S = \frac{d^2 \times h}{3\cdot82}, \text{ or } = \frac{c^2 \times h}{37\cdot7}, \text{ as in Ex. 259.}$$

$$h = \frac{S \times 3\cdot82}{d^2}, \text{ or } = \frac{S \times 37\cdot7}{c^2}, \text{ as in Ex. 260.}$$

$$d = \frac{\sqrt{S} \times \sqrt{3\cdot82}}{h}, \text{ or } c = \frac{\sqrt{S} \times \sqrt{37\cdot7}}{\sqrt{h}},$$

as in Ex. 262.

For the two lines C, D. (The "check number" for 1·954 is 6·18, and for 6·14 it is 19·42. See page 90.)

$$S = \frac{d^2 \times h}{1{\cdot}954^2}, \text{ or } = \frac{d^2 \times h}{6{\cdot}14^2}, \text{ as in Ex. 256.}$$

$$h = \frac{S \times 1{\cdot}954^2}{d^2}, \text{ or } = \frac{S \times 6{\cdot}14^2}{c^2}, \text{ as in Ex. 256.}$$

$$d = \frac{\sqrt{S} \times 1{\cdot}954}{\sqrt{h}}, \text{ or } c = \frac{\sqrt{S} \times 6{\cdot}14}{\sqrt{c}}, \text{ as in Ex. 257.}$$

Here 3·82, 37·7, are "Divisors," and 1·954, 6·14 (their Square Roots), are "Gauge Points." See APPENDIX K.

N.B. If the length of *the Slope* side is given, the height must be found, as explained in the *Note* after Ex. 452, below.

For a "right-angled" Cone, see next page.

**Ex. 450.**—How many Cubic inches are there in a Cone, the axis of which is 18 inches high, and the diameter $3\frac{1}{2}$ inches.

$$\frac{3{\cdot}5^2 \times 18}{1{\cdot}954^2}\ \frac{\text{C}}{\text{D}}\ \frac{18 \text{ height}}{1{\cdot}954 \text{ G. P.}}\ \frac{57{\cdot}7 \text{ cub. in.}}{3{\cdot}5 \text{ diam.}}, \text{ as in Ex. 256.}$$

**Ex. 451.**—How many Cubic feet are there in a Cone $10\frac{1}{2}$ feet high, the circumference of whose base is 9 feet?

$$\frac{9^2 \times 10{\cdot}5}{6{\cdot}14^2}\ \frac{\text{C}}{\text{D}}\ \frac{10{\cdot}5 \text{ height}}{6{\cdot}14 \text{ G. P.}}\ \frac{22{\cdot}56 \text{ cub. ft.}}{9 \text{ circumf.}}, \text{ as in Ex. 256.}$$

**Ex. 452.**—Required the height of a Cone whose content is 265 Cubic inches, and circumference 20 inches.

$$\frac{265 \times 6{\cdot}14^2}{20^2}\ \frac{\text{C}}{\text{D}}\ \frac{25 \text{ in.}}{6{\cdot}14 \text{ G. P.}}\ \frac{265 \text{ Content}}{20 \text{ Circumf.}}$$

*Note.*

When the length of *the slope side* of a Cone is given, together with the base (or circumference), the height may be found, as in Ex. 377; taking the slope length as the Hypothenuse, and $\frac{1}{2}$ diameter as the Base, of a Right-angled triangle. If the circumference is given, the diameter is found by Formula I., page 122.

## Right-angled Cones

Or such as have an apex angle = 90°. The preceding rules apply to these, but a still shorter process is available, in consequence of the diameter being always double the height; so that *both* need not be given: or if one is found the other is found. The following are the Formulæ:

$$\text{Content}\begin{cases} d^3 \times \cdot 1309 \text{ for the line E.} & \text{Ex. 311.} \\[2ex] \dfrac{d^2 \times d}{2{\cdot}764^2} \text{ for lines C, D.} & \text{Ex. 256.} \end{cases}$$

$$\text{Diameter}\begin{cases} \sqrt[3]{\dfrac{\text{Content}}{\cdot 1309}} \text{ for the line E.} & \text{Ex. 314.} \\[2ex] \sqrt[3]{\text{Content} \times 2{\cdot}764^2} \text{ for C, D.} & \text{Ex. 443.} \end{cases}$$

In a Right-angled Cone whose diameter at base is 2·11 inches, the content is 1·23 Cubic inches, and height 1·055 inches. So if the content is 67 Cubic inches, the diameter is 8 inches, and height 4 inches.

N.B. Since the use of $\frac{\text{ꓭ}}{\text{D}}$ inverted requires consideration, (N.B. to Ex. 322) an Example will be given, **Ex. 452½.** Find the diameter of a Right-angled Cone whose content is 226 Cubic inches.

$$\sqrt[3]{226 \times 2{\cdot}764^2}\ \frac{\text{ꓭ}}{\text{D}}\ \frac{12}{12}\text{————————————}\frac{226}{2{\cdot}764}$$

If we set the 226 of the *first* radius of ꓭ, over 2·764 on D, it will soon be evident that we cannot make two coinciding quantities on ꓭ and D; but we shall see $\frac{\text{ꓭ}}{\text{D}}\ \frac{10}{13{\cdot}15}$. Then shift the Slide till 10 in the middle of ꓭ is over 13·15 on D, and reading backwards a little on D, we come to 12 under 12 on ꓭ.

## PYRAMIDS. (Solid Content.)

$$\text{Solidity} = \frac{\text{Area of base} \times \text{perp. height}}{3}.$$

But if the sides of the base are equal, as they generally are in practice, use the following Formulæ :—

| | "Divisors" for ABCD. Ex. 259. | | "Gauge Points" for CD. Ex. 256. | Check numbers. |
|---|---|---|---|---|
| Square | $\frac{s^2 \times h}{3}$, | or | $\frac{s^2 \times h}{1{\cdot}732^2}$ . . . | 5·477. |
| Triangle | $\frac{s^2 \times h}{6{\cdot}925}$, | or | $\frac{s^2 \times h}{2{\cdot}631^2}$ . . . | 8·321. |
| Pentagon | $\frac{s^2 \times h}{1{\cdot}743}$, | or | $\frac{s^2 \times h}{1{\cdot}32^2}$ . . . | 4·175. |
| Hexagon | $\frac{s^2 \times h}{1{\cdot}154}$, | or | $\frac{s^2 \times h}{1{\cdot}074^2}$ . . . | 3·397. |
| Octagon | $\frac{s^2 \times h}{{\cdot}6213}$, | or | $\frac{s^2 \times h}{{\cdot}7882^2}$ . . . | 2·492. |
| Decagon | $\frac{s^2 \times h}{{\cdot}390}$, | or | $\frac{s^2 \times h}{{\cdot}6244^2}$ . . . | 1·975. |

**Ex. 453.**—What is the solidity of a square Pyramid, whose perpendicular height is 14 feet, and each side of its base $3\frac{1}{2}$ feet ?

$\frac{3{\cdot}5^2 \times 14}{1732^2}$ $\frac{\text{C}}{\text{D}}$ $\frac{14 \text{ height}}{1{\cdot}732}$ $\frac{57{\cdot}2 \text{ cub. ft.}}{3{\cdot}5 \text{ side}}$, as in Ex. 256.

**Ex. 454.**—How many Cubic inches are there in an hexagonal Pyramid whose perpendicular height is 9 inches, and each side of its base $2\frac{1}{2}$ inches ?

$\frac{2{\cdot}5^2 \times 9}{1.074^2}$ $\frac{\text{C}}{\text{D}}$ $\frac{2 \text{ height}}{1{\cdot}074}$ $\frac{(90)}{(3{\cdot}4 \text{ check})}$ $\frac{48{\cdot}9 \text{ cub. in.}}{2{\cdot}5 \text{ side}}$

The same remarks apply to the sloping sides of an equal-sided Pyramid compared with its height, as apply to "Cones" (*Note*, page 178); half of one of the sides on which it stands being the Base, and the sloping side the Hypothenuse.

## PRISMS. (SOLID CONTENT.)

**Solidity** = Area of base × length.

[I.] *When the base is not equal-sided.*

If the end is Triangular, its *area* must be found out in Ex. 373. If the end is a Rhombus, Rhomboid, Trapezoid or Trapezium, its *area* must be found as in Examples 365, 366, 367, 368.

Let there be a Triangular Prism,* the sides of the base of which are 2·21, 2·55, and 2·38 inches, and length 14 inches. The area of the base is found as in Ex. 373, to be $\sqrt{1{\cdot}36 \times 1{\cdot}02 \times 1{\cdot}19 \times 3{\cdot}57} = \sqrt{5{\cdot}89} = 2{\cdot}43$ Square inches; and this 2·43 multiplied by 14, the length, gives 34 Cubic inches as the Solid content.

So again, if the end of the Prism is a Trapezoid, of which the parallel sides are 18 and 12 feet, perpendicular 5 feet, and length 24 feet;—the area of the base, as by Ex. 367, is 75 Square feet, and the Solid content = 75 × 24 = 1,800 Cubic feet.

[II.] *If the base of the Prism is equal sided.*

Let $s$ = any one side; and $l$ = height.

The Solid content is found by the Formulæ following: column *a* with its "Divisors," being adapted to the *four* lines ABCD, as in Ex. 259; and column *b* with its "Gauge Points," for the *two* lines CD, as in Ex. 256.

* A "Wedge" differs from a Triangular Prism, in that its ends are not parallel. A "Prismoid" differs from a Prism, though its ends (or longer sides) are parallel and of the same figure, in that they are not of the same area.

| | (*a*) | | (*b*) | Check number. |
|---|---|---|---|---|
| Triangular | $\frac{s^2 \times l}{2{\cdot}31}$, | or | $\frac{s^2 \times l}{1{\cdot}519^2}$ . . . . | 4·81. |
| Pentagonal | $\frac{s^2 \times l}{{\cdot}581}$, | or | $\frac{s^2 \times l}{{\cdot}762^2}$ . . . | 2·41. |
| Hexagonal | $\frac{s^2 \times l}{{\cdot}385}$, | or | $\frac{s^2 \times l}{{\cdot}620^2}$ . . . | 1·96. |
| Octagonal | $\frac{s^2 \times l}{{\cdot}207}$, | or | $\frac{s^2 \times l}{{\cdot}455^2}$ . . . | 1·44. |
| Decagonal | $\frac{s^2 \times l}{{\cdot}130}$, | or | $\frac{s^2 \times l}{{\cdot}360^2}$ . . . | 1·14. |

**Ex. 455.**—Required the content in Cubic feet, of an equilateral triangular Prism, 12 feet long, each side of its base being 2½ feet.

$\frac{2{\cdot}5^2 \times 12}{1{\cdot}52^2}$ $\frac{\text{C}}{\text{D}} \frac{12 \text{ length}}{1{\cdot}52 \text{ G. P.}} \frac{32{\cdot}5 \text{ cub. ft.}}{2{\cdot}5 \text{ side of base}}$, as in Ex. 256.

**Ex. 456.**—What is the content in Cubic inches, of an equilateral Hexagonal Prism, 8 feet long, each side of its base being 1½ inches?

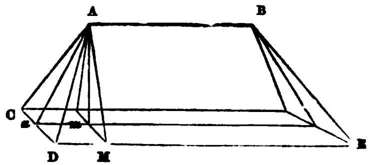
$\frac{1{\cdot}5^2 \times 8}{{\cdot}620^2}$ $\frac{\text{C}}{\text{D}} \frac{8 \text{ length}}{{\cdot}620 \text{ G. P.}} \frac{46{\cdot}8 \text{ cub. in.}}{1{\cdot}5 \text{ side of base}}$, as in Ex. 255.

*Note.*

If the height is given in *feet*, and the length of a side in *inches*, multiply the "Gauge Points" in column *b* by 12. Thus in a Pentagonal Prism, 20 *feet* high, and 9 *inches* to a side, the content is 19·36 Cubic feet, or $\frac{\text{C}}{\text{D}} \frac{19{\cdot}36 \text{ cub. ft.}}{9} \frac{20 = \text{height}}{9{\cdot}14 = {\cdot}762 \times 12}$.

## WEDGES. (Solid Content.)

Let $e$ = edge AB.
$l$ = length of base = DE.
$w$ = width of base = DC.
$h$ = perpendicular height Am.




The base is always the line parallel to the edge.

$$\text{Solidity} = \frac{(2l + e) \times w \times h}{6},$$

Or if the dimensions are given in inches, and the content required in *Square feet*, the "Divisor" is 10368 (= 6 × 1728). Or if the dimensions are given in feet, and the content required in *Square yards*, the "Divisor" is 162 (= 6 × 27).

**Ex. 457.**—What is the content in Cubic inches, of a Wedge whose base is 18 by 8 inches, its edge at top 12 inches, and its perpendicular height 9 inches?

$$x = \frac{(36 + 12) \times 8 \times 9}{6} = \frac{48 \times 72}{6} = \frac{A}{B} \quad \frac{48}{6} \quad \frac{576 \text{ cub. in.}}{72}$$

N.B. It is often required *in practice*, to find the perpendicular height from the measured slope, as follows: measure straight down AM, or anywhere else along the slope, and call this $s$. Measure $\frac{1}{2}$ the width, or $a$ D, and call this $d$. Then we have the Hypothenuse and Base of a right-angled Triangle to find the perpendicular A$m$ = $\sqrt{s^2 - d^2}$, or $\sqrt{(s + d) \times (s - d)}$ as in Ex. 277.

Thus if AM measure 9·85 feet, and $a$D (= $m$M) 4 feet,

$$x = \sqrt{13{\cdot}85 \times 5{\cdot}85} = \frac{C}{D} \quad \frac{13{\cdot}85}{13{\cdot}85} \quad \frac{5{\cdot}85}{9 = Am}.$$

## Regular Solids, equal Facets.

Let $s$ = side or edge of any one Facet. Then the Solidity is as follows :

| Tetraedron or Triangular Pyramid. 4 sides. | Hexaedron or Cube. 6 sides. | Octaedron or Cube with corners cut off. |
|---|---|---|
| $s^3 \times \cdot 11785$, Ex. 311. | $s^3$, Ex. 321. | $s^3 \times \cdot 4714$, Ex. 311. |
| $\frac{s^2 \times s}{8 \cdot 486}$, Ex. 259. | | $\frac{s^2 \times s}{2 \cdot 121}$, Ex. 259. |
| $\frac{s^2 \times s}{2 \cdot 913^2}$, Ex. 256. | | $\frac{s^2 \times s}{1 \cdot 456^2}$, Ex. 256. |

**Ex. 458.**—Required the content in Cubic inches of an Octaedron, of which the side or edge of any one Facet is 4 inches.

| | | | |
|---|---|---|---|
| Either $4^3 \times \cdot 4714$ | E | ·4714 | 30·2 cub. in. |
| | D | 1 | 4 |
| or $\frac{4^2 \times 4}{1 \cdot 456^2}$ | C | 4 | 30.2 cub. in. |
| | D | 1·456 | 4 |

## FRUSTRUMS. (Solid Content.)

### [I.] Frustrums of Square Pyramids.

Let A = area of greater end.
$a$ = area of smaller end.
$l$ = perpendicular distance between the ends.
S = length of a side at base.
$s$ = length of a side at top.

$$\text{* Solidity} = \frac{(A + d) + (S \times s) \times l}{3}.$$

* The general Formula for *all* Frustrums, is

$$\text{Solidity} = \frac{(A + a + \sqrt{A \times a}) \times l}{3},$$

**Ex. 459.**—Required the content in Cubic feet of the Frustrum of a Square Pyramid, whose perpendicular height is 5 feet; its base a square of 3 feet to a side; and its upper end a square of $2\frac{1}{2}$ feet to a side.

$$\text{Here Solidity} = \frac{(3^2 + 2\cdot5^2) \times (3 \times 2\cdot5) \times 5}{3} = \frac{(9 + 6\cdot25 + 7\cdot5) \times 5}{3}$$

$$= \frac{22\cdot75 \times 5}{3} = \frac{\text{A} \qquad 22\cdot75 \qquad\qquad 37\cdot9 \text{ cub. ft.}}{3 \qquad\qquad 5}.$$

## [II.] Conical Frustrums.

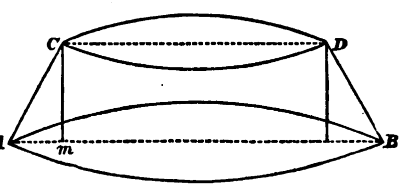
Let D = diameter (or C = circumference) at the greater end, and $d$ = diameter (or $c$ = circumference) at the lesser end; and $l$ = perpendicular height.

$$\textbf{Solid Content} = \frac{(D^2 + d^2 + Dd) \times l}{3\cdot82}, \text{ or } \frac{[(D + d)^2 - Dd] \times l}{3\cdot82}.$$

$$\text{or} = \frac{(C^2 + c + Cc) \times l}{37\cdot7}, \text{ or } \frac{[(C + c^2) - Cc] \times l}{37\cdot7}.$$

N.B. If the dimensions are given in inches, and the content required in Cubic *feet*, the Divisors will be 6600, and 65144, respectively.

**Ex. 460.**—The perpendicular† height of a Conical frustrum is 4 inches. At the lower end the diameter is 20 inches, and at the upper end it is 14 inches. Required the content in Cubic inches.




but the Formulæ for Frustrums of Pyramids and Cones given in pages 184 185, are more convenient, as it is sometimes troublesome to get $\sqrt{A \times a}$, when A and $a$ are large numbers.

†See "Remarks," page 187.

$$\text{Solid content} = \frac{(20^2 + 14^2 + 20 \times 14) \times 4}{3 \cdot 82}$$

The Slide Rule, as in page 83, gives at once $20^2 = 400$, and $14^2 = 196$. Then we have $\frac{(400 + 196 + 280) \times 4}{3 \cdot 82} = \frac{876 \times 4}{3 \cdot 82}$, solved as follows:

| | | |
|---|---|---|
| A | 4 | 917 cub. in. |
| B | 3·82 Divisor | 876 |

The exact answer is 917·3472.

**Ex. 461.**—How many Cubic feet of timber are there in a tapering rounded log, whose length is 14 feet: the circumference at one end being 11 feet, and at the other $5\frac{1}{2}$ feet?

$$\text{Solid content} = \frac{(11^2 + 5 \cdot 5^2 + 11 \times 5 \cdot 5) \times 14}{37 \cdot 7}.$$

The Slide Rule gives at once the squares of 11 and of 5·5, as in page 83; and also $11 \times 5 \cdot 5 = 60 \cdot 5$. Then we have $\frac{(121 + 30 \cdot 25 + 60 \cdot 5) \times 14}{37 \cdot 7} = \frac{211 \cdot 75 \times 14}{37 \cdot 7}$, solved as follows:

| | | |
|---|---|---|
| A | 78·6 cub. ft. | 211·75 |
| B | 14 | 37·7 Divisor |

The exact answer is 78·6356. (See "Remarks," page 187.)

**Ex. 462.**—Required the content in Cubic *feet* of a Conical frustrum, whose dimensions in *inches* are as follows: Height 20. Lower diameter 28. Upper diameter 20.

Here, the squares of 28 and 20, also $28 \times 20 = 560$, being easily found (as in the previous Examples) by the Slide Rule, we have Solid content $= \frac{(784 + 480 + 560) \times 20}{6600} = \frac{1744 \times 20}{6600}$, solved as follows:

| | | |
|---|---|---|
| A | 5·28 cub. ft. | 1744 |
| B | 20 | 6600 Divisor |

The exact answer is 5·2845 Cubic feet.

### [III.] Regular Polygonal Frustrums.

Let S = length of a side at the greater end ; $s$ = length of a side at the lesser end; and $l$ = the perpendicular height between the ends. Then

$$\text{Solid content} = \frac{(S^2 + s^2 + Ss) \times l}{T}.$$

The values of T are as follows : *

| | |
|---|---|
| Trigon 69·28.<br>Pentagon 1·744.<br>Hexagon 1·155.<br>Octagon ·6213. | N.B. If the dimensions are given in inches, and the content required in Cubic *feet*, the Divisors will be 11970, 3013, 1995, and 1073, respectively. |

**Ex. 463.**—What is the content of a Pentagonal frustrum, the height of which is 5 feet, each side of its base 1 foot 6 inches, and each side of the upper end 6 inches ?

The Slide Rule easily finds $18^2 = 324$, and $18 \times 6 = 108$. Then we have the Solid content $= \frac{(324 + 36 + 108) \times 60}{1·744} = \frac{468 \times 60}{1·744}$ solved as follows :

| | | |
|---|---|---|
| A | 468 | 16100 cub. in. |
| B | 1·744 Divisor | 60 |

Or, with reference to the N.B. above, we can obtain the Cubic *feet* at once by using the Divisor 3013.

| | | |
|---|---|---|
| A | 468 | 9·32 cub. ft. |
| B | 3013 Divisor | 60 |

The *exact* answer is 16103·664 Cubic inches, or 9·31925 Cubic feet.

### "Remarks."

It frequently happens, in the case of Conical frustrums, that the perpendicular height is not given, but has to be computed from the given *slope* height. In such a case we have a right-angled triangle whereof the Hypothenuse (the slope) is given, as also the Base (half the difference of the top and bottom diameters), to find the Perpendicular, as in Ex. 277.

* The divisors are found by dividing 3 by ·433, 1·7205, 2·598, 3·634, the Tubular constants shown in page 146.

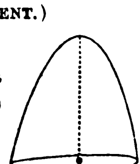
Thus in the figure to Ex. 460, the measured slope AC was 5 inches, and A$m$ (half the difference between the top and bottom diameters) 3 inches. Hence C$m$, the perpendicular height $= \sqrt{(5 + 3) \times (5 - 3)} = \sqrt{16} = 4$, Ex. 277.

When the angle AC$m$ is small, that is, when A$m$ bears a small proportion to AC, as in long conical castings or rounded timber, no practical error arises from using the slope length for the perpendicular length. For instance, in Ex. 461, the height assumed is 14 feet, whereas in reality it was 13 feet $11\frac{2}{3}$ inches, or 13·973 feet. The error of *content* arising from using 14, is only $\frac{1}{1230}$. In that Example (461), the Base of the Right-angled triangle $= \frac{3·5 - 1·75}{2} =$ ·875 feet (3·5 and 1·75 are the *diameters*), and the Hypothenuse or slope side is 14 feet. Again, if the slope height is 20 feet, and the diameters 2 feet and 4 feet, the Perpendicular would only be $\sqrt{240^2 + 12^2}$, or $\frac{3}{10}$ of an inch less than 20 feet, and this is a very common slope. Here $\frac{D - d}{2}$ is $\frac{1}{20}$ of Slope, and $\frac{1}{800}$ of Slope is deducted to find the Perpendicular.

If the Base, or $\frac{D - d}{2}$, is $\frac{1}{5}$ of the Slope, deduct from the Slope $\frac{1}{50}$.

| | | | | |
|---|---|---|---|---|
| " | $\frac{1}{7}$ | " | " | $\frac{1}{100}$. |
| " | $\frac{1}{10}$ | " | " | $\frac{1}{200}$. |
| " | $\frac{1}{14}$ | " | " | $\frac{1}{400}$. |
| " | $\frac{1}{16}$ | " | " | $\frac{1}{520}$. |
| " | $\frac{1}{20}$ | " | " | $\frac{1}{800}$. |

## PARABOLIC CONOIDS. (Solid Content.)



Since the content of a Parabolic Conoid is that of a Cylinder of half the length, we have (as in page 167)

$$S = \frac{d^2 \times \frac{1}{2} l}{1·128}.$$

| | | | |
|---|---|---|---|
| C | ½ length | ½ length × 10 | Cubic content |
| D | 1·128 | 3·568 | Diameter |

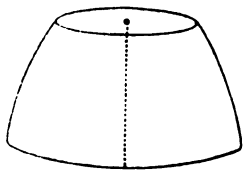
**Ex. 464.**—Required the content in Cubic inches, of a vessel shaped as above, whose diameter at the base is 40 inches, and depth 45 inches.

| | | |
|---|---|---|
| C | 22·5 | 2828 cub. in. |
| D | 1·128 | 40 |

## FRUSTRUMS OF PARABOLIC CONOIDS.

When the difference of the diameters is 6 inches or under, multiply it by ·55 ; but if it exceed 6 inches, by ·57. Add the product to the less diameter, and the sum will be the "mean diameter" of a Cylinder of equal height.

**Ex. 465.**—Required the content in Cubic inches of the figure, whose base diameter is 35 inches, and upper diameter 28 inches ; the depth being 18 inches. (Mean diameter = 31·99, or 7 × ·57.)



| | | | | |
|---|---|---|---|---|
| C | 18 | 1800 | 14468 cub. in. | , as in Ex. 424. |
| D | 1·128 | 11·28 | 32 | |

### [IV.] Frustrums of Wedges.

The Solidity is generally computed as a "Prismoid." See Ex. 466.

## PRISMOIDS

Are solids, regularly tapering or otherwise, of which the two parallel ends are Rectangles, Trapezoids, or Triangles ; the parallel surfaces, though of the same *shape*, have different *areas*. The *length* of a

Prismoid is the perpendicular distance between its two parallel surfaces or *ends.**

Let A = area of greater end.
† $a$ = area of lesser end.
$l$ = length between ends.
$m$ = area of "Middle section" as explained farther on.

$$\text{Solidity} = \frac{(A + a + 4m) \times l}{6}.$$

N.B. If the measurements are taken in inches, and the content required in Cubic feet, the "Divisor" is 10368 (= 6 × 1728). If the measurements are taken in feet, and the content required in Cubic yards—as in Railway Cuttings, Embankments, &c. (see APPENDIX F), the "Divisor" is 162 (= 6 × 27). If the length is given in feet and the other dimensions in inches, and the content required in Cubic feet, the "Divisor" is 864 (= 6 × 144).

## To find *m*.

*m* is the area of the "middle section," *i.e.* a section parallel to, and equidistant from, the two parallel surfaces or ends of a Solid.

(I.) If the ends are *Rectangles* as in Examples 466, 467, 468, *m* is the mean of their breadths multiplied by the mean of their depths.
(II.) If the ends are *Trapezoids* as in Ex. 470,

$$m = \frac{\text{Sum of 4 breadths}}{4} \times \frac{\text{Sum of 2 perpendiculars}}{2}.$$

(III.) If the ends are Triangles as in Ex. 469, *m* is the area of a triangle whose base is the mean of their Bases, and whose perpendicular is the mean of their Perpendiculars.

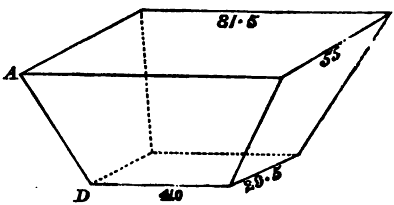
---

* Frustrums of Pyramids and of Cones may be considered regularly tapering Prismoids, and the contents may be found by the Prismoidal Formula, which, however, is not so convenient as that generally used. (See pages 184 and 185.)

† Sometimes the "area" of the lesser end = o, as in Ex. 471.

N.B. The whole $4m$ required in the Formula, may be obtained *at once*, by multiplying the *sum* of the mean breadths by the sum of the mean depths. This shortens the process, as will be seen in the following Examples.

**Ex. 466.**—What is the content of a Railway coal waggon of which the top measures $81\frac{1}{2}$ by 55 inches, sloping to 41 by $29\frac{1}{2}$ inches at the bottom; the perpendicular depth being $47\frac{1}{4}$ inches?



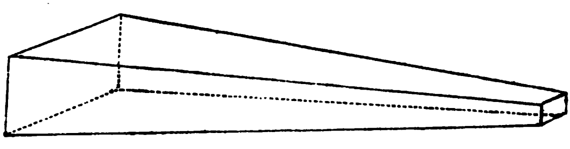

$$\left.\begin{array}{rcrcl} 81{\cdot}5 \times 55 & = & 4482{\cdot}5 & = & A \\ 41{\cdot}0 \times 29{\cdot}5 & = & 1209{\cdot}5 & = & a \\ \hline 122{\cdot}5 \times 84{\cdot}5 & = & 10351{\cdot}25 & = & 4m \\ & & \overline{16043{\cdot}25} & & \end{array}\right\}$$

In this Example $m = \dfrac{81{\cdot}5 + 41}{2} \times \dfrac{55 + 29{\cdot}5}{2} = 2587{\cdot}8125.$

Then $\dfrac{16043{\cdot}25 \times 47{\cdot}25}{6} = \dfrac{A}{B}$ $\dfrac{126000 \text{ cub. in.}}{47{\cdot}25}$ $\dfrac{16043{\cdot}25}{6 \text{ divisor}}$

Or with reference to N.B. page 190 $\dfrac{A}{B}$ $\dfrac{72{\cdot}9 \text{ cub. ft.}}{47{\cdot}25}$ $\dfrac{16043{\cdot}25}{10368 \text{ divisor}}$

**Ex. 467.**—Required the content in Cubic feet of a Prismoidal iron casting, whose lower surface is at right angles to the ends, which are Rectangles of 16 by 12, and 7 by 4, inches; the perpendicular length being 13 *feet*.



$$\left.\begin{array}{l} 16 \times 12 = 192 = A \\ \ \,7 \times \ \ 4 = \ \ 28 = a \\ \hline 23 \times 16 = 368 = 4m \\ \qquad\qquad\quad \underline{588} \end{array}\right\} \text{In this Example } m = \frac{16+7}{2} \times \frac{12 \times 4}{2} = 92.$$

$$\text{Then } \frac{588 \times 156}{6} = \frac{A}{B} \ \frac{588}{6} \text{------------} \frac{15288 \text{ cub. in.}}{156}$$

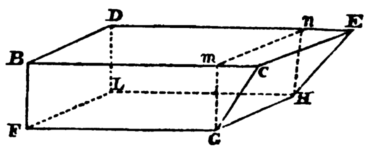
$$\text{Or } \frac{588 \times 156}{10368} = \frac{A}{B} \ \frac{588}{10368} \text{------------} \frac{8{\cdot}8472 \text{ cub. ft.}}{156}$$

Or if we keep the length in feet, *i.e.* 13, and the rest in inches,

$$\frac{588 \times 156}{864} = \frac{A}{B} \ \frac{588}{864} \text{------------} \frac{8{\cdot}8472 \text{ cub. ft.}}{156}$$

**Ex. 468.**—In the figure, where one of the sides is perpendicular to the ends,

Let BD or FL = 4 feet.
BF or DL = 3 feet.
DE or BC = 12 feet.
FG or LH = 8 feet.
B$m$ or D$n$ = 8 feet.




Required the content in Cubic feet.

$$\left.\begin{array}{l} 12 \times 4 = \ \ 48 = A \\ \ \ 8 \times 4 = \ \ 32 = a \\ \hline 20 \times 8 = 160 = 4m \\ \qquad\qquad\ \ \underline{240} \end{array}\right\} \text{Or } m = \frac{12+8}{2} \times \frac{4+4}{2} = 40.$$

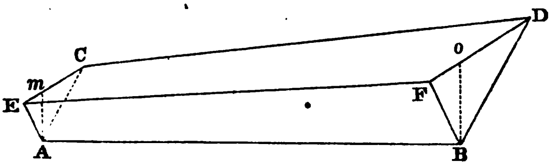
$$\text{Then } \frac{240 \times 3}{6} \ \frac{A}{B} \ \frac{240}{6} \text{------------} \frac{120 \text{ cub. ft.}}{3}$$

This may be tested by computing first, the content of the Parallelopiped FBD$nm$G, and then the Wedge $nm$GHE ; as follows

I. Parallelopiped = $3 \times 4 \times 8 = 96$ Cubic feet.

II. Wedge, where length of base $mn = 4$; edge $CE = 4$; width of base $mG = 3$; and perpendicular height $mC = 4$. (See Ex. 457.) $\frac{(8 + 4) \times 3 \times 4}{6} = 24$ Cubic feet. Then $24 + 96 = 120$ Cubic feet, as before.

**Ex. 469.**—Required the content in Cubic *yards* of an excavation in the form of a triangular trench, the triangular ends being parallel to each other, and perpendicular.




The measurements are as follows :

$$\left.\begin{array}{llll} EC = 60 \text{ ft.} & & FD = 105 \text{ ft.} \\ mA = 20 \text{ ft.} & & oB = 35 \text{ ft.} \end{array}\right\} \text{Length 594 feet from E to F, or A to B.}$$

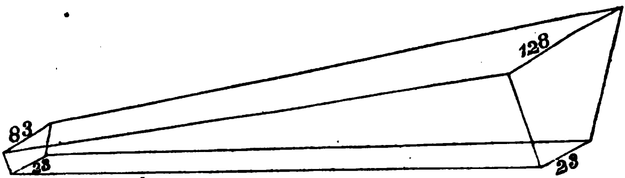
$$\left.\begin{array}{rcl} 20 \times 30 & = & 600{\cdot}0 = a \\ 35 \times 52\tfrac{1}{2} & = & 1837{\cdot}5 = A \\ \hline 55 \times 82{\cdot}5 & = & 4537{\cdot}5 = 4m \\ & & \overline{6975{\cdot}0} \end{array}\right\}$$

Here $m$ = area of a triangle whose base is the mean of 105 and 60, and whose perpendicular is the mean of 35 and 20; or $m = \frac{82{\cdot}5 \times 27{\cdot}5}{2} = 1134{\cdot}375$.

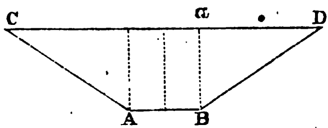
Then $\frac{6975 \times 594}{162} = \frac{A}{B} \frac{6975}{162 \text{ “divisor”}} \frac{25575 \text{ cub. yds.}}{594}$

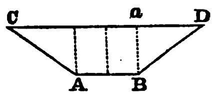
With the Formula “divisor” 6, we should have $\frac{6975 \times 594}{6}$ $= 690525$ Cubic *feet*. (N.B. page 190.)

**Ex. 470.**—Let the figure represent one of the sections of a Railway cutting, the perpendicular ends being Trapezoids, and the Roadway 594 feet long, and 23 feet wide throughout.




Smaller Trapezoid 83 feet wide at top, and perpendicular depth 20 feet. Larger Trapezoid 128 feet wide at top, and perpendicular depth 35 feet.



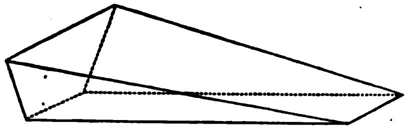




Required the content in Cubic *yards*.

$$\left.\begin{array}{llll} \text{Mean width.} & \text{Depth.} & & \\ 53 & \times\ 20 & = 1060 & = a \\ 75{\cdot}5 & \times\ 35 & = 2642{\cdot}5 & = A \\ \hline 128{\cdot}5 & \times\ 55 & = 7067{\cdot}5 & = 4m \\ & & \overline{1077{\cdot}0} & \end{array}\right\} m = \frac{83+128+23+23}{4}\times\frac{35+20}{2} = 64{\cdot}25\times27{\cdot}5 = 1766{\cdot}875.$$

$$\text{Then } \frac{10770\times594}{162} = \frac{A}{B}\quad\frac{10770}{162}\quad\frac{39490 \text{ cub. yards}}{594}$$

The "divisor" 162 is taken with reference to N.B. page 190. Had the content been required in Cubic *feet*, $x = \dfrac{10770\times594}{6} = 1066230$ Cubic *feet*.



**Ex. 471.**—Let the figure represent the last "section" of a Railway cutting, when the "area" at one end = 0. Length 720 feet. Roadway 30 feet wide throughout. One end a Trapezoid 230 feet wide at the top, 30 at the base, and perpendicular height 150 feet. Required the content in Cubic *yards.*

$$\begin{array}{rcl} 155 \times 50 = 7750 & = & A \\ 30 \times 0 = 0 & = & a \\ \hline 185 \times 50 = 9250 & = & 4m \\ \hline 17030 & & \end{array}$$

Then $\dfrac{17030 \times 720}{162} = \dfrac{A}{B} \quad \dfrac{17030}{162} \quad \dfrac{75620 \text{ cub. yards}}{720}$

Had the usual "divisor" 6 (N.B. page 190) been used, the content would be Cubic *feet* 2040000.

(See "Railway Cuttings and Embankments." Appendix F.)

## GAUGING (for Liquid Contents).

| | | Parallelopipeds. | | | Cylinders. | | Spheres. | |
|---|---|---|---|---|---|---|---|---|
| | | F.F.F. | F. I. I. | I. I. I. | F. I. | I. I. | F. | I. |
| Bushels | Divisor | 1·2837 | 184·85 | 2218·2 | 235·36 | 2824·3 | — | — |
| | G. P. | 1·133 | 13·595 | 47·10 | 15·34 | 53·14 | — | — |
| | *Check* | 3·583 | 42 94 | 148·9 | 48·51 | 168·1 | — | — |
| Gallons | Divisor | ·16046 | 23·106 | 277·274 | 29·419 | 353·036 | ·30645 | 529·55 |
| | G. P. | ·4005 | 4·806 | 16·65 | 5·424 | 18·79 | ·5535 | 23·01 |
| | *Check.* | 12·66 | 15·20 | 52·66 | 17·15 | 59·42 | 1·750 | 7·277 |
| Quarts | Divisor | ·04011 | 5·776 | 69·318 | 7·335 | 88·258 | — | 132·39 |
| | G. P. | — | 2·403 | 8·326 | 2·712 | 9·395 | — | 11·51 |
| | *Check.* | — | 7·600 | 26·33 | 8·546 | 29·71 | — | 36·38 |
| Ounces | Divisor | ·00100 | ·1444 | 1·733 | ·1839 | 2·206 | — | 3·308 |
| | G. P. | — | ·380 | 1·316 | ·4288 | 1·485 | — | 1·819 |
| | *Check.* | — | 1·202 | 4·163 | 1·356 | 4·697 | — | 5·752 |

N.B. 1. The "Divisors" are to be used with the *four* lines A, B, C, D. The "Gauge Points" are to be used with the *two* lines C, D. The "Check numbers" are to be used with C, D, when the Gauge Points are inconvenient. (See page 90.) For explanation of "Divisors" and "Gauge Points" see Appendix K.

N.B. 2. The letters F. F. I. I. refer to the *measurements* of the vessels to be gauged, whether in Feet or Inches: for instance, if we have a Parallelopiped whose length is given in Feet, and depth and breadth in Inches, and we want the content in *Bushels*, we look under F. I. I., and use 184·85 as a "Divisor," with the lines A, B, C, D; or

13·595 as a "Gauge Point" (its *check number* being 42·94) with the lines C, D. If we have to find the *Gallons* in a CYLINDER whose depth is given in Feet, and diameter in Inches, we look under F. I., and use 29·42 as a "Divisor," or 5·424 as a "Gauge Point."

---

## Formulæ for PARALLELOPIPEDS.

| A | Product of any 2 sides | Content |
|---|---|---|
| B | "Divisor" | 3d side |

But if any two of the sides are *the same*, take the following:

$$\text{Content} = \frac{\text{Sq. side}^2 \times \text{3d side}}{\text{Divisor}}$$

| A | "Divisor" | |
|---|---|---|
| B | 3d side | |
| C | | Content |
| D | | Square side |

OR

$$\text{Content} = \frac{\text{Sq. side}^2 \times \text{3d side}}{\text{G. P.}^2}$$

| C | 3d side | 3d side × 10 | Content |
|---|---|---|---|
| D | G. P. | Check No. | Square side |

---

## Formulæ for CYLINDERS.

$$\text{Content} = \frac{\text{diam.}^2 \times \text{length}}{\text{Divisor}}$$

| A | "Divisor" | |
|---|---|---|
| B | Length | |
| C | | Content |
| D | | Diameter |

OR

$$\text{Content} = \frac{\text{diam.}^2 \times \text{length}}{\text{G. P.}^2}$$

| C | Length | Length × 10 | Content |
|---|---|---|---|
| D | G. P. | Check No. | Diameter |

also

| C | 44 | 490 | Gallons for every foot deep |
|---|---|---|---|
| D | 3 | 10 | Feet diameter of Cylinder |

## Formulæ for SPHERES.

$$\text{Content} = \frac{\text{Diam.}^2 \times \text{diam.}}{\text{Divisor}} \left\{ \begin{array}{llll} \text{A} & \text{“Divisor”} & & \\ \hline \text{B} & \text{Diameter} & & \\ \text{C} & & & \text{Content} \\ \hline \text{D} & & & \text{Diameter} \end{array} \right.$$

OR

$$\text{Content} = \frac{\text{diam.}^2 \times \text{diam.}}{\text{G. P.}^2} \quad \begin{array}{llll} \text{C} & \text{Diam.} & \text{Diam.} \times 10 & \text{Content} \\ \hline \text{D} & \text{G. P.} & \text{Check No.} & \text{Diameter} \end{array}$$

**Ex. 472.**—Required the content in *Bushels*, of a rectangular floor of malt, measuring 6 feet by 48 inches, and 5 inches deep.

$$\begin{array}{lll} \text{A} & 30 \ (= 6 \times 5) & 7{\cdot}78 \text{ Bushels} \\ \hline \text{B} & 184{\cdot}85 \text{ Divisor} & 48 \end{array}$$

**Ex. 473.**—How many Gallons are there in a ship's tank which is 4 feet 1 inch square at the top, and 6 feet 1 inch deep? (inside measure.)

$$\begin{array}{lll} \text{C} & 73 \text{ inches} & 6320 \text{ gallons} \\ \hline \text{D} & 16{\cdot}65 \text{ G. P.} & 49 = \text{Square side inches} \end{array}$$

The *exact* content is 6321·3 gallons.

**Ex. 474.**—To what depth in inches will 3 Fluid ounces fill a rectangular vessel that measures $6\frac{1}{2}$ by $5\frac{1}{4}$ inches?

$$\begin{array}{lll} \text{A} & 34{\cdot}125 \ (= 6\tfrac{1}{2} \times 5\tfrac{1}{4}) & 3 \text{ ounces} \\ \hline \text{B} & 1{\cdot}733 \text{ “Divisor”} & {\cdot}152 \text{ inch} = 3\text{d side} \end{array}$$

***Ex. 475.**—What is the content in *Gallons* of a cylindrical vessel, 40 inches deep, and 27 inches in diameter?

---

* For practice, the learner may try Ex. 475 with the *four* lines, as follows:

$$\left\{ \begin{array}{lll} \text{A} & 353 \text{ Divisor} & \\ \hline \text{B} & 40 & \\ \text{C} & & 82{\cdot}6 \text{ Ans.} \\ \hline \text{D} & & 27 \end{array} \right.$$

| | | |
|---|---|---|
| C | 40 inches deep | 82·6 gallons |
| D | 18·79 G. P. | 27 in. diam. |

***Ex. 476.**—How many *Gallons* of water are lifted at each stroke of a $9\frac{1}{2}$ inch pump, the length of the stroke being $2\frac{1}{4}$ feet ?

| | | |
|---|---|---|
| C | 2·25 feet deep | 6·9 gallons |
| D | 5·424 G. P. | 9·5 in. diam. |

**Ex. 477.**—Required the diameter in inches of a pipe 20 inches long, to contain 5 *gallons*.

| | | | |
|---|---|---|---|
| C | 200 | 2 | 5 |
| D | 59·42 check | 5·942 | 9·4 inches |

**Ex. 478.**—Required the length in feet of a pipe 3 inches in diameter, to contain 2 gallons.

| | | |
|---|---|---|
| C | 2 gallons | 6·53 feet |
| D | 3 | 5·42 G. P. in. diam. |

**Ex. 479.**—Required the *Quarts* in a cylindrical measure, 15 inches deep, and 11 inches in diameter.

| | | | |
|---|---|---|---|
| C | 15 | 26·6 Quarts | 150 |
| D | 9·395 G. P. | 11 | 29·71 check |

N.B. In this Example, as in Ex. 424, it is necessary to use the *check number* with 10 times 15 inches over it. See page 90.

**Ex. 480.**—There are 4 Cylinders whose diameters are respectively 2, $3\frac{1}{2}$, $3\frac{3}{4}$, and 4 feet. How many Gallons would there be in *a foot depth* of each Cylinder? (See last line, page 197.)

---

* For practice, the learner may try Ex. 476 with the *four* lines, as follows :

| | | |
|---|---|---|
| A | 29·42 Divisor | |
| B | 2·25 | |
| C | | 6·9 Ans. |
| D | | 9·5 |

| | | | | | |
|---|---|---|---|---|---|
| C | 19·6 | 44 | 60 | 68·8 | 78·3 Gallons |
| D | 2 | 3 | 3·5 | 3·75 | 4 |

**Ex. 481.**—Required the diameter in inches of a Cylinder, which when filled to the height of 28 feet, contains 8566 *Gallons*.

| | | |
|---|---|---|
| C | 28 ft. long | 8566 Gallons |
| D | 5·424 G. P. | 30 in. diam. |

Here $x = \sqrt{\dfrac{8566 \times 5{\cdot}424^2}{28}}$, as in Ex. 257.

**Ex. 482.**—I have a phial 1·8 inches in diameter inside. How high must I fill it to have 4 *Fluid ounces?*

| | | |
|---|---|---|
| C | 2·72 inches | 4 Fl. oz. |
| D | 1·485 G. P. | 1·8 |

**Ex. 483.**—How many *Gallons* of gas are there in a spherical balloon 50 feet in diameter?

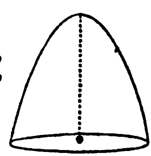
| | | |
|---|---|---|
| C | 50 | 407800 Gallons |
| D | ·5535 G. P. | 50 |

**Ex. 484.**—How many *Quarts* of liquid are there in a globular bottle 9 inches diameter inside, filled to the neck?

| | | | |
|---|---|---|---|
| C | 9 | 5·5 Quarts | 90 |
| D | 11·51 | 9 | 36·8 *check* |

---

## PARABOLIC CONOID (GALLONS).

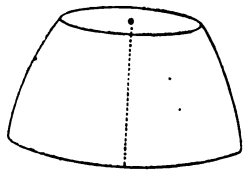


**Ex. 485.**—Take the dimensions in Ex. 464, and use the "Divisor" 18·79, or its "Gauge Point" 59·42.

| | | |
|---|---|---|
| C | 22·5 ½ depth | 102 Gallons |
| D | 18·79 | 40 diam. at base |

## FRUSTRUM OF PARABOLIC CONOID (GALLONS).

**Ex. 486.**—Take the dimensions in Ex. 465, and use the "Divisor" 18·79, or its "Gauge Point" 59·42.



| | | |
|---|---|---|
| C | 18 depth | 5·22 Gallons |
| D | 18·79 | 32 Mean diam. |

## SPECIFIC GRAVITY.

| Quantity. | Cubic inches. | Weight. |
|---|---|---|
| 1 Gallon | 277·274 | 10 lbs. avoir. |
| 1 Quart | 69·318 | 2½ lbs. avoir. |
| 1 Pint | 34·659 | 1¼ lbs. avoir. |
| 1 Fl. oz. | 1·733 | 1 oz. avoir. |

1 Cubic foot of *fresh* water = 6·2321 Gallons, or 62·321 lbs., or ·02782 Tons.

1 Cubic foot of *sea* water weighs 64 lbs., or ·02857 Tons.

·01605 Cubic foot of water, or $\frac{1}{10}$ Gallon, weighs 1 lb.

35·945 Cubic feet of water, or 224 Gallons, weighs 1 Ton.

1 Gallon a minute is 231 Cubic feet in a day of 24 hours.

Weight of anything in lbs. *per Cubic foot* $= \dfrac{\text{Sp. Gr.}}{\cdot 01605}$.

*Substance heavier than water.*

**Ex. 487.**—A lump of Indian standard silver weighed 3293 grains in air, and 2973 grains in water. Required its specific gravity.

*Answer* $\dfrac{3293}{320 \text{ loss in grains}} = 10{\cdot}39.$

*Substance lighter than water.*

**Ex. 488.**—A piece of Elm weighed 15 ounces in air, and when a piece of metal weighing 16 ounces in water was attached to it, the whole weighed *in water* 6 ounces. Required the specific gravity.

*Answer* $\frac{15}{(15 + 16) - 6} = \frac{15}{25} = \cdot 600$ the specific gravity.

**Ex. 489.**—If the anchor of a man-of-war weighs 95 Cwt. of which the wooden stock is 5 Cwt. and not affected in weight by submersion, what is the *loss of weight* on the 90 Cwt. when sunk in sea water? the specific gravity of the iron being 7·788, and of the water 1·026. Answer $= \frac{90 \times 1\cdot026}{7\cdot788} =$ 11·86 Cwt. less than when out of the water. [Or since 90 Cwt. = 161280 ounces, we have $\frac{161280}{7788} =$ 20·7 Cubic feet of iron, which multiplied by 64·2, the lbs. weight of a Cubic foot of sea water, gives 1329 lbs. or 11·86 Cwt. as before.]

**Ex. 490.**—The topmast of a man-of-war contained 159 Cubic feet of Norway fir, whose specfic gravity is ·580, and it weighed 5772 lbs. or 2·577 Tons. What weight would it float in sea water?

Here ·02857 (the weight in tons of a Cubic foot of sea water) × 159 = 4·543 Tons: from which, if 2·577 Tons (the weight of the mast) is deducted, the remainder is 1·966 Tons, the weight the mast would support in sea water.

*To find the quantity of each ingredient in a given Compound.*

Let $m$ = weight of the mass.

$x$ = weight of the heavier metal.

$(m - x)$ weight of the lighter metal.

$a$, $b$, $c$ the specific gravities respectively of the heavier ingredient —the lighter—and the Compound.

$$\text{Then } x = \frac{(c - b) \times a}{(a - b) \times c} \times m.$$

**Ex. 491.**—A mass of gold and silver weighs 63 oz. and its specific gravity is 16·126. What is the quantity of each ingredient, taking the specific gravity of gold at 19·64 and silver 11·091?

$$\left.\begin{array}{r}(16{\cdot}126 - 11{\cdot}091) \times 19{\cdot}64 = 98{\cdot}888 \\ (19{\cdot}64 - 11{\cdot}091) \times 16{\cdot}126 = 137{\cdot}861\end{array}\right\} \frac{98{\cdot}888 \times 63}{137{\cdot}861} = 45{\cdot}19 \text{ oz. of gold, } 17{\cdot}81 \text{ oz. of silver.}$$

---

# SPECIFIC GRAVITIES.

| | |
|---|---|
| Acid, muriatic | 1·170 |
| ,, nitric | 1·500 |
| ,, sulphuric | 1·848 |
| Alcohol (absolute) | ·794 |
| Rectified Spirit | ·838 |
| Proof Spirit | ·920 |
| Ash | ·800 |
| Basalt | 2·800 |
| Bath Stone | 2·200 |
| Beech | ·800 |
| Beer | 1·028 |
| Bone | 1·660 |
| Brandy | ·837 |
| Brass (wrought) | 8·396 |
| Brick | 2·000 |
| Brickwork | 1·800 |
| Bronze | 8·220 |
| Butter | ·042 |
| Chalk | 2·33 to 2·450 |
| Clay | 2·100 |
| Coal (solid) | 1·270 |
| ,, (loose) | ·81 to ·845 |
| Coke | ·432 |
| Copper (cast) | 8·788 |
| ,, (wrought) | 8·915 |
| Cork | ·240 |
| Deal (Memel) | ·400 |
| ,, (Norway) | ·680 |
| Fir (see Pine) | |
| Flint | 2·600 |
| Glass (flint) | 3·200 |
| ,, (plate) | 2·760 |
| ,, (crown) | 2·500 |
| Gold (pure) | 19·24 to 19·640 |
| ,, (standard) | 17·700 |
| Granite | 2·650 |
| Gravel | 1·700 |
| Gun-metal | 8·780 |
| Gunpowder (shaken) | ·930 |
| Gypsum | 2·280 |
| Honey | 1·450 |
| Ice | ·940 |
| Indigo | ·800 to ·900 |
| Iron (cast) | 7·207 |
| ,, (bar and round) | 7·808 |
| ,, (wrought) | 7·788 |
| Ivory | 1·8 to 1·900 |
| Jet | 1·300 |
| Lead | 11·352 |
| Limestone | 2·6 to 2·700 |
| Mahogany (Honduras) | ·580 |
| ,, (Spanish) | ·900 |
| Maple | ·680 to ·750 |
| Marble | 2·720 |
| Mercury (at 52°) | 13·570 |
| Milk | 1·032 |
| Millstone | 2·480 |
| Mortar | 1·700 |
| Nickel (wrought) | 8·660 |
| Norway spar | ·580 |
| Oak | ·750 to ·930 |
| Oil (olive) | ·915 |
| ,, (whale) | ·900 to ·920 |
| ,, (linseed) | ·940 |
| Opium | 1·340 |
| Paving stone | 2·420 |
| Pewter | 7·470 |
| Pine (red) | ·580 to ·650 |
| ,, (white) | ·450 to ·550 |
| ,, (yellow) | ·510 |
| Pitch | 1·150 |
| Platinum (hammered) | 20·000 |
| Portland stone | 2·4 to 2·500 |
| Pumice | ·915 |
| Purbeck stone | 2·600 |
| Quartz | 2·900 |
| Rottenstone | 1·980 |
| Salt (solid) | 2·130 |
| ,, (loose) | ·826 |
| Sand | 1·6 to 1·800 |
| Sandstone | 2·3 to 2·400 |
| Silver (pure) | 11·09 to 10·470 |
| ,, (standard) | 10·320 |
| Slate | 2·7 to 2·800 |
| Soap (hot) | ·990 |
| ,, (white soft) | 1·085 |
| ,, (cold hard) | 1·022 |
| Spermaceti | ·943 |
| Starch | ·800 |
| Steel | 7·820 |
| Sugar (white) | 1·600 |
| Sulphur (fused) | 2·000 |
| Tallow | ·950 |
| Tar | 1·015 |
| Teak | ·725 to ·750 |
| Tin | 7·300 |
| Treacle | 1·290 |
| Vinegar | 1·010 |
| Water (sea) | 1·026 |
| ,, (62°) | 1·000 |
| ,, (85°) | ·9977 |
| ,, (100°) | ·9954 |
| Wax | ·964 |
| Zinc | 7·100 |

In the following Formulæ, the set of the Slide may be changed, according to the "proportion" required. See page 18, Examples 495, 497.

| | | | |
|---|---|---|---|
| (I.) | A | Sp. Gr. | Grains weight |
| | B | ·0396 | Cubic inches |
| (II.) | A | Sp. Gr. | Oz. (avoir.) weight |
| | B | 1·733 | Cubic inches |
| (III.) | A | Sp. Gr. | lbs. (avoir.) weight |
| | B | 27·7274 | Cubic inches |
| (IV.) | A | Sp. Gr. | lbs. (avoir.) weight |
| | B | ·01605 | Cubic feet |
| (V.) | A | Sp. Gr. | Cwt. weight |
| | B | 1·797 | Cubic feet |
| (VI.) | A | Sp. Gr. | Tons weight |
| | B | 35·945 | Cubic feet |
| (VII.) | A | Sp. Gr. | lbs. (avoir.) weight |
| | B | 35·304 | Cylindrical inches |
| (VIII.) | A | Sp. Gr. | 5·9 |
| | B | 4·7 | Cubic inches in 1 lb. |
| (IX.) | A | Sp. Gr. | 6·3 |
| | B | 5·7 | Cubic feet in 1 Ton |

N.B. If the weight in lbs. of any number of "Gallons" of an article is given, the Sp. Gr. = lbs. ÷ Gallons × 10. If it is "Bushels" Sp. Gr. = lbs. ÷ Bushels × 80. Thus if 391·6 gallons of soft white soap weigh 4249 lbs., its Sp. Gr. $= \frac{4249}{3916} = 1{\cdot}085$.

If the Slide is *inverted*, we have as follows :

| | | | | | |
|---|---|---|---|---|---|
| (X.) | A | Sp. Gr. | ·0396 | 1·733 | 27·7274 |
| | Ɔ | Cubic inches | grains | oz. avoir. | lbs. avoir. |

| | | | | | |
|---|---|---|---|---|---|
| (XI.) | A | Sp. Gr. | ·01605 | 1·797 | 35·945 |
| | Ɔ | Cubic feet | lbs. | Cwt. | Tons |

| | | | | |
|---|---|---|---|---|
| (XII.) | A | Sp. Gr. | 27·73 | 5·9 |
| | Ɔ | Cubic inches in 1 lb. | 1 | 4·7 |

| | | | | |
|---|---|---|---|---|
| (XIII.) | A | Sp. Gr. | 35·945 | 6·3 |
| | Ɔ | Cubic feet in 1 Ton | 1 | 5·7 |

For "*Spheres*" only. (See Ex. 498.)

| | | | |
|---|---|---|---|
| (XIV.) | E | Sp. Gr. | lbs. |
| | D | 3·755 | inches diameter |

**Ex. 492.**—The Sp. Gr. of wrought copper being 8·915, how many lbs. would 6·22 Cubic inches weigh?

| | | | |
|---|---|---|---|
| (III.) | A | 2·0 lbs. | 8·915 |
| | B | 6·22 | 27·73 |

| | | | |
|---|---|---|---|
| or else (X.) | A | 8·915 | 27·73 |
| | Ɔ | 6·22 | 2·0 lbs. |

**Ex. 493.**—The Sp. Gr. of lead being 11·352, how many Cubic inches are there in 1 lb.?*

| | | | |
|---|---|---|---|
| (VIII.) | A | 5·9 | 11·352 |
| | B | 2·442 cub. in. | 4·7 |

| | | | |
|---|---|---|---|
| or (XII.) | A | 6·3 | 11·352 |
| | Ɔ | 5·7 | 2·442 cub. in. |

**Ex. 494.**—In Wallace's Tables of "Wrought *round* iron," the weight of a cylinder 12 inches diameter, and 12 inches long, or

* Formulæ VIII. and XII. only apply to cases of *one* lb. So IX. and XIII. when *one* Ton is in question.

1728 *cylindrical* inches, is given at 382·2 lbs., whereas by Templeton's Tables it is 374·4 lbs. Required the specific gravity according to each.

| | | | | |
|---|---|---|---|---|
| (VII.) | A | 7·65 T. | 7·81 W. | 35·304 |
| | B | 374·4 lbs. | 382·2 lbs. | 1728 cylindr. in. |

**Ex. 495.**—In the above Tables, the weight of a Cubic foot of "Wrought *square* iron," or 12 inches square and 12 deep, is given by Wallace at 486$\frac{2}{3}$ lbs., and by Templeton at 478 lbs. Required the Specific gravity according to each; also the Cubic inches in 1 lb.

| | | | | |
|---|---|---|---|---|
| (IV.) | A | ·01605 | 7·67 T. | 7·81 W. |
| | B | 1 | 478 | 486·67 |

| | | | |
|---|---|---|---|
| (III.) | A | 11·352 | 27·73 |
| | B | 1 | 2·44 cub. in. |

| | | | |
|---|---|---|---|
| or (VIII.) | A | 5·9 | 11·352 |
| | B | 2·44 cub. in. | 4·7 |

| | | | |
|---|---|---|---|
| or (XII.) | A | 11·352 | 5·9 |
| | C | 2·44 cub. in. | 4·7 |

**Ex. 496.**—What is the weight in tons of a main topmast of Norway spar, Sp. Gr. ·580, containing 159 Cubic feet? (Ex. 490.)

| | | | | |
|---|---|---|---|---|
| (VI.) | A | 2·577 tons | ·580 | (or by XI.) |
| | B | 159 | 35·945 | |

**Ex. 497.**—A bushel of loose coal, containing 1·63 Cubic feet, is found to weigh from 83 to 86 lbs. What specific gravity does that give?

| | | | | |
|---|---|---|---|---|
| (IV.) | A | ·01605 | ·817 | ·845 Sp. Gr. |
| | B | 1·63 | 83 | 86 |

**Ex. 498.**—Supposing a 9 lb. shot to have a diameter of

4 inches, what is its specific gravity? [See (XIV.) page 205. Diameter$^3$ : lbs. :: $3{\cdot}755^3$ (or 52·955) : specific gravity.]

| (XIV.) | | | |
|---|---|---|---|
| | E | 7·447 sp. gr. | 9 lbs. |
| | D | 3·755 | 4 in. diam. |

---

## GAUGING OF THE WEIGHT AND DIMENSIONS OF METALS.

The two following Examples will show the *usual* method: (the specific gravity of the metal being known). The subsequent Examples will explain the use of the "Table," constructed for five metals in constant use; its Divisors and Gauge Points may be computed for any other substances, as shown in Appendix K.

**Ex. 499.**—Required the weight in lbs. of a cylinder of "Cast iron," specific gravity 7·207, three feet long, and ten inches in diameter.

This cylinder, if of *water*, would contain 10·2 *gallons*, as shown in Ex. 476, as follows:

| | | |
|---|---|---|
| C | 3 | 10·2 galls. |
| D | 5·424 G. P. | 10 |

But as a gallon of water weighs 10 lbs., the *weight in water* would be 1020 lbs. This multiplied by 7·207, the specific gravity of cast iron, gives the weight of the iron cylinder 735 lbs. (Compare Ex. 506.)

**Ex. 500.**—What is the circumference of a brass rod weighing 2·1 lbs., and 18 inches long; the specific gravity being 8·4?

Here $\frac{2{\cdot}1 \text{ lbs.}}{8{\cdot}4} = {\cdot}25$; that is to say, the space occupied by ·25 lbs. of water would, if occupied by brass, weigh 2·1 lbs. But since (page 201) 27·724 Cubic inches of water weigh 1 lb., the above ·25 lbs. is equivalent to a content of 6·932 Cubic inches. Then by Formula (*c*), page 167

| | | |
|---|---|---|
| C | 6·932 Cub. in. | 18 in. long |
| D | 2·2 in. circumf. | 3·545 G. P. |

The above computation may be compared with the speedy solution of the same question in Ex. 507.

| | | Parallelopipeds. | | | Cylinders. | | | | Globes. | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | (d) | | (c) | | (d) | | (c) | |
| | | F. F. F. | F. I. I. | I. I. I. | F. I. | I. I. | F. I. | I. I. | F. I. | I. I. | F. I. | I. I. |
| Lead Sp. Gr. 11·352 | Divisor | ·00142 | ·2035 | 2·442 | ·2592 | 3·110 | 2·558 | 30·69 | ·00270 | 4·665 | ·0837 | 144·6 |
| | G. P. | ·0376 | ·4512 | 1·564 | ·5091 | 1·763 | 1·599 | 5·540 | ·0519 | 2·160 | ·2893 | 12·03 |
| | *Check* | ·1189 | 1·427 | 4·942 | 1·610 | 5·577 | 5·058 | 17·52 | ·1643 | 6·830 | ·9149 | 38·03 |
| Wr. Copper Sp. Gr. 8·915 | Divisor | ·00180 | ·2592 | 3·110 | ·3300 | 3·960 | 3·257 | 39·08 | ·00344 | 5·940 | ·1066 | 184·2 |
| | G. P. | ·0424 | ·5091 | 1·764 | ·5745 | 1·990 | 1·805 | 6·252 | ·0586 | 2·437 | ·3265 | 13·57 |
| | *Check* | ·1342 | 1·610 | 5·577 | 1·817 | 6·293 | 5·707 | 19·77 | ·1854 | 7·707 | 1·032 | 42·92 |
| Wr. Brass Sp. Gr. 8·396 | Divisor | ·00191 | ·2752 | 3·302 | ·3504 | 4·205 | 3·458 | 41·50 | ·00365 | 6·307 | ·1132 | 195·6 |
| | G. P. | ·0437 | ·5246 | 1·817 | ·5919 | 2·051 | 1·860 | 6·442 | ·0604 | 2·511 | ·3364 | 13·98 |
| | *Check* | ·1382 | 1·659 | 5·747 | 1·872 | 6·484 | 5·881 | 20·37 | ·1910 | 7·942 | 1·064 | 44·22 |
| Wr. Iron Sp. Gr. 7·808 | Divisor | ·00205 | ·2959 | 3·551 | ·3768 | 4·521 | 3·719 | 44·63 | ·00392 | 6·782 | ·1217 | 210·3 |
| | G. P. | ·0453 | ·5439 | 1·885 | ·6138 | 2·126 | 1·928 | 6·680 | ·0626 | 2·604 | ·3489 | 14·50 |
| | *Check* | ·1434 | 1·720 | 5·959 | 1·941 | 6·724 | 6·098 | 21·12 | ·1981 | 8·235 | 1·103 | 45·86 |
| Cast Iron Sp. Gr. 7·207 | Divisor | ·00203 | ·3206 | 3·847 | ·4082 | 4·898 | 4·029 | 48·35 | ·00425 | 7 348 | ·1319 | 227·8 |
| | G. P. | ·0472 | ·5662 | 1·961 | ·6389 | 2·213 | 2·007 | 6·953 | ·0652 | 2·711 | ·3632 | 15·09 |
| | *Check* | ·1492 | 1·790 | 6·203 | 2·020 | 6·999 | 6·346 | 21·99 | ·2062 | 8·572 | 1·148 | 47·73 |

The "Divisors" &c. for weight of Water may be taken from "Gallons," page 196, remembering 10 lbs. of water = 1 gallon.

*The mode of constructing such Table is shown in Appendix K.*

N.B. 1. Under "Cast iron" PARALLELOPIPEDS I. I. I., see 3·847. This shows that 3·847 Cubic inches of "Cast iron" weigh 1 lb.; *also* that a Square foot of "Cast iron" to weigh 144 lbs. must have a thickness of 3·847 inches.

N.B. 2. Under "Lead" PARALLELOPIPEDS F. F. F., see ·00142. This shows that ·00142 Cubic foot of Lead weighs 1 lb.

N.B. 3. Under CYLINDERS,* (*d*) are the Divisors, &c., where the *diameters* are in question. Where *circumferences* are in question, as in Examples 506 and 507, see under (*c*). The same applies to SPHERES. The N.B.'s in page 196 will further elucidate.

*The set of the Slide is as follows:*

## PARALLELOPIPEDS.

| | | |
|---|---|---|
| A | Product of any 2 sides | Content |
| B | "Divisor" | 3d side |

But if any two sides are *the same*, use the following:

| | | |
|---|---|---|
| A | "Divisor" | |
| B | 3d side | |
| C | | Content |
| D | | Square side |

| | | | | |
|---|---|---|---|---|
| or else | C | 3d side | 3d side × 10 | Content |
| | D | G. P. | Check number | Square side |

* Under the "Divisors" for CYLINDERS, Diameters I. I., we have the number of *cylindrical* inches in 1 lb. of the article. A cylindrical inch is 1 inch long and 1 inch in diameter, and exceeds a cubic inch as 1 exceeds ·7854; or 2200 cylindrical inches in a cubic foot. See Formula IV. page 148.

## CYLINDERS.

### (I.) *Diameters.*

| | | |
|---|---|---|
| A | "Divisor" | |
| B | Length | |
| C | | Content |
| D | | Diameter |

or

| | | | |
|---|---|---|---|
| C | Length | Length × 10 | Content |
| D | G. P. | Check No. | Diameter |

### (II.) *Circumferences.*

| | | |
|---|---|---|
| A | "Divisor" | |
| B | Length | |
| C | | Content |
| D | | Circumference |

or

| | | | |
|---|---|---|---|
| C | Length | Length × 10 | Content |
| D | G. P. | Check No. | Circumference |

## SPHERES.

### (I.) *Diameters.*

| | | |
|---|---|---|
| A | "Divisor" | |
| B | Diameter | |
| C | | Content |
| D | | Diameter |

or

| | | | |
|---|---|---|---|
| C | Diameter | Diameter × 10 | Content |
| D | G. P. | Check No. | Diameter |

### (II.) *Circumferences.*

| | | |
|---|---|---|
| A | "Divisor" | |
| B | Circumference | |
| C | | Content |
| D | | Circumference |

| | | | |
|---|---|---|---|
| or C | Circumf. | Circumf. × 10 | Content |
| D | G. P. | Check No. | Circumf. |

When once the proper use of the two lines C D alone, is acquired, it is better to use them, in preference to the four lines A, B, C, D. See pages 91 to 94.

**Ex. 501.**—Required the weight in lbs. of a bar of wrought iron $2\frac{1}{4}$ inches square, and 108 inches long.

| | | |
|---|---|---|
| C | 108 | 154 lbs. |
| D | 1·885 G. P. | 2·25 |

**Ex. 502.**—Required the weight in lbs. of a bar of flat wrought iron, 20 feet long, $\frac{3}{8}$ inches thick, and $2\frac{3}{4}$ inches broad. $\left(x = \frac{20 \times 2{\cdot}75 \times {\cdot}375}{{\cdot}2959}.\right)$*

| | | |
|---|---|---|
| A | 55 (= 20 × 2·75) | 69·7 lbs. |
| B | ·2959 Divisor | ·375 (= $\frac{3}{8}$) |

**Ex. 503.**—What is the thickness of a Square foot of copper that weighs 2 lbs.? (By N.B. 1st, page 209, a Square foot 3·11 inches thick, weighs 144 lbs.)

| | | |
|---|---|---|
| A | 144 | 2 lbs. |
| B | 3·11 | ·043 inch |

**Ex. 504.**—How many lbs. per Square foot does sheet lead of the following thickness weigh? $\frac{1}{10}$ inch, $\frac{1}{8}$ inch, $\frac{1}{6}$ inch, and $\frac{1}{5}$ inch? (By N.B. 1st, page 209, a Square foot of lead 2·442 inches thick, weighs 144 lbs.)

| | | | | | |
|---|---|---|---|---|---|
| A | 144 | 5·9 | 7·37 | 9·83 | 11·8 lbs. |
| B | 2·442 | ·1 | ·125 | ·1667 | ·2 inch |

---

* Or, if the "square side" is found = 1·015, as in Ex. 266, we may use

| | | |
|---|---|---|
| C | 69·7 lbs. | 20 × 10 |
| D | 1·015 "mean side" | 1·72 check number |

**Ex. 505.**—A rod of wrought iron 18 feet long, weighs 26·88 lbs. Required its diameter in *inches.*

| C | 18 | 26·88 |
|---|---|---|
| D | ·6138 G. P. | ·75 inch |

**Ex. 506.**—Required the weight in lbs. of a cast iron pillar, 10 inches diameter, and 3 feet long. (See Ex. 499.)

| C | 3 | 30 (= 3 × 10) | 73 lbs. |
|---|---|---|---|
| D | ·6389 G. P. | 2·02 check | 10 |

**Ex. 507.**—What is the circumference of a piece of brass rod, weighing 2·1 lbs. and 18 inches long? (See Ex. 500.)

| C | 2·1 | 18 |
|---|---|---|
| D | 2·2 inches | 6·442 G. P. |

**Ex. 508.**—What is the weight in lbs. of a cast iron sphere 4 inches diameter?

| C | 4 | 8·707 lbs. |
|---|---|---|
| D | 2·711 G. P. | 4 |

N.B. The specific gravity generally used for shot and shell is somewhat different, being 7·447. (See Ex. 498.)

**Ex. 509.**—Required the diameter of a cylinder 27 inches long, to contain 43 lbs. weight of water? (Here, as per Note at foot of the Table, page 208, 43 lbs. = 4·3 gallons.) As in Ex. 477.

| C | 4·3 gallons | 27 inches long |
|---|---|---|
| D | 23·71 inches diam. | 59·42 |

---

* For practice, Ex. 506 may be solved by the *four* lines A, B, C, D, using the "Divisor" from the Table.

| A | ·4082 "Divisor" | |
|---|---|---|
| B | 3 | |
| C | | 735 lbs. answer |
| D | | 10 |

*Remarks.*

It is often useful to know, when we see in works on Mechanics such Tables as that on page 208, what Specific Gravities have been used. To do this we look to see what is under PARALLELOPIPEDS I. I. I. (*i.e.* the Cubic inches in 1 lb. of the material).

**Ex. 510.**—In Carroll's 2-foot Rule, we find for "cast iron" 3·806—"wrought iron" 3·592—"gun-metal" 3·417—"copper" 3·103. What Specific Gravities do these denote?

Here we have to divide 27·274 (the Cubic inches in 1 lb. of water) consecutively by 3·806, 3·592, 3·417, and 3·103. We therefore *invert* the Slide, as in Examples 75 to 79, or, what is the same thing, use Formula (XII.) page 205.

| A | 1 | 5·9 | 7·285 C. I. | 7·719 W. I. | 8·115 G. M. | 8·936 Co. |
|---|---|---|---|---|---|---|
| D | 27·724 | 47 | 3·806 | 3·592 | 3·417 | 3·103 |

(See also Examples 494, 495.)

So also, since 35·3036 *Cylindrical* inches (see footnote page 202) of water weigh 1 lb., if in such Tables as that in page 209 we look under CYLINDERS (*d*) I. I., we see the Cylindrical inches in 1 lb. of the article. Hence, if we divide 353·036 by these "Divisors," we see what Specific Gravity has been used. Thus in the TABLE in page 208, if we divide 35·3036 successively by 3·110, 3·960, 4·205, 4·521, we have as follows:

| A | 1 | 11·35 L. | 8·91 W. C. | 8·4 W. B. | 7·81 W. I. |
|---|---|---|---|---|---|
| D | 35·3 | 3·110 | 3·960 | 4·205 | 4·521 |

## SHOT AND SHELL.

A 9 lb. shot is generally considered 4 inches diameter. This by Ex. 498 gives a Specific Gravity of 7·447, which, with reference to the Table on page 208, would make the "*Diameter*" Divisor for Spheres I. I. = 7·11; its G. P. = 2·667; and its "*Circumference*" Divisor = 220·5; with a G. P. = 14·85. (Ex. 498.)

Let D = external diameter of a shell, and $d$ = internal diameter. Then weight in lbs. = $(D^3 - d^3) \times {\cdot}1358$.*

**Ex. 511.**—What is the weight in lbs. of a shell 11·1 inches external, and 8 inches internal diameter? $11{\cdot}1^3$ = 1368 lbs. and $8^3$ = 512, as in Ex. 321.

Then (1308 − 512) × ·1358 = 856 × ·1358 = 117 lbs.

---

## METAL PIPES.

Let $s$ = sum of outer and inner diameters, *in inches*. Let $d$ = difference of diameters *in inches*.

Weight in lbs. per foot length $= \dfrac{s \times d}{\text{Divisor}}$.

The "Divisor" is found in the Table, page 208, under Cylinders, Diameters, F. I. (See also N.B. after Ex. 514.)

**Ex. 512.**—What is the weight in lbs. *per foot*, of a cast iron pipe of 8 inches bore, and ¾ inch thick? (Here $s$ = 9·5 + 8, and $d$ = 9·5 − 8.)

| | | |
|---|---|---|
| A | 17·5 = $s$ | 6·43 lbs. |
| B | ·4082 Divisor | 1·5 = $d$ |

**Ex. 513.**—What is the weight in lbs. *per foot*, of a copper pipe of 7½ inches bore, and ⅛ thickness?

| | | |
|---|---|---|
| A | | 15·25 = $s$ |
| B | ·25 inch = $d$ | ·3300 |

**Ex. 514.**—What is the weight in lbs. *per foot*, of a lead pipe 2 inches bore, and ·23 inch thickness? $\left(x = \dfrac{4{\cdot}46 \times {\cdot}46}{{\cdot}2592}.\right)$

---

* The above is assuming the Specific Gravity = 7·207, but the usual multiplier is $\frac{9}{64}$, or ·1406, with Specific Gravity = 7·447. Ex. 498.

| | | |
|---|---|---|
| A | 4·46 | 7·9 lbs. |
| B | ·2592 | ·46 |

Weight in lbs. of lead pipe = bore × 4 nearly.

Also the following Formula may be used :

"*Divisors.*"

Cast iron ·102
Lead ·0649
Copper ·0825

| | | |
|---|---|---|
| A | Inner diameter + thickness | lbs. per foot |
| B | "Divisor" | inches thickness |

Thus Ex. 514 would be

| | | |
|---|---|---|
| A | 2 | 7·9 lbs. |
| B | ·0649 | ·23 |

N.B. The above divisors are the Cylinder F. I. (*d*) divisors of page 208, *divided by* 40.

---

## BRICKWORK.

A "Rod" of Brickwork, including mortar, is generally reckoned at 272 square feet of *surface*, with a "Standard" thickness of $1\frac{1}{2}$ bricks, or three half-bricks. If the thickness is 2 bricks (or 4 half-bricks), the number of rods in a piece of work, is increased in the ratio of $1\frac{1}{2}$ to 2, or 3 to 4. If $2\frac{1}{2}$ bricks thick, the increase is in the ratio of 3 to 5, and so on.

A Rod of Brickwork of "Standard" thickness, contains, with mortar, about 306 Cubic feet or $11\frac{1}{3}$ Cubic yards, and requires from 4400 to 4500 bricks. It weighs about 16 tons. At £15·1 per rod (Standard) the price per Square yard is 10*s*. A stock brick weighs about 5 lbs., or 450 to a ton.

(I.) *Standard thickness.*

| | | | |
|---|---|---|---|
| (*a*) | A | Height in feet | Rods of brickwork |
| | B | 272 | Length in feet |

| | | | |
|---|---|---|---|
| (*b*) | A | Height in feet | Cubic yards |
| | B | 24 (= $272 \div 11\frac{1}{3}$) | Length in feet |

(II.) *Above Standard thickness.*

In this case the *number of Square feet* of surface must first be found.

| | | | |
|---|---|---|---|
| (*c*) | A | Square feet of surface | Rods |
| | B | 408 | Bricks thick |

| | | | |
|---|---|---|---|
| (*d*) | A | Square feet of surface | Quarter rods* |
| | B | 102 | Bricks thick |

| | | | |
|---|---|---|---|
| (*e*) | A | Square feet of surface | Cubic yards |
| | B | 36 ($= 408 \div 11\frac{1}{3}$) | Bricks thick |

**Ex. 515.**—How many rods of brickwork are there in a wall 40 feet 6 inches long, by 22 feet 9 inches high, the thickness being "Standard."

| | | | |
|---|---|---|---|
| (*a*) | A | 3·39 rods | 22·75 |
| | B | 40·5 | 272 |

**Ex. 516.**—Supposing in the preceding Example the thickness were $2\frac{1}{2}$ bricks, what would be the number of rods?

Here it is necessary to find $40{\cdot}5 \times 22{\cdot}75 = 921{\cdot}375$ Square feet.

| | | | |
|---|---|---|---|
| (*c*) | A | 5·65 rods | 921·4 |
| | B | 2·5 | 408 |

It will be seen that 5·65 : 3·39 :: 5 : 3 : or Ex. 515 to Ex. 516.

---

## TIMBER MEASURING.

(I.) *Squared Timber.*

See Ex. 418.

---

* It is sometimes convenient, in calculating the price, to have the content in "quarter" rods.

### (II.) *Round Timber.*

The "customary" rule for finding the content, is to take the "quarter-girt" in *inches* :* multiply the square of this by the length in *feet*, and divide by $12^2$ (or 144); or $\dfrac{\left(\frac{g}{4}\right)^2 \times l}{12^2}$ = content in Cubic feet. For this, the setting of the Slide Rule is as follows :

| | | | |
|---|---|---|---|
| (*a*) | C | Length in feet | Content in Cubic feet |
| | D | 12 | Quarter-girt in inches |

But if we use the *whole* girt instead of the *quarter* girt, we have $\dfrac{g^2 \times l}{48^2}$, which on the Slide Rule is as follows :

| | | | |
|---|---|---|---|
| (*b*) | C | Length in feet | Content in Cubic feet |
| | D | 48 | Quarter girt in inches |

**Ex. 517.**—Required the Cubic feet (customary measure) in a rounded log of timber, 24 feet long and 39 inches in girt, or $9\frac{3}{4}$ inches "Quarter girt."

| | | | |
|---|---|---|---|
| By (*a*) | C | 24 feet long | 15·844 Cub. ft. |
| | D | 12 | 9·75 = $\frac{1}{4}$ girt |

| | | | |
|---|---|---|---|
| By (*b*) | C | 15·844 Cub. ft. | 24 feet long |
| | D | 39 = girt | 48 |

### *Remarks.*

The content thus found by the "customary" rule is much less than the real content, as calculated by the "true" method for content of solid cylinders. In fact, we should have to add more than $\frac{1}{4}$ of the "customary" content to get the "true." That is, we should have to multiply it by 1·27324 (Ex. 159). Or, if the "true" content as a cylinder were calculated, we should have to deduct 21·46 per cent. to

* If the bark is on when the girth is taken, it is usual to deduct $\frac{1}{12}$ of the measured girt. So, if measured 48 inches, the content is calculated as if measured 44 inches.

reduce it to "customary" (Ex. 124). Thus, in the preceding Example, $15{\cdot}844 \times 1{\cdot}27324 = 20{\cdot}17$ is the "true" content as found by Ex. 425. Still, the "customary" is used in *practice*, as it allows for *squaring.* The true section, if fully squared, would be as in Ex. 334. The area of a square inscribed in a circle is less than the area of the circle, as ·7854 is less than 1. See Ex. 123, and Formula V. page 149.

By the following Formula, *both* the "true" and the "customary" content can be found together. (Invert the Slide.)

| | | | |
|---|---|---|---|
| ꓭ | Length in feet | "True" | "Customary" |
| D | Girt in inches | 42·54 | 48 |

**Ex. 518.**—In a cylindrical log, such as described in Ex. 517, what is the "true," and what is the "customary" content in Cubic feet?

| | | | |
|---|---|---|---|
| ꓭ | 20·17 Cub. ft. | 15·84 Cub. ft. | 24 = length in ft. |
| D | 42·54 | 48 | 39 = girt in in. |

If the rounded timber tapers regularly, it is the Frustrum of a Cone, but in practice its content is found by the "customary" formula, using the mean of the two end circumferences, as the "mean girt." The "mean girt," thus derived, is too small. For instance, if the timber had been 24 feet long, and the two ends respectively 45 and 33 inches in girt, or $9\frac{3}{4}$ mean quarter girt, the "customary" content would be 15·844 Cubic feet; whereas if calculated as in Ex. 461, as a Conical Frustrum, it would be 20·235 Cubic feet.

So also in what is called "squared" timber, the ends are seldom similar, but generally rectangles of different sizes, and thus the solid is, properly speaking, a *Prismoid,* though it is "customary" to take the mean widths and breadths and calculate the contents as "Parallelopipeds." For all practical purposes the content is the same.

---

## CASK GAUGING.

Let L = length; H = head diameter; B = bung diameter: all in inches, and all *inside* measurement.

$$\text{Content in Gallons} = \frac{L \times (H^2 + 2 \,.\, B^2)}{33^2}$$

**Ex. 519.**—Required the gallons in a pipe of Port, measuring $L = 47$; $H = 25{\cdot}3$; $B = 32$. (Here $x = \dfrac{47 \times 25{\cdot}3^2 + 32^2 + 32^2}{33^2}$.

| | | | | |
|---|---|---|---|---|
| C | 27·7 | 44·1 | 44·1 | 47 |
| D | 25·3 | 32 | 32 | 33 |

and $27{\cdot}7 + 44{\cdot}1 + 44{\cdot}1 = 115{\cdot}9$ gallons.

The result by this Rule is generally about 2 per cent. too little, if the cask bulges much.

---

## WEIGHT OF CATTLE.

For the weight of the *four quarters* (which is $\frac{6}{10}$ of the weight of the living animal), take the *girt* in feet, close behind the shoulder, and the *length* in feet from the fore part of the shoulder-blade to the bone of the tail above the end of the buttock. Then the weight in stone of 8 lbs. $= \dfrac{g^2 \times l}{1{\cdot}55^2}$; in stone of 14 lbs. $= \dfrac{g^2 \times l}{2{\cdot}05^2}$; and the weight in cwt. $= \dfrac{g^2 \times l}{5{\cdot}79^2}$; so that the weight in each kind can be found at one setting of the Slide Rule as follows: (Invert the Slide.)

| Ɔ | St. of 8 lbs. | St. of 14 lbs. | Cwt. | Length |
|---|---|---|---|---|
| D | 1·55 | 2·05 | 5·79 | Girt |

**Ex. 520.**—What is the weight in each kind of weight of the four quarters of an ox, whose length is 5 feet 10 inches, and girt $6\frac{1}{2}$ feet?

| | | | | |
|---|---|---|---|---|
| B (inverted) | 1·03 St. of 8 lbs. | 55·7 St. of 14 lbs. | 7.33 cwt. | 5·83 Length |
| D | 1·55* | 2·05 | 5·79 | 6·5 Girt |

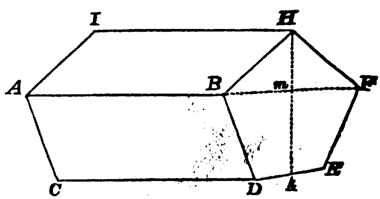
## MENSURATION OF HAY.

A "Load" of *new* hay weighs 19·3 cwt. or ·964 tons, if we consider it to be 36 trusses of 60 lbs. to the truss. Its weight is about 17 Cubic yards to a ton.

A "Load" of *old* hay weighs 18 cwt. or ·9 tons, if we consider it to be 36 trusses of 56 lbs. to the truss. Its weight is about 11·3 Cubic yards to a ton.

**Ex. 521.**—The following measurements were taken in feet.

AB = 38. CD = 30. BF = 18. DE = 14. IH = 37.
$km$ = 12. $n$H = 9.




$(38 + 30) \times (18 + 14) = 2176$
$38 \times 18 = 684$
$30 \times 14 = 420$

3280

Multiply by $\frac{km}{6}$ $\times 2$

6560

$18 \times 9 \times \frac{37}{2} = 2997$

27)9557 Cubic feet.

353·93 Cubic yards.

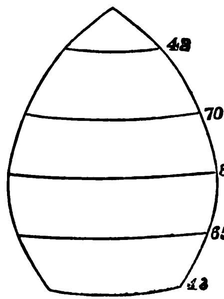
The check number (see page 90) for 1·55 is 4·9; for 2·05 it is 6·48; and for 5·79 it is 18·3. If we use the lines C, D, *not* inverted, we only find the answer in *one* kind. Thus:

| | | | |
|---|---|---|---|
| C | 5·83 | 58·3 | answer in St. of 8 lbs. |
| D | 1·55 | 4·9 | 6·5 |

**Ex. 522.**—In a globular stack, the following *girths* were taken in feet; the four spaces from the base being *equal*, *i.e.* 6 feet. The last girth to the crown being $5\frac{1}{4}$ feet.

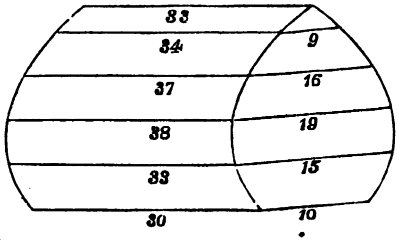
N.B. The four spaces from the base must be equal to each other.

| | | |
|---|---|---|
| Odd girths | $65^2 =$ | 4225 |
| | $70^2 =$ | 4900 |
| | | 9125 |
| Constant | = | 4 |
| | | 36500 |
| Highest and lowest girths | $43^2$ | 1849 |
| | $42^2$ | 1764 |
| | | 40113 |
| | $80^2 \times 2 = +$ | 12800 |
| | | 52913 |
| Distance between girths | | $\times$ 6 feet |
| | | 317478 |
| | $42^2 \times 5\frac{1}{4}^2 =$ | 9261 |
| | | 326739 |
| | $\times$ | ·00098 |
| | | 320·20 Cubic yards |




The "Stone" of hay is 22 lbs. The best way to ascertain weight, is to cut out a rectangular piece, and after measuring its cubic content, to weigh it, and calculate the weight of the whole stack from it.

**Ex. 523.**—In a bulged stack (see next figure), let measurements be taken from the base, so as to form five spaces, of which four are 6 feet apart, and the *upper space* 5 feet. The measurements as shown in the figure.




$30 \times 10 = 330$
$34 \times 9 = 306$
606

$33 \times 15 = 495$
$27 \times 16 = 592$

1087
Constant $\times$ 4

4348

4954
$(38 \times 19) \times 2 = 1444$

6398
2

12796

Upper section or triangular prism $5 \times \frac{9}{2} \times 33$ } = 742·5

27 ) 13538·5 Cubic feet

501·82 Cubic yards

## PILING OF SHOT.

### (I.) *Triangular pile.*

Let $a$ = number of balls in bottom row. Then the total number of balls $= \frac{a \times (a + 1) \times a + 2}{6}$. So if $a = 46$, the total number of balls is 17296.

### (II.) *Square pile.*

Let $a$ = number of balls in bottom row. Then the total number $= \frac{a \times (a + 1) \times (2\,a + 1)}{6}$. So if $a = 24$, the number of balls is 4900.

(III.) *Rectangular pile.*

Let $a$ = number of balls at the longest side of the base; and let $b$ = number at the short side of the base. Total = $[3a - (b - 1)] \times b \times (b + 1)$. So if $a = 30$, and $b = 12$, the total is 2054.

---

## WROUGHT IRON BEAMS.

If Rectangular, and supported at both ends, and loaded in the middle the weight they would bear in lbs. is $\frac{(850 \times b) \times d^2}{l}$; where $l$ = length in feet, and $b$ and $d$ the breadth and depth in *inches.* The Slide Rule facilitates the computation.

**Ex. 524.**—What weight in lbs. will a wrought iron bar bear, that measures 20 feet in length, $\frac{3}{8}$ inch breadth, and $2\frac{3}{4}$ inches depth?

Here $x = \frac{31{\cdot}875 \times 2{\cdot}75^2}{20}$.

| | | |
|---|---|---|
| A | | 20 |
| B | | 31·875 |
| C | | |
| D | 2·75 | |

The weight itself of the bar (as in Ex. 502) will be 69·7 lbs.

---

## LAND MEASURING (General).

1 link = 7·92 inches; 100 links = 22 yards or 66 feet = 1 chain; 625 square links = 1 perch.

| | | | | |
|---|---|---|---|---|
| (I.) | A | ·62 | 1 | Acres |
| | B | 27000 | 43560 | Square feet |

| | | | | |
|---|---|---|---|---|
| (II.) | A | ·62 | 1 | Acres |
| | B | 3000 | 4840 | Square yards |

(III.)

| | | | |
|---|---|---|---|
| A | 1 | [220] 220 | [Yards] Feet to 1 inch |
| D | 5280 | [8] 24 | Inches to 1 mile |

(IV.)

| | | | |
|---|---|---|---|
| A | 4 | | Chains to 1 inch |
| D | 20 | | Inches to 1 mile |

(V.)

| | | | |
|---|---|---|---|
| A | 208·7 | 360 | Side of sq. acre in inches |
| D | 1 | ·58 | Feet to 1 inch |

(VI.)

| | | | |
|---|---|---|---|
| C | 1 | 10 | Acres in 1 square inch |
| D | 208·7 | 660 | Feet to 1 inch |

(VII.)

| | | | |
|---|---|---|---|
| C | ·1 | 2.5 | Acres in 1 square inch |
| D | 1 | 5 | Chains to 1 inch |

N.B. If "chains per inch" are given, square the number of chains, and divide by 10. The result is "acres in 1 square inch."

(VIII.)

| | | | |
|---|---|---|---|
| C | 1 | 1·6 | Acres in 1 square inch |
| D | 22 | 88 | Yards to 1 inch |

(IX.)

| | | | |
|---|---|---|---|
| B | 2·5 | | Acres to 1 square inch |
| D | 16 | | Inches to 1 mile |

(X.)

| | | | |
|---|---|---|---|
| A | [66] 22 [1] | | [Feet] Yards [chains] wide |
| B | 8 | | Acres per running mile |

(XI.)

| | | | |
|---|---|---|---|
| A | ·125 | | Chains wide |
| B | Miles long | | Acres |

(XII.)

| | | | |
|---|---|---|---|
| A | Chains length | | Gradient |
| D | 66 | | Feet of fall |

N.B. The "Gradient" here, is the *slope* divided by the perpendicular fall; whereas, strictly speaking, it is the *base* divided by the perpendicular; but practically the above can lead to no error in road making &c., (see Ex. 529).

(XIII.)

| | | | |
|---|---|---|---|
| A | 88 | | £ per mile |
| B | 12 | | Pence per running yard |

| | | | |
|---|---|---|---|
| (XIV.) | A | Pence per Cubic yard | Cost in £ |
| | B | 240 | Cubic yards |

| | | | |
|---|---|---|---|
| (XV.) | A | 121 | £ per Acre |
| | B | 6 | Pence per square yard |

**Ex. 525.**—The Crystal Palace of 1851, in Hyde Park, covered an area of 772784 square feet. What is this in acres? (Ex. 205.)

| | | | |
|---|---|---|---|
| (I.) | A | ·62 | 17·74 acres |
| | B | 27000 | 772784 |

**Ex. 526.**—On a scale of 16 perches to 1 inch, how many inches represent a mile; and how many acres are there in 1 square inch? (16 perches = 88 yards, or 4 chains.)

| | | | |
|---|---|---|---|
| (III.) | A | 220 | 88 yards |
| | D | 8 | 20 inches in 1 mile |

| | | | |
|---|---|---|---|
| (VII.) | C | 2·5 | 1·6 acres in 1 square inch |
| | D | 5 | 4 |

**Ex. 527.**—Given the scale of 330 feet to 1 inch. Required the number of inches to a mile; the side of a square acre in inches; and the number of acres in 1 square inch. (See Examples 79 and 203.)

| | | | | |
|---|---|---|---|---|
| (III.) | A | 220 | 330 | $= \frac{5280}{330}$ |
| | D | 24 | 16·0 inches | |

| | | | | |
|---|---|---|---|---|
| (V.) | A | 360 | 330 | $= \frac{\sqrt{43560}}{330}$ |
| | D | ·58 | ·6325 inch | |

| | | | | |
|---|---|---|---|---|
| (VI.) | C | 10 | 2·5 acres | $= \frac{330^2}{43560}$ |
| | D | 660 | 330 | |

N.B. The Ordnance Survey is $\frac{1}{2500}$ of reality; that is, every mile of ground occupies 25·344 inches on the Map; and every $208\frac{1}{3}$ feet occupies 1 inch. This is ·3168 inches to 1 chain; 1 square inch = ·99369 acre; and 1·0018 square inch = 1 acre.

**Ex. 528.**—In road making, if the breadth is 41 feet, how many acres are cut up in each mile of road?

| (X.) | | | |
|---|---|---|---|
| | A | 66 | 41 |
| | B | 8 | 4·97 acres |

**Ex. 529.**—In Old Holborn Hill, the fall was 300 feet in 70 chains of ascent. What was the gradient?

| (XII.) | | | | |
|---|---|---|---|---|
| | A | 70 | (one in) 15·4 | $= \frac{70 \times 66}{300}$ |
| | D | 66 | 3 | |

N.B. The actual angle of ascent was 3° 43′ 20″, and the *base* 4610·3 feet, giving the *true* gradient $\frac{4610{\cdot}3}{300} = 15{\cdot}37$.

**Ex. 530.**—Suppose a tract of land has been measured with a 50 feet chain, and the area recorded as so many acres: and subsequently the chain was found to be too short by 9 inches. How much per cent. of the area will have to be deducted?

| | | |
|---|---|---|
| C | 97 or 3 per cent. to deduct | 100 |
| D | 49·25 | 50 |

**Ex. 531.**—Three fields have been measured with a Gunter's chain, and entered as 8·6, 8·4, and 8·1 acres respectively; but it is afterwards found that the chain was 15 inches too long. What would be the *true* content? (Here, if we only wanted to correct *one*, say the first, we should set the Slide as in Ex. 294 $\frac{67{\cdot}3^2 \times 8{\cdot}6}{66^2}$; but when we have a "series" and two squares are constant, it is necessary to find the Square of 66, which is seen instantly on the D line, as explained in page 83, and then we have $\frac{67{\cdot}3^2 \times 8{\cdot}6}{4356}$, $\frac{67{\cdot}3^2 \times 8{\cdot}4}{4356}$, $\frac{67{\cdot}3^2 \times 8{\cdot}1}{4356}$, where $67{\cdot}3^2$ and 43560 are constant, and therefore the whole "series" solvable at once, as in Ex. 294.

| | | | |
|---|---|---|---|
| A | 8·42 acres | 8·73 acres | 8·94 acres |
| B | 8·1 | 8·4 | 8·6 |
| C | | | 43560 |
| D | | | 67·3 |

**Ex. 532.**—Required the measure in inches, of 1000 feet, on each of three scales, viz. : 2, $2\frac{1}{2}$, and 6 inches to a mile, respectively.

| A | 1000 feet | ·379 inch | ·473 inch | 1·136 inch |
|---|---|---|---|---|
| B | 5280 feet | 2 | 2·5 | 6 |

In the ordinary "military" Protractors, the distance between two *red* parallel lines represents 100 yards. This on a scale of 4 inches to a mile is $\frac{10}{44}$ or $\cdot 2\dot{2}\dot{7}$ inch.

---

## LAND MEASURING (Squares).

| (I.) | C | 1 | | Area in perches |
|---|---|---|---|---|
| | D | 5·5 | | Side of square in yards |

| (II.) | C | 1 | 10 | Area in acres |
|---|---|---|---|---|
| | D | 69·57 | 220 | Side of square in yards |

| (III.) | C | 1 | 10 | Area in acres |
|---|---|---|---|---|
| | D | 208·7 | 660 | Side of square in feet |

**Ex. 533.**—Lincoln's Inn Fields is a square of $13\frac{1}{2}$ acres. Required the length in yards of each side.

| (II.) | C | 10 | 13·5 |
|---|---|---|---|
| | D | 220 | 256 yards |

**Ex. 534.**—In South India there is a native land measure which is a square of 152 feet to a side. Required its equivalent in acres.

| (III.) | C | ·53 acres | 10 |
|---|---|---|---|
| | D | 152 | 660 |

N.B. 32 of the above measures make what is called a Kuttee, and the whole equivalent can be obtained, without first finding the equivalent of the smaller measure ; for $x = \frac{32 \times 152^2}{43560}$, as in Ex. 259 (43560 square feet = 1 acre) as follows :—

| | | |
|---|---|---|
| A | 43560 | |
| B | 32 | |
| C | | 16·97 acres |
| D | | 152 |

**Ex. 535.**—The Irish "perch" is 7 yards linear. Required the *side* of an Irish "acre" in yards. (1 acre = 160 square perches.) Here $x = \sqrt{7^2 \times 160}$, as in Ex. 253.

| | | |
|---|---|---|
| C | 1 | 160 |
| D | 7 | 88·55 yard |

N.B. If the square yards in an Irish acre had been required, $x = 7^2 \times 160$, (as in Ex. 252) = 7840 ; or 1·62 Imperial acres.

**Ex. 536.**—A plot of land in India is said to contain exactly 6400 square *cubits.* Measured in English measure, it is found to contain 1936 square yards. Required the length of the *cubit* in inches.

(Here $x = 36 \times \sqrt{\frac{1936}{6400}}$, as in Ex. 257.)

| | | |
|---|---|---|
| C | 1936 | 6400 |
| D | 19·8 inches | |

---

## LAND MEASURING (Rectangles).

N.B. 1st. If the sides of a Rectangle are given, the *diagonal* may be found as in Ex. 287.

N.B. 2d. When the measurements are made in chains and links, remember that 100,000 square links = 1 acre ; so that if the acre is 562483 square links, it is equivalent to 5·62483 acres ; so 7350 square links = ·07335 acres. It is simply cutting off, as decimals, the last five figures.

| | | | |
|---|---|---|---|
| (I.) | A | Yards one way | Square miles |
| | B | 3097600 | Yards the other way |

$$(\text{II.})\ \frac{\text{A}\quad [\text{Yards}]\ \text{Feet one way}\qquad\qquad \text{Acres}}{\text{B}\quad [4840]\quad 43560\quad [\text{Yards}]\ \text{Feet the other way}}$$

$$(\text{III.})\ \frac{\text{A}\quad \text{Fee}\ \ [\text{Links}]\ \text{one way}\qquad\qquad \text{Perches}}{\text{B}\quad 272{\cdot}5\ \ [625]\qquad \text{Feet}\ [\text{Links}]\ \text{the other way}}$$

$$(\text{IV.})\ \frac{\text{A}\quad \text{Chains of 50 ft. one way}\qquad\qquad \text{Acres}}{\text{B}\qquad\qquad 17{\cdot}42\quad \text{Chains of 50 ft. the other way}}$$

$$(\text{V.})\ \frac{\text{A}\quad 22\qquad\qquad \text{Yards one way}}{\text{C}\quad 220\qquad\qquad \text{Yards the other way}} = 1\ \text{acre}$$

$$(\text{VI.})\ \frac{\text{A}\quad 66\qquad\qquad \text{Feet one way}}{\text{C}\quad 660\qquad\qquad \text{Feet the other way}} = 1\ \text{acre}$$

N.B. With reference to Formula (IV.) with chains of 33 feet, the constant would be 40, or $\frac{43560}{33^2}$, instead of 17·42, or $\frac{43560}{50^2}$.

**Ex. 537.**—From the scale attached to a map, we find that it is a rectangle of 2000 yards by 1000. Required the area in square miles.

$$(\text{I.})\ \frac{\text{A}\quad {\cdot}646\ \text{Square miles}\qquad\qquad 2000}{\text{B}\quad 1000\qquad\qquad 3097600}$$

**Ex. 538.**—Required the number of acres in the "Champs de Mars" in Paris, which is a rectangular piece of ground 1320 English feet by 2700.

$$(\text{II.})\ \frac{\text{A}\quad 8{\cdot}18\ \text{acres}\qquad\qquad}{\text{B}\quad 2700\qquad\qquad 43560}$$

**Ex. 539.**—A rectangular field, known to contain $1\frac{1}{2}$ acre, has one side 110 yards long. Required the length in yards of each of the shorter sides.

$$(\text{II.})\ \frac{\text{A}\quad 1{\cdot}5\qquad\qquad 110}{\text{B}\quad 66\ \text{yards}\qquad\qquad 4840}$$

**Ex. 540.**—A rectangular slip of land measures 320 links by 56. Required the content in perches.

(III.)

| | | |
|---|---|---|
| A | 2·87 Perches | 320 |
| B | 56 | 625 |

**Ex. 541.**—A rectangular field of $2\frac{1}{2}$ acres has had one of its sides measured with a 50 feet chain, and recorded as $8\frac{1}{2}$ chains long. What is the length of each of the shorter sides in similar chains?

(IV.)

| | | |
|---|---|---|
| A | 2·5 | 8·5 |
| B | 5·13 chains | 17·42 |

**Ex. 542.**—What must be the *breadth* of a roadway 1452 feet long, to make up one acre; and what must be the *length* of another which is 36 feet wide, also to make up one acre?

(VI.)

| | | | |
|---|---|---|---|
| A | 66 | 30 ft. | 36 ft. |
| D | 660 | 1452 | 1210 |

**Ex. 543.**—From three long strips of land, whose widths are respectively 9, 12, and 16 yards, it is required to take off lengths equal to *two* acres. What lengths in yards must be marked off on each? Here $\frac{2 \times 4840}{9}$, $\frac{2 \times 4840}{12}$, $\frac{2 \times 4840}{16}$, (as in Ex. 35.)

| | | | | |
|---|---|---|---|---|
| A | 605 yards | 807 yards | 1076 yards | 4840 |
| D | 16 | 12 | 9 | 2 |

N.B. Had it been *three* acres to cut off, we should place 3 on D under 4840 on A.

**Ex. 544.**—Lay out 3·2 acres in the form of a rectangle, one of whose sides shall be *three* times as long as the other at right angles to it, and give the length of the shorter side in feet. First by Formula (I.), 3·2 acres = 139600 square feet, and secondly, as in Ex. 251. $\sqrt{\frac{139600}{3}}$.

| | | |
|---|---|---|
| C | 3 | 139600 |
| D | 1 | 215·7 feet |

---

## LAND MEASURING. (Triangles.)

| | | | |
|---|---|---|---|
| (I.) | A | Area in acres | Perp. in chains |
| | B | Base in chains | 20 |

| | | | |
|---|---|---|---|
| (II.) | A | Area in acres | Perp. in chains |
| | Ɔ | 20 | Base in chains |

| | | | |
|---|---|---|---|
| (III.) | A | Area in acres | Perp. in feet |
| | B | Base in feet | 87120 |

| | | | |
|---|---|---|---|
| (IV.) | A | Area in acres | Perp. in feet |
| | Ɔ | 87120 | Base in feet |

**Ex. 545.**—Required the content in acres of a triangular piece of land, whose base is 700 links, and perpendicular 210 links.

| | | | |
|---|---|---|---|
| (I.) | A | 2·10 chains | ·0735 acres |
| | B | 20 | 7 chains |

**Ex. 546.**—The base of a triangular piece of land is 165 feet. How many feet in length must a perpendicular be raised, so as to set out a triangle whose area shall be $\frac{1}{4}$ acre? or if the perpendicular is 180 feet, what must the base be, for the same $\frac{1}{4}$ acre content?

| | | | | |
|---|---|---|---|---|
| (IV.) | A | ·25 acre | 132 perp. | 180 perp. |
| | Ɔ | 87120 | 165 | 121 base |

N.B. The *inverted* forms (II.) and (IV.) are used when the content is constant, but either the base or perpendicular varying, as in Ex. 75.

---

## LAND MEASURING. (Trapezoids.)

(See Ex. 367.)

| | | | |
|---|---|---|---|
| (I.) | A | Area in acres | Sum of parallel sides |
| | B | Perp. in chains | 20 |

| | | | |
|---|---|---|---|
| (II.) | A | Area in acres | Sum of parallel sides |
| | B | Perp. in feet | 87120 |

**Ex. 547.**—In the figure underneath, let the portion C$mn$D be a Trapezoid, in which C$m$ = 100 feet; D$n$ = 175 feet; and the perpendicular distance $mn$ = 300 feet. Required its content in acres.

| | | | |
|---|---|---|---|
| (II.) | A | ·947 acres | 275 |
| | B | 300 | 87120 |

---

## LAND MEASURING. (Trapeziums.)

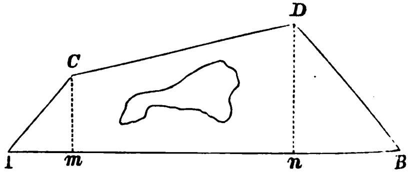
(See Examples 368, 369.)

| | | | |
|---|---|---|---|
| (I.) | A | Acres | Sum of perps. in Chains |
| | B | Diagonal in chains | 20 |

| | | | |
|---|---|---|---|
| (II.) | A | Acres | Sum of perps. in feet |
| | B | Diagonal in feet | 87120 |

For *Examples*, see further on under "Irregular Figures."

N.B. Sometimes, as n the figure, there is an obstruction in measuring a diagonal, or sometimes the perpendicular from one of the corners falls on the diagonal outside the boundary.




In such cases measure A$m$, $mn$, and $n$B; also the perpendiculars $m$C and $n$D. We have then the data for the two end triangles, as in Ex. 367, and for the intervening Trapezoid C$mn$D, as in Ex. 547.

## LAND MEASURING. (Circles.) Ex. 383, &c.

The diameter of a Circle containing 1 acre, is 235·5 feet, or 78·5 yards. 1 mile circumference encloses 50·9 acres.

| | | | | |
|---|---|---|---|---|
| (I.) | C | 1 | 4·5 | Area in acres |
| | D | 235·5 | 500 | Diam. in feet |

| | | | | |
|---|---|---|---|---|
| (II.) | C | 1 | 30 | Area in acres |
| | D | 78·5 | 430 | Diam. in yards |

| | | | | |
|---|---|---|---|---|
| (III.) | C | 1 | 10 | Area in acres |
| | D | 3·57 | 11·28 | Diam. in Imp. chains |

| | | | | |
|---|---|---|---|---|
| (IV.) | C | 1 | 10 | Area in acres |
| | D | 4·71 | 14·9 | Diam. in chains of 50 ft. |

| | | | | |
|---|---|---|---|---|
| (V.) | C | 1 | 10 | Area in acres |
| | D | 740 | 2340 | Circumf. in feet |

| | | | | |
|---|---|---|---|---|
| (VI.) | C | 1 | 10 | Area in acres |
| | D | 246·6 | 780 | Circumf. in yards |

| | | | | | |
|---|---|---|---|---|---|
| (VII.) | C | 3·18 | 50·93 | 815 | Acres |
| | D | ·25 | 1 | 4 | Miles circumf. |

**Ex. 548.**—Required the area in acres, of each of two Circles, whose diameters respectively are 1320 and 2970 feet.

| | | | | |
|---|---|---|---|---|
| (I.) | C | 4·5 | 31·4 acres | 159 acres |
| | D | 500 | 1320 | 2970 |

**Ex. 549.**—What length of rope, in yards—one end being fixed in the ground, and at the other end a horse tethered—will allow the animal to graze ¼ acre?

| | | | |
|---|---|---|---|
| (II.) | C | ·25 acre | 30 |
| | D | 39·3 = diam. or 19·6 = answer | 430 |

**Ex. 550.**—Required the areas in acres of three circular ponds, whose circumferences are 710, 1170, and 1480 feet.

| (V.) | | | | | |
|---|---|---|---|---|---|
| C | 1 | ·92 acres | 2·5 acres | 4·0 acres |
| D | 740 | 710 | 1170 | 1480 |

**Ex. 551.**—A person is said to have an estate 7 miles round. How many acres does it contain?

| (VII.) | | |
|---|---|---|
| C | 815 | 2500 acres |
| D | 4 | 7 |

---

## LAND MEASURING. (Ellipses.) Ex. 393.

| | | |
|---|---|---|
| A | One semi-axis in feet | Area in acres |
| B | 13865 | Other semi-axis in feet |

Or if we use the axes instead of the semi-axes, the divisor is 55462.

---

## LAND MEASURING. (Circular Rings.) Ex. 400.

| | | | |
|---|---|---|---|
| (I.) | A | Sum of diams. in feet | Area in acres |
| | B | 55462 | Diff. of diams. in feet |
| (II.) | A | Sum of diams. in chains | Area in acres |
| | B | 12·732 | Diff. of diams. in chains |
| (III.) | A | Sum of circumf. in feet | Area in acres |
| | B | 54·74 | Diff. of Circumf. in feet |

**Ex. 552.**—Let the figure Ex. 400, be a circular field, in which AB measures 45 chains, and DB 12·5 chains (making the diameter of the *inner circle* 20 chains). Required the area of the *outer zone* in acres.

| | | |
|---|---|---|
| A | 65 (= 45 + 20) | 127·6 acres |
| B | 12·73 | 25 (=45 − 20 |

N.B. Compare this with Ex. 548, where the area of each circle is given, and 127·6 is their difference, or outer zone.

**Ex. 553.**—The circular fences on each side of a gravel walk round a shrubbery are 800, and 714 measuring round. Required the area in acres (compare Ex. 401).

| | | |
|---|---|---|
| A | 4·38 acres | 1514 |
| B | 86 | 54·74 |

## LAND MEASURING. (IRREGULAR FIGURES.)

The total area is obtained by measuring diagonals with their perpendiculars (*i.e.* offsets), and so dividing the figure into Trapeziums and Triangles, as in figure 1 below; or into Trapezoids and Triangles, as figure 2; and then computing the content of each portion by the Formulæ in pages 229 and 230.

N.B. Where the figure is divided into Trapeziums and Triangles, as in figure 1, the content may be computed without knowing whereabouts on the diagonals the perpendiculars are raised; but the figure cannot be *drawn* without this being noted; so it is always done by Surveyors. In cases like figure 2 (Ex. 556), it is necessary to note where the offsets are made, both for computation and for drawing; for unless this is done, we should not know the length of the bases of the triangles.

**Ex. 554.**—Let fig. 1 be a piece of land, on which the following measurements have been made in *chains.*

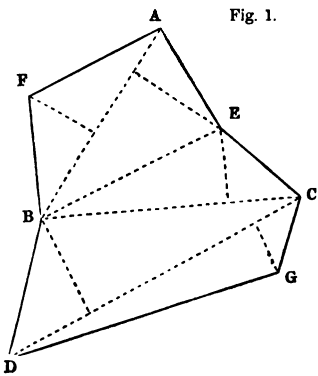


Fig. 1.

AB = 6·20. Offset to E = 2·70; to F = 2·00 (Trapezium).

BC = 7·00. Offset to E = 2·10 (Triangle).

CD = 8·90. Offset to G = 1·50; to B = 2·50 (Trapezium).

| | | | |
|---|---|---|---|
| (I.) p. 232 | A | 1·46 acre | 4·70 sum of perps. |
| | B | 6·20 | 20 "Divisor" |
| (I.) p. 231 | A | ·0735 acre | 2·10 Perp. |
| | B | 7·00 | 20 |
| (I.) p. 232 | A | 1·78 acre | 4·00 sum of perps. |
| | B | 8·90 | 20 |

Then the sum of these three portions = 3·3135 acres, the total area.*

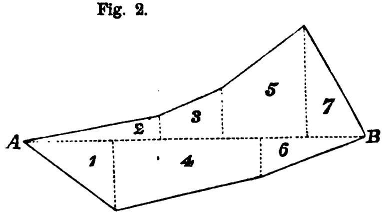
**Ex. 555.**—Suppose the Surveyor had measured the above field in *feet*, as follows :

A to B = 409. Offset to E = 178 ; offset to F = 132.
B to C = 462. Offset to E = 134.
C to D = 588. Offset to G = 99 ; to B = 165.

Then by Formula (II.), page 232, and Formula (III.), page 231, we should have :

1st (409 × 310) ÷ 87120 = 1·46 acres. 2d (462 × 134) ÷ 87120 = ·0735 acres ; and 3d (588 × 264) ÷ 87120 = 1·78 acres. Total 3·315 acres as in Ex. 554.

Fig. 2.




**Ex. 556.**—Let fig. 2 be a piece of land surveyed by measuring AB with its Offsets in *links*, as per extract from the "Field-book" as under ; beginning at A ⊙.

The calculation would be by Formula (XV.), page 142, and Formula (IX.), page 141.

* The figure is drawn to the Scale of 5 chains, or 330 feet, to 1 inch.

| | B | | Plot |
|---|---|---|---|
| | 600 | | $(160 \times 120) \div 2 = 9600$ = 1 Triangle. |
| 200 | 490 | | $(240 \times 50) \div 2 = 6000$ = 2 Triangle. |
| | 420 | 70 | $(100 \times 130) \div 2 = 6500$ = 3 Trapezoid. |
| 80 | 340 | | $(260 \times 190) \div 2 = 24700$ = 4 Trapezoid. |
| 50 | 240 | | $(150 \times 280) \div 2 = 21000$ = 5 Trapezoid. |
| | 160 | 120 | $(180 \times 70) \div 2 = 6300$ = 6 Triangle. |
| | ⊙A | | $(100 \times 200) \div 2 = 11000$ = 7 Triangle. |

Total 85100 Sq. links, or ·851 acres

*Narrow Irregular Figures.*

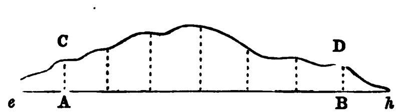
Such as occur at the boundaries of fields, the *main* portions of which have been surveyed in Triangles, Trapeziums, &c.

1st. If the line is a general curve, as in Ex. 557, the method of "equidistant ordinates" is used; that is, the base line is divided into an *equal* number of parts, by an *odd* number of offsets. Then to the sum of the 1st and last offsets, add 4 times the sum of all the even offsets (*i.e.* the 2d, 4th, &c.) and twice the sum of all the *odd* offsets (not including the 1st and last). Multiply the total sum by $\frac{1}{3}$ of the distance between any two offsets.

(N.B. 1st. If it is not convenient to measure first the whole base line, so as to be sure of dividing it into *equal* parts, measure equal parts from one end, and calculate the odd piece at the other end, separately.

N.B. 2d. When the base *is* divided into an equal number of parts, it does not matter at which end we begin to reckon the 2d, 4th, 6th, &c. offsets. The odd offsets and the even offsets will be the same, from whichever end we begin.

**Ex. 557.**—Let the piece of land represented in the figure, A C D B, be measured thus: 6 spaces each of 16 links, the 1st




offset A C being 10, the next 14, 18, 22, 16, 12, and BD 12 links.

$$\left.\begin{array}{r}\text{Then 1st offset} = 10\\ \text{last offset} = 12\end{array}\right\}\text{Total 22.}$$

| Even. | Odd. | |
|---|---|---|
| 14 | 18 | |
| 22 | 16 | Then $22 + (48 \times 4) + (34 \times 2) = 282$. |
| 12 | — | |
| — | 34 | And $\frac{282 \times 16}{3} = 1504$ Sq. links, or ·01504 acre. |
| 48 | | |

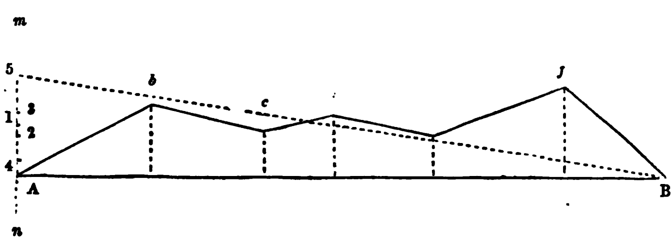
**Ex. 558.**—In the above figure, $eCDh$, where the curve terminates in a point, $e$ and $h$ both $= 0$; the other offsets as before. Here the first *even* offset will be AC = 10; and 14 the first *odd* offset.

$$\left.\begin{array}{l}\text{1st offset} = 0\\ \text{last do.} \;\;= 0\end{array}\right\}\text{Total} = 0.$$

| Even. | Odd. | |
|---|---|---|
| 10 | 14 | |
| 18 | 22 | Then $0 + (56 \times 4) + (48 \times 2) = 320$. |
| 16 | 12 | |
| 12 | — | |
| — | 48 | And $\frac{320 \times 16}{3} = 1707$ Sq. links, or ·01707 acre. |
| 56 | | |

*Where the boundary of a field is irregular.*

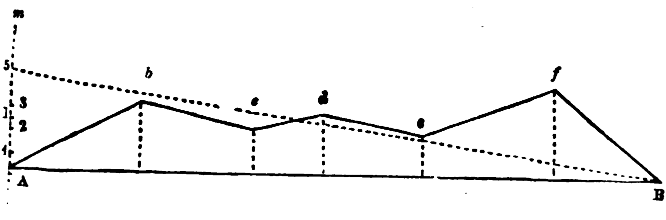
**Ex. 559.**—Let the annexed figure represent a piece of a wavy irregular boundary. Measure to each bend or corner—without reference to equal distances—a series of offsets from the base AB, which is 5·76 chains in length.



The following will be the entry from the "Field-book," beginning at ⊙A ; and also the computation of contents.

| | links. | Squ. links. |
|---|---|---|
| | ⊙B | (120 × 62) ÷ 2 = 3720 |
| | 576 | (100 × 102) ÷ 2 = 5100 |
| 82 | 478 | (60 × 90) ÷ 2 = 2700 |
| 38 | 364 | (84 × 88) ÷ 2 = 3696 |
| 50 | 280 | (114 × 120) ÷ 2 = 6840 |
| 40 | 220 | (98 × 82) ÷ 2 = 4018 |
| 62 | 120 | |
| | ⊙A | 26074 Squ. links = ·26074 acres. |

This computation is of course made on the supposition that the turns of the boundary are a succession of *straight* lines, such as A*b*, *bc*, &c. whereas they are generally slightly curved ; but as the curves are sometimes *in*, and sometimes *out*, the average result is as near the truth as is required in practice ; and if there is a group of such fields, the projections of one are cancelled by the indentations of the next.




*In practice*, the content of this kind of boundary areas is often quickly made, by plotting the figure on paper, on a large scale (about 1 chain to 1 inch), and using a Parallel Ruler, as follows : (The reference is to the preceding figure, which is on a scale of about 1⅔ chains to 1 inch.

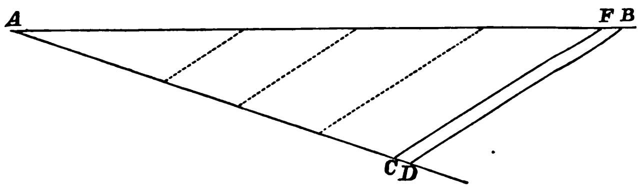
At A draw a line *n*A*m* at right angles to AB ; lay a parallel ruler from A to *c*, the 3d point ; move the upper part of the ruler to *b*, and mark where it cuts A*m*, at 1 ;—from 1 lay the ruler to *d*, bring

its lower edge down to *c*, and mark where it cuts A*m* at 2 ;—from 2, lay the ruler to *e*, and move the upper edge to *d*, and mark where it cuts A*m* at 3 ;—from 3, lay the ruler to *f*, and bring the lower edge down to *e*, and mark where it cuts A*m* at 4 ;—from 4, lay the ruler to B, and raise the upper edge to *f*, and mark where it cuts A*m* at 5.

From 5 draw the line 5B ; then will the triangle AB5 be equal in area to the whole irregular figure ; for A5 will be found to measure 90·5 links ; and (576 × 90·5) ÷ 2 = 26074, as in Ex. 559.

N.B. The line *n*A*m* is drawn below A, because the marks will sometimes fall below A.

As connected generally with the subject, it may be well to notice the most simple method of drawing, at the foot of a Plan, a scale to *feet* when the plan is so many "inches to 1 mile." The following figure and Example 560 will explain.




**Ex. 560.**—To prepare a scale of *feet* corresponding to a scale of $3\frac{1}{8}$ inches to 1 mile. Draw AB $3\frac{1}{8}$ inches long ; it will represent 5280 feet. From A draw AD, at an angle of about 20° or 25°, but make AD = 5280 from *any* "scale of Equal Parts." On the same line AD, cut off AC = 5000 parts from the *same scale* of Equal parts. Join DB, and from C draw CF parallel to DB. Then AF will represent 5000 feet, and can be subdivided into portions of 1000 feet by marking on AC spaces of 1000 feet from the scale used, and drawing from each parallels to touch AF.

## DEPRESSION OF THE HORIZON.

In observing at long distances along the surface of the earth, the curvature of the earth, or *depression of the horizon,* must be taken into consideration. The drop of the horizon in *feet,* when D = distance in Statute miles, is $\frac{2}{3}$ D²; which at the distance of one mile would be a fall of 8 inches; but as the effect of "Refraction" is to *raise* a distant object, the *apparent* difference is $\frac{5}{9}$ D², or $6\frac{2}{3}$ inches in one mile. So that if the horizontal wire of a Levelling instrument is in a line with the door-step of a house one mile off, that step is not 8 inches, but $6\frac{2}{3}$ inches, higher from the ground than the wire of the instrument.

The two following Formulæ show the ratio of fall in inches, to *yards* of distance. No, I. is the *true* depression, or actual curvature of the earth; No. II. is the *apparent* depression, including the effect of "Refraction."

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| (I.) | C | Inches fall (true) | ·1 | 1 | 2·59 | 8 | 10 |
| | D | Yards distance | 196·8 | 622 | 1000 | 1760 | 1968 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| (II.) | C | Inches fall (apparent) | ·1 | 1 | 6·67 | 10 | 60 |
| | D | Yards distance | 215·5 | 682 | 1760 | 2155 | 5280 |

Where D = distance in Statute *miles,* and $h$ = apparent depression of the horizon (or difference of level) in *feet,* we use the following Formula, which includes the effect of Refraction: $h = \frac{5}{9} D^2$, as in Ex. 259; or $h = \frac{5 \times D^2}{3^2}$, as in Ex. 256. $D = \sqrt{\frac{9}{5} \text{ of } h}$, as in Ex. 262; or $D = 3 \times \sqrt{\frac{h}{5}}$, as in Ex. 257.

The following settings of the Rule are applicable:

| | | | |
|---|---|---|---|
| either | A | 9 | |
| | B | 5 | |
| | C | | Feet depression |
| | D | | Dist. in Stat. miles |

| | | | |
|---|---|---|---|
| or | C | 5 | Feet depression |
| | D | 3 | Dist. in Stat. miles |

**Ex. 561.**—At what height of the eye, in feet, above the level of the sea, can I see the water line of a ship $3\frac{1}{4}$ Statute miles distant?

| | | |
|---|---|---|
| C | 5 | 5·9 feet |
| D | 3 | 32·5 Stat. miles |

**Ex. 562.**—The eye being 22 feet above the sea, I can just see on the horizon a light which I know to be at an elevation of 125 feet. What is its distance in Statute miles?

| | | | |
|---|---|---|---|
| C | 5 | 22 feet | 125 feet |
| D | 3 | 6·3 miles | 15 miles |

The *answer* is 6·3 + 15 = 21·3 Statute miles.

N.B. A *Nautical* mile = 1·15 Statute; or 23 Nautical = 20 Statute. So for the above Formula, we may use the following:—

| | | |
|---|---|---|
| C | 9 | Feet depression of horizon |
| D | 3·5 | Distance in *Nautical* miles |

*Inaccessible Distances.*

Where the object does not subtend a greater angle than 5°.

| | | | |
|---|---|---|---|
| (I.) | A | Angle in ″ | Height of object |
| | B | 206264 | Distance |
| (II.) | A | Angle in ′ | Height of object |
| | B | 3438 | Distance |
| (III.) | A | Angle in ′ | Height in feet |
| | B | 1146 | Dist. in yards |
| (IV.) | A | Angle in ′ | Height in feet |
| | B | ·651 | Dist. in Statute miles |
| (V.) | A | Angle in ′ | Height in feet |
| | B | ·566 | Dist. in Nautical miles |

N.B. In the preceding Formulæ, the space subtended by the angle is the *chord* of that angle, whereas in those cases where the space measured is at right angles to one of the sides, it is the *tangent*. However, in small angles there is no appreciable difference.

**Ex. 563.**—What angle will a foot subtend at a mile distance?

| | | | |
|---|---|---|---|
| By Form. (I.) | A | 1 | 39″ |
| | B | 5280 feet | 206264 |

| | | | |
|---|---|---|---|
| By Form. (IV.) | A | ·651″ = ·39″ | 1 |
| | B | ·651 | 1 mile |

**Ex. 564.**—A man 6 feet high subtends an angle of 6·9′ or 412″. Required his distance in yards.

| | | | |
|---|---|---|---|
| By Form. (III.) | A | 6 feet | 6·9′ |
| | B | 1000 yards | 1146 |

| | | | |
|---|---|---|---|
| By Form. (I.) | A | 2 yards | 412″ |
| | B | 1000 yards | 206264 |

**Ex. 565.**—What angle in minutes will a vessel riding at anchor subtend, whose length is known to be 150 feet, and distance 2·3 Statute miles (or 2 Nautical)?

| | | | |
|---|---|---|---|
| (IV.) | A | 42·5′ | 150 |
| | B | ·651 | 2·3 |

**Ex. 566.**—A vessel of about 1000 tons is observed just hull down, and an angle taken from the sea horizon to her main-topsail yard is 8′ 30″. Supposing the yard to be 100 feet above deck, what is her distance in miles?

| | | | |
|---|---|---|---|
| (IV.) | A | 8·5′ | 100 |
| | B | ·651 | 7·66 |

The *answer* is 7·66 Statute, or 6·66 Nautical miles. An error of 10 feet in the assumed height, or of 50″ in the angle, would cause an error of $\frac{3}{4}$ Statute mile in the *Distance.*

*Distance by Sound.*

The average is 1142 feet per second, or 4·62 seconds for a Statute mile of distance: so that if the sound of a cannon is heard 16 seconds after the flash is seen, the gun is $3\frac{1}{2}$ miles off.

### *Heights by Barometer.*

Let B = Lower barometer. T = Lower thermometer.
$b$ = Upper barometer. $t$ = Upper thermometer.

$$\text{Height in feet} = \frac{B - b}{B + b} \times f\text{; where } f \text{ is as under :—}$$

When T + $t$ is between 130° and 110°, $f$ = 5700
110 and 130, $f$ = 5550
100 and 110, $f$ = 5450
under 100, $f$ = 5400

$$\text{Example} \begin{cases} B = 30{\cdot}24, \text{ diff.} = 4{\cdot}05 \\ b = 26{\cdot}19, \text{ sum} = 56{\cdot}43 \end{cases} \quad \begin{cases} T = 64{\cdot}0 \\ t = 54{\cdot}8 \text{ sum } 118°. \end{cases}$$

Then height in feet $= \frac{4{\cdot}054}{56{\cdot}43} \times 5550 = 3987$ feet. (The true height was 3964.) On an average, we may assume 93 feet for each *tenth* of an inch as high as 2000 feet, or 1·86 inches fall of barometer.

### *Heights by Temperature.*

The temperature falls about 1° for about 290 feet of altitude; or more correctly, add to this 290, another ·6 feet for every degree that the *mean* of T and $t$ exceeds 32°. For example, T = 54°; $t$ = 46°. Mean = 50°; and Mean − 32° = 18°. Then for each degree of difference between T and $t$ we have 290 + (18 × ·6), or 300·8; and 300·8 × 8 = 2406 feet.

### *Height by Boiling Point.*

Till we reach a height of about 4000 feet, the Boiling point falls 1° per 510 to 515 feet. From 4000 to 6000 feet, it falls 1° per 515 to 520 feet. From 6000 to 8000 feet, it falls 1° per 520 to 530 feet. At an elevation of 8000 feet, water boils at about 195°.

## MECHANICAL POWERS.

F = fulcrum ; W = weight ; P = power.

LEVER.* Dist. of P from F : Dist. of W from F :: W : P.

WHEEL AND AXLE. Same as above ; the *radius* of the *Wheel* being the same as "Dist. of P from F ;" and the radius of the Axle, the same as "Dist. of W from F."

SCREW. Same as LEVER ; the *circumference* of the moving power being the "Dist. of P from F," and the Pitch (or distance between the threads) the same as "Dist. of W from F."

PULLEY. P = W divided by the number of ropes on the stretch, not counting the one that is pulled by the hand.

**Ex. 567.**—If two men carry a burden of 200 lbs. suspended from a pole 4 feet long, the ends of which rest on their shoulders ; how many inches *from the middle* of the pole must the weight be suspended, so as to give one man 75 lbs. to bear, and the other 125 lbs. ? (Lever of 3d kind.)

| | | | |
|---|---|---|---|
| A | 48 inches | 18 | 30 |
| B | 200 | 75 | 125 |

The weight will then be hung 18 inches from one end and 30 inches from the other, or 6 inches *from the middle.*

**Ex. 568.**—Let there be a crane, the winch of which describes a circle 30 inches diameter. The pinion makes 8 revolutions for 1 of the Wheel (*i.e.*, has $\frac{1}{8}$ the number of teeth) ; and the Axle (or barrel) is 11 inches diameter. Required the weight in lbs. that a Power of 36 lbs. would raise.

$$\text{Here } \left.\begin{matrix} 11 : 30 :: 36 : y \\ 1 : 8 :: y : x \end{matrix}\right\} = \frac{30 \times 36 \times 8}{11} = \frac{240 \times 36}{11}.$$

| | | |
|---|---|---|
| A | 36 | 785·5 lbs. |
| B | 11 | 240 |

* In the 1st kind of Lever, F is between P and W ; in the 2d, W is between P and F ; and the 3d, P is between F and W.

N.B. In this and similar Examples we use the *diameter* proportions 30 : 11, which is the same as if we used their *radii* proportions 15 : $5\frac{1}{2}$.

**Ex. 569.**—Required the effective force (W) of a Screw of $\frac{7}{8}$ pitch, moved by a Power of 50 lbs. at the extremity of a lever 30 inches in length. (First find by Formula 1st, page 122, the *circumference* of a diameter of 60, to be 188)—Then,

| | | | |
|---|---|---|---|
| $\frac{188 \times 50}{\cdot 875}$ | A | 188 | 10760 lbs. |
| | B | ·875 | 50 |

N.B. In practice the friction lessens much of the advantage of the lever handle.

**Ex. 570.**—What is the proportion between the Power and the Weight in a "luff tackle" purchase, where the fixed block has 2 sheaves, and the movable one 1?

Answer 1 to 3; there being 3 ropes on the stretch, besides the one held in the hand.

## WHEEL AND MILL WORK.

The "pitch *line*," where the wheel has cogs or teeth, is a line of circumference joining those points where one cog bears most on the cog of the other wheel; and when a "diameter" is spoken of, it refers to the diameter from pitch line across to pitch line, being somewhat more than the diameter of the wheel would be supposing the cogs knocked off. The "pitch" itself, is the distance between the centres of two contiguous teeth.

N.B. As the "number of teeth" is in proportion to the diameter of the wheels, either may be used to calculate by. So in "Pulleys," or "Shaft and Drum," the relative diameters are used, just as the relative number of teeth in cog wheels.

| | | | |
|---|---|---|---|
| (I.) | A | Diam. of wheel in feet | Velocity in ft. per sec. |
| | B | 1·91 | Revolutions per minute |

| | | | |
|---|---|---|---|
| (II.) | A | Inches pitch | Inches diameter |
| | B | 3·1416 | No. of teeth or cogs |

| | | | |
|---|---|---|---|
| (III.) | A | Veloc. of larger wheel | No. of teeth (or diam.) of smaller |
| | B | Veloc. of smaller wheel | No. of teeth (or diam.) of larger |

**Ex. 571.**—Suppose a wheel to be 34½ inches, or 2·87 feet diameter, as A in Ex. 578, and revolving 16 times in a minute; at what rate is its circumference travelling, in feet, per second?

| | | | |
|---|---|---|---|
| (I.) | A | 2·87 | 2·42 ft. per sec. |
| | B | 1·91 | 16 |

**Ex. 572.**—What is the "pitch" of each tooth in a wheel 70 inches diameter (to the pitch line), and with 146 teeth?

| | | | |
|---|---|---|---|
| (II.) | A | 1.5 inches | 70 |
| | B | 3·1416 | 146 |

**Ex. 573.**—Suppose, as in Ex. 578, the wheel A to have 54 teeth of 2 inch pitch;—B 24,—C 40,—and D 18 teeth, also of 2 inch pitch. Required the diameters of the respective wheels in inches.

| | | | | | | |
|---|---|---|---|---|---|---|
| (II.) | A | 2 | 11·5 | 15·26 | 25·5 | 34·4 |
| | B | 3·1416 | 18 | 24 | 40 | 54 |

**Ex. 574.**—If a wheel, as A in Examples 571 and 578, is 34·5 inches diameter, and makes 16 revolutions a minute, what will be the diameter of another wheel B, that works in it, and makes 35·8 revolutions in the same time?

| | | | |
|---|---|---|---|
| (III.) | A | 15·42 inches | 16 velocity of larger |
| | B | 34·5 | 35·8 velocity of smaller |

**Ex. 575.**—The upright shaft in a Mill makes 51 revolutions a minute, and the *line* shaft 30. The wheel on the upright is 15 inches diameter. What is the diameter of the *line* shaft wheel?

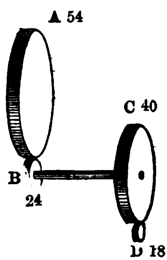
| | | | |
|---|---|---|---|
| (III.) | A | 30 velocity of larger | 15 |
| | B | 51 velocity of smaller | 25·5 diam. |

**Ex. 576.**—The fly-wheel shaft of a Steam engine makes 33 revolutions a minute, having a wheel upon it of 54 teeth. Required the number of teeth in the drum wheel, so that the shaft may make 49 revolutions a minute.

| | | | |
|---|---|---|---|
| (III.) | A | 33 | 70 teeth |
| | B | 49 | 104 |

**Ex. 577.**—If a Shaft making 68 revolutions a minute has six drums upon it, whose diameters are respectively 24, 16, 18, 32, 14, 30, inches; each of which drives a separate machine; what will be the diameters of six others to replace them, when the speed of the Shaft cannot be kept up above 62 revolutions per minute? (Here a reduction of speed is equivalent to wheels of larger diameter.)

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| A | 14 | 16 | 18 | 24 | 32 | 30 | 68 |
| B | 15·3 | 17·5 | 19·7 | 26·4 | 35·1 | 32·7 | 62 |




**Ex. 578.**—In the figure, A is a driving wheel of 54 teeth, working a smaller one B of 24 teeth, which again by a shaft, revolves C of 40 teeth, and this drives D of 18 teeth. Required the ratio of velocity between A and D.

Here $\frac{54 \times 40}{24 \times 18} = 5$. That is, D revolves 5 times as fast as A; so that if A revolves 16 times in a minute, D will revolve 80 times.

**Ex. 579.**—In the preceding figure let the driving wheel A revolve 16 times in a minute, and have 54 teeth. It is by means of a smaller wheel B, attached to a shaft working another driving wheel C of 40 teeth, to which is attached another small wheel D. Required the *number of teeth* in the small wheels B and D, so as to give a final velocity to D, of 80 revolutions per minute.

(Here the number of revolutions per minute of the wheel B will be the Geometrical mean of 16 and 80, or (as in Ex. 258), 35·8 revolutions per minute.)

| | | | |
|---|---|---|---|
| 1st. (III.) | A | 16 | 24 teeth in B |
| | B | 35·8 | 54 |

| | | | |
|---|---|---|---|
| 2d. (III.) | A | 80 | 40 |
| | B | 35·8 | 18 teeth in D |

N.B. If C had been a driving Pulley or Drum, of 25 inches diameter, attached by a belt to a smaller pulley or drum B, and it were required to find the *number of teeth* in B, and the *diameter* in inches of D; B would be found as before to revolve 35·8 times per minute, and then

| | | | |
|---|---|---|---|
| (III.) | A | 80 | 25 |
| | B | 35·8 | 11·2 inches diam. of D |

*Miscellaneous.*

**Ex. 580.**—If the diameter of a Horse track is 24 feet, how many times ought the horse to go round in 1 minute?

| | | |
|---|---|---|
| A | 35 Feet diam. | 24 |
| D | 2 Turns to 1 min. | 2·9 times |

This gives a speed of 220 feet per minute, or $2\frac{1}{2}$ miles an hour.

The velocity of *Millstones* ought to be about 1540 feet per minute. Or divide 49 (the diameter to 1540 circumference) by the feet diameter of the Millstone.

**Ex. 581.**—Required the force in lbs. of a 6 inch Hydraulic Ram when a Power of 50 lbs. is applied to the end of a lever which is as 12 to 1 in effect; the diameter of the Pump or plunger being $\frac{7}{8}$ inch. (The areas of the Pump and Ram surfaces are found as in Ex. 383.) Then ·6013 : 28·27 : : 50 : 2350 lbs. And as the Power is 12 to 1, the answer is $2350 \times 12 = 28200$ lbs. But it is not necessary to find the areas at all, as the given *diameters* are enough.

| | | |
|---|---|---|
| C | 50 | 2350 lbs. |
| D | ·875 inch | 6 inch |

Then $2350 \times 12 = 2800$ lbs. as before.

## LATERAL PRESSURE OF WATER.

Pressure on all 4 sides together in lbs. $= \dfrac{\text{length in ft.} \times \text{depth}^2}{\cdot 0321}$,

or (to suit lines C and D) $\dfrac{\text{length in ft.} \times \text{depth}^2}{\cdot 0179^2}$.

**Ex. 582.**—Required the pressure of water in lbs. on the 4 sides of a cistern 18 by 13 feet, and 9 feet deep.

| | | | |
|---|---|---|---|
| Either | A | ·0321 "Divisor" | |
| | B | 62 length of 4 sides | |
| | C | | 156400 lbs. |
| | D | | 9 depth |

| | | | |
|---|---|---|---|
| or | C | 62 | 156400 lbs. |
| | D | ·0179 | 9 |

N.B. For the answer in *Cwts.* use the "Divisor" 3·59; and for *Tons*, 71·9.

*Shut sluice below surface.*

Pressure in lbs. = area × depth from surface of water to centre of sluice × 62·32. (Measurement in *feet.*)

**Ex. 583.**—Required the pressure in lbs. on a Sluice gate 3 feet square, whose centre is 30 feet below the surface of the water. Answer 3 × 3 × 30 × 62·32 = 16826 lbs.

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## DISCHARGE THROUGH ORIFICES.

*Cubic feet per second.*

Rectangular. Open to surface. $\dfrac{\text{Area in Sq. ft.} \times \sqrt{\text{depth in ft.}}}{\sqrt{\cdot 0864}}$.

Rectangular. All below the surface. $\dfrac{\text{Area in Sq. ft.} \times \sqrt{\text{depth in ft.}}}{\sqrt{\cdot 0384}}$.

**Ex. 584.**—Required the discharge in Cubic feet per second, through an opening in a dam or weir $6\frac{1}{2}$ feet wide, and extending 9 inches below the surface. (Area = 6·5 × ·75 = 4·875 Square feet.)

| | | | |
|---|---|---|---|
| C | ·75 | ·0864 | as in Ex. 257. |
| D | 14·4 Cub. ft. | 4·875 area in Sq. ft. | |

**Ex. 585.**—Required the discharge in Cubic feet per second, through a similar opening to the above, but whose centre is 4 feet *below* the surface. (Here area = 4·875 Square feet, as before.)

| | | | |
|---|---|---|---|
| C | ·0384 | 4 | as in Ex. 257. |
| D | 4·875 area in Sq. ft. | 49·72 Cub. ft. | |

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# APPENDIX A.

## PRINCIPLES ON WHICH THE LINES ARE LAID DOWN ON THE SLIDE RULE.

(For "Logarithms" see Appendix B.)

It is not at all *necessary* to know what will be explained in this Section, and it may be omitted by those who are not interested in the matter. To those who *are*, it may be observed that the line A (and the same remarks apply throughout to B and C) is a *Logarithmic Scale;* that is, on it, Logarithmic *spaces* take the place of Logarithmic *numbers.*

Let either Radius on A, B, or C, represent a space = 1·0. Then the distance from 1 to 2 = ·301 (log. of 2); from 1 to 3 is a distance = ·477 (log. of 3); 1 to 4 = ·602 (log. of 4), &c.*

But we know that instead of multiplying two numbers for a product, we may *add* their Logarithms, and the number corresponding to that Logarithmic sum is their numerical product. Say 2 × 4 = 8.

---

* In a 12 inch Rule, each radius on A, B, or C, is $5\frac{1}{2}$ inches; from 1 to 4 therefore, would be 5·5 × ·602 = 3·31 inches; and so on.

Now Log. of 2 *plus* Log. of 4 = Log. of 8. To try this on the Slide Rule, take a pair of compasses, or slip of paper. Measure the space from 1 to 2 on the line A, and add to it a space measured from 1 to 4. It will reach to 8. Instead of using compasses, we may use the *Slide* B; for if we set 1 of B under 2 of A, the line B itself will measure onwards from 2 of A, a space equal to 1 to 4 on A (for the divisions on A and B are identical). Then over 4 on B will be seen 8 on A. "Division" is just the reverse of the above operation.

But it may be asked "How is this made applicable to a *Rule of Three sum*, which after all is *the* sum of the Slide Rule?" Simply thus: Let us take it in the form $x = \frac{bc}{a}$, or as a numerical example $x = \frac{3 \times 8}{6}$. With the compasses measure the space from 1 to 8 on the line A, and add it *on* to the right of 3. It will reach to 24. (So far it is multiplication.) Then take a space equal to 1 to 6, and measure this *back* from 24. (This is division.) It will reach back to 4; and this is the answer; but the Slide B supersedes the compasses. It may simplify the explanation, by writing the equation $x = 3 \times \frac{8}{6}$. Take on B the distancc between 6 and 8, *i.e.*, their *difference* (which arithmetically would be dividing 8 by 6), and add this (which is the same as multiplying) to 3 on A, by setting 6 on B under 3 on A, and looking forward to 8 on B; over which, on A, is 4 the answwr. The Slide Rule, when set, will then appear thus:

| A | 3 | 4 Answer |
|---|---|---|
| B | 6 | 8 |

the space between 6 and 8 on B being added to the 3 on A.

Since, then, the Slide Rule can solve any question given in the form $x = b \times \frac{c}{a}$, or $x = \frac{bc}{a}$, it will solve any "Rule of Three" sum; for $x = \frac{bc}{a}$ is the same as $a : b :: c : x$.

With regard to the D line, it will be observed that the one Radius on that line is exactly twice the length of either of the Radii on A, B, or C. Take, for instance, the number 4. From 1 to 4 on C is 5·5 × ·602 inches, or 3·31; but from 1 to 4 on D is 11 × ·602, or

6·62 inches. A space therefore (say 1 to 4) on C, has the value of twice the same space on D; or in other words, a space on C is double *in value* a similar space on D. Now as these spaces are *logarithmic* values, and as to double a logarithm is to square its natural number, the numbers on C are the *squares* of those under them on D, when the two lines are set even.

In those Rules which have the line called "NUMBERS" or "LOG," on the back of the Slide, it will be seen that this line is 11 inches in length, and divided into 100 parts of 5 subdivisions each. If laid even with the line D, it answers to show the logarithmic values of the Radius on D, which is also 11 inches; so that over 4 on D will be ·602, which is the logarithm of 4. (See Appendix B.)

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## APPENDIX B.

### LOGARITHMS.

On the back of the Slide is a line marked "Num." or "Numbers," or sometimes "Logs." If this line is shut in even with the line D, the numbers on it will represent the *Logarithms* of the numbers under them on D. (See last paragraph of Appendix A.)

The following will exemplify; and if some of these equivalents are marked on the Rule with a pen and ink, it will be a guide to the accurate setting of the Slide.

| N | 1 | 0·301 | 0·380 | 0·544 | 0·716 | 0·903 | Logs. |
|---|---|---|---|---|---|---|---|
| D | 1 | 2·0 | 2·4 | 3·5 | 5·2 | 8·0 | Nat. numbers |

The *indices* to the Logarithms must be added, just as we should do in using "Tables." Thus the log. of 5200 would not be ·716 (which is all the line N shows), but 3·716. So also, the log. of ·52 is $\bar{1}$·716, or 9·716; and the log. of ·052 is $\bar{2}$·716, or 8·716. So also if the logarithm 3·064 is given; under ·064 we read 116; but the right natural number must have *four* figures, and is therefore 1160.

The following few Logarithms may be useful, for checking the Slide, to see if it is correctly divided.

| | | |
|---|---|---|
| 2 = ·301030 | 6 = ·778151 | 1·66 = ·2201 |
| 3 = ·477121 | 7 = ·845098 | 1·95 = ·2900 |
| 4 = ·602060 | 8 = ·903090 | 3·2 = ·5051 |
| 5 = ·698970 | 9 = ·954242 | 7·5 = ·8750 |

The line N will not read nearer than to three places of figures, unless the eye is experienced; still it will be sufficiently near, to be useful in the five following cases :—

1. Involution and Evolution.
2. Areas of Triangles whose 3 sides are given.
3. Geometrical Progression.
4. Compound Interest.
5. Annuities and Leases.

## [I.] INVOLUTION.

**Ex. 586.**—Raise 8 to the sixth power; or $8^6$.

| 1st. Log. of 8 | N | 1 \| | ·903 |
|---|---|---|---|
| | D | 1 \| | 8 |

| 2d. ·903 × 6 | A | ·903 | 5·418 |
|---|---|---|---|
| | B | 1 | 6 |

| 3d. Nat. num. of ·418 | N | 1 \| | 418 |
|---|---|---|---|
| | D | 1 \| | 262 |

The "index" being 5, the answer is 26200. The *true* answer is 262144. The error of the Rule being only $\frac{1}{1820}$.

**Ex. 587.**—What is $\sqrt[5]{17382}$?

| 1st. Log. of 17382 | N | 1 \| | ·241 |
|---|---|---|---|
| | D | 1 \| | 17382 |

To the ·241 add the "index" 4; and since there are 5 figures in the Natural number, we get the log. 4·241 as the log. of 17382. Then

2d. $\frac{4{\cdot}241}{5}$

| | | | |
|---|---|---|---|
| A | ·848 | | 1 |
| B | 4·241 | | |

3d. Nat. num. of ·845

| | | | |
|---|---|---|---|
| N | 1 \| | | ·848 |
| D | ·1 \| | | 7·05 Ans. |

### [II.] AREAS OF TRIANGLES.

**Ex. 588.**—What is the area in Square feet, of a Triangle whose 3 sides are 45, 53, and 61 feet? (See Ex. 373.)

The half sum of the sides is 79·5, and the area is

$$\sqrt{18{\cdot}5 \times 26{\cdot}5 \times 34{\cdot}5 \times 79{\cdot}5}.$$

| | | | | | |
|---|---|---|---|---|---|
| N | 1 \| | ·267 | ·423 | ·538 | ·900 |
| D | 1 \| | 18·5 | 26·5 | 34·5 | 79·5 |

Each log. requires the "index" 1. Then add the four, and the sum is 6·128. Half of this logarithm (equivalent to extracting the Square Root of its Natural number) is 3·064. Then under ·064 on N, we find 116 on D, but the "index" being 3, the answer must contain *four* figures, or 1160. The true answer is 1159·8. The error is $\frac{1}{5800}$.

Ex. 373 may be tried for practice.

So again if the three sides are 126, 247, and 296 feet. The area is $\sqrt{208{\cdot}5 \times 87{\cdot}5 \times 38{\cdot}5 \times 334{\cdot}5}$.

| | | | | | |
|---|---|---|---|---|---|
| N | 1 \| | ·586 | ·942 | ·319 | ·524 |
| D | 1 \| | 38·5 | 87·5 | 208·5 | 334·5 |

Then 1·586
1·942
2·319
2·524
2)8·371
4·185

| | | | |
|---|---|---|---|
| N | 1 \| | ·185 | |
| D | 1 \| | 153 | |

(see over)

And with an Index of 4, the answer would be 15300 Square feet. The true result is 15328 feet.

N.B. The above logarithms were actually taken out from a 12-inch Rule of Messrs. Elliot.

## [III.] GEOMETRICAL PROGRESSION.

(For "*Arithmetical* Progression," see next page.)

$$\left.\begin{aligned} \text{Let } a &= \text{first term} \\ z &= \text{last term} \\ r &= \text{ratio} \\ s &= \text{sum of series} \\ n &= \text{number of terms} \end{aligned}\right\} \begin{aligned} z &= (r^{n-1}) \times a. \\ s &= a \times \frac{r^n - 1}{r - 1}. \\ s &= \frac{(z \times r) - a}{r - 1}. \end{aligned}$$

**Ex. 589.**—The first term in a Geometrical series is 5; the last term is 3645, and the ratio is 3. What is the *sum* of the series?

Here $s = \dfrac{(3645 \times 3) - 5}{3 - 1} = \dfrac{10935 - 5}{2} = 5465$. The Slide Rule will assist in the solution, but "Logarithms" are not required.

**Ex. 590.**—A farmer agreed to purchase 20 oxen on the following terms: 3 farthings for the first, 9 farthings for the second, 27 farthings for the third, and so on trebling on each one to the 20th, and to pay the price of that last or 20th. What would he have to pay?

Here $a = 3$, $r = 3$, $n = 20$; and the last term ($z$) would be $(3^{19}) \times 3$.

As shown in page 253, the Slide Rule gives the logarithm of 3 to be ·477, and this multiplied by 19 (also by the Slide Rule) is 9·065. The Slide Rule (as in page 253) shows the natural number on D to be 116, or ten figures for the Index 9, gives 1160000000 as the value of $3^{19}$. This multiplied by 3 gives 3480,000,000 farthings = £3,625,000. (The true answer is 3,486,784,401 farthings = £3,632,067 1*s.* 8¼.)

**Ex. 591.**—Let $a = 6$, $r = 3$, $n = 12$. Required the *last* term, as also the *sum* of the series.

$z = (3^{11}) \times 6 = 177147 \times 6 = 1062882 =$ last Term.

$$s = \frac{3^{12} - 1}{3 - 1} \times 6 = \frac{531441 - 1}{2} \times 6 = 531440 \times 6 = 1594320$$

$=$ Sum.

**Ex. 592.**—A person asked as the price of his horse, the value of the *last* nail, taking 1 farthing for the first, $\frac{1}{2}d.$ for the second, $1d.$ for the third, and so on doubling at each nail. The horse had 8 nails in each shoe, or 32 nails. Required the price.

The Slide Rule gives ·301 as the Log. of 3. Then $·301 \times 31 = 9·332$. It also shows on D that the natural number for 332 is 2147; so with an Index of 9, ten figures in the answer, or 2,147,000,000 farthings = £2,240,000. (The true answer is £2,236,962 2*s.* 8*d.*, or an error of $\frac{1}{667}$.)

**Ex. 593.**—In the above case, what would have been the price if the *sum* of the series had been asked? Here $s = \frac{(r^n) - 1}{r - 1} \times a$ $= \frac{(2^{32}) - 1}{2 - 1} \times 1.$

$2^{32} = 9·6329600$ which is the Log. of 4294968317; and deducting 1, leaves 4,294,968,315 farthings = £4,473,924 5*s.* $3\frac{3}{4}d.$ The Slide Rule would make it 4,270,000,000 farthings, or dividing by 960, = £4,450,000.

[*Arithmetical* Progression.]

The Slide Rule does not give the answer *direct*, but may be made useful in working out the computations.

$$\left.\begin{array}{rl} \text{Let } a = & \text{first term} \\ z = & \text{last term} \\ d = & \text{common difference} \\ n = & \text{number of terms} \\ s = & \text{sum of terms} \end{array}\right\} \begin{array}{l} (1.)\ a = \frac{s}{n} - \frac{(n-1) \times d}{2}. \\ (2.)\ a = z - (n - 1 \times d). \\ (3.)\ s = \left(a + \frac{n - 1 \times d}{2}\right) \times n. \end{array}$$

$$(4.)\ s = \frac{(a + z) \times n}{2}.$$

N.B. When $a = 1$, and $d = 2$, then $s = n^2$.

(5.) $d = \dfrac{z - a}{n - 1}$.

(6.) $n = \dfrac{z - a}{d} + 1$.

(7.) $z = \dfrac{\sqrt{(8ds + (2a - d)^2}}{2} - \frac{1}{2}d$.

**Ex. 594.**—A man is to receive 360£ at 12 several payments, each to exceed the former by 4£. What must be the *first* instalment?

Here by (1) $\dfrac{360}{12} - \dfrac{11 \times 4}{2}$, or $30 - 22 = 8$£.

**Ex. 595.**—A man takes out of his pocket at 8 several times, so many different numbers of shillings, every one exceeding the former by 6. The last was 64. What was the *first*?

Here, by (2) $64 - (7 \times 6) = 64 - 42 = 22$ answer.

**Ex. 596.**—Suppose 400 stones placed in a straight line, at intervals of 1 yard from each other, and a basket placed 1 yard from the first; how many *miles* must a man travel to bring back each singly to the basket? Here $a$ the first term $= 1$ yard going, and 1 returning, or *two* yards. By (3) $\left(2 + \dfrac{399 \times 2}{2}\right) \times 400$ $= (2 + 399) \times 400 = 160400$ *yards*; or $\dfrac{(2 + 399) \times 400}{1760}$ $= 91{\cdot}136$ *miles*.

**Ex. 597.**—What is the sum of a series of terms whose first term is 5, last term 29, and number of terms 7?

Here by (4) $\dfrac{(5 + 29) \times 7}{2} = \dfrac{34 \times 7}{2} = 119$.

**Ex. 598.**—A person is to travel to a certain place in 12 days, and to go but 3 miles the first day, increasing every day by some regular excess, so that the last day's journey may be 58 miles. What is this daily excess, and total length of the journey? Here the excess by (5) $= \dfrac{58 - 3}{11} = 5$; and length of journey by (4) $= \dfrac{(58 + 3) \times 12}{2} = 366$ miles.

**Ex. 599.**—A man being asked how many children he had, said that the youngest was 4 years old, and the oldest 32; and that his family had increased by 1 every 4 years. How many children had he? Here by (6) $x = \frac{32 - 4}{4} + 1 = 8.$

**Ex. 600.**—Let the first term be 7; the common difference 2; and the sum of the series 567. Required the last term.

$$\text{Here by (7) } x = \frac{\sqrt{(8 \times 2 \times 567) + \overline{14 - 2}|^2}}{2} - \tfrac{1}{2}2$$

$$= \frac{\sqrt{9216}}{2} - 1 = 47.$$

## [IV.] COMPOUND INTEREST.

(For "Simple Interest," see page 59.)

Let $p$ = Principal

$r$ = Interest of 1£ for 1 year in £. Thus at at 5 per cent. $r$ = ·05£

$t$ = Number of years

$m$ = Amount at end of $t$ years being principal *plus* interest

R = 1 + $r$ or the amount of 1£ at the end of the first year

$i$ = Accumulated interest

$$m = p \times R^t.$$

$$p = \frac{m}{R^t}.$$

$$i = m - p.$$

$$t = \sqrt[R]{\frac{m}{p}}.$$

$$R = \sqrt[t]{\frac{m}{p}}.$$

$$r = R - 1.$$

The following Logarithms are useful, and some might be written on the back of the Slide Rule. Tables I., II., III., need not be used, except as *comparison* with Slide Rule work:—

| | | | |
|---|---|---|---|
| 1·03 | log. ·0128372 | 1·0575 | log. ·0242804 |
| 1·035 | log. ·0149403 | 1·06 | log. ·0253059 |
| 1·04 | log. ·0170333 | 1·065 | log. ·0273496 |
| 1·045 | log. ·0191163 | 1·07 | log. ·0293838 |
| 1·05 | log. ·0211893 | 1·075 | log. ·0314085 |
| 1·055 | log. ·0232526 | 1·08 | log. ·0334238 |

## TABLE I.

### Amount of 1£ at "Compound Interest" = $R^t$.

| Years. | 3 per cent. | 3½ per cent. | 4 per cent. | 4½ per cent. | 5 per cent. |
|---|---|---|---|---|---|
| 1 | 1·0300 | 1·0350 | 1·0400 | 1·0450 | 1·0500 |
| 2 | 1·0609 | 1·0712 | 1·0816 | 1·0920 | 1·1025 |
| 3 | 1·0927 | 1·1087 | 1·1248 | 1·1411 | 1·1576 |
| 4 | 1·1255 | 1·1475 | 1·1698 | 1·1925 | 1·2155 |
| 5 | 1·1593 | 1·1876 | 1·2166 | 1·2461 | 1·2763 |
| 6 | 1·1940 | 1·2292 | 1·2653 | 1·3022 | 1·3400 |
| 7 | 1·2299 | 1·2722 | 1·3159 | 1·3608 | 1·4071 |
| 8 | 1·2668 | 1·3168 | 1·3685 | 1·4221 | 1·4774 |
| 9 | 1·3048 | 1·3628 | 1·4233 | 1·4860 | 1·5513 |
| 10 | 1·3439 | 1·4106 | 1·4802 | 1·5529 | 1·6289 |
| 11 | 1·3842 | 1·4599 | 1·5394 | 1·6228 | 1·7103 |
| 12 | 1·4258 | 1·5110 | 1·6010 | 1·6958 | 1·7958 |
| 13 | 1·4685 | 1·5639 | 1·6650 | 1·7721 | 1·8856 |
| 14 | 1·5126 | 1·6186 | 1·7316 | 1·8519 | 1·9799 |
| 15 | 1·5580 | 1·6753 | 1·8009 | 1·9352 | 2·0789 |
| 16 | 1·6047 | 1·7339 | 1·8729 | 2·0223 | 2·1829 |
| 17 | 1·6528 | 1·7946 | 1·9479 | 2·1133 | 2·2920 |
| 18 | 1·7024 | 1·8574 | 2·0258 | 2·2084 | 2·4066 |
| 19 | 1·7535 | 1·9225 | 2·1068 | 2·3078 | 2·5269 |
| 20 | 1·8061 | 1·9897 | 2·1911 | 2·4117 | 2·6533 |
| 21 | 1·8603 | 2·0594 | 2·2787 | 2·5202 | 2·7860 |
| 22 | 1·9161 | 2·1315 | 2·3699 | 2·6336 | 2·9253 |
| 23 | 1·9736 | 2·2061 | 2·4647 | 2·7521 | 3·0715 |
| 24 | 2·0328 | 2·2833 | 2·5633 | 2·8760 | 3·2251 |
| 25 | 2·0938 | 2·3632 | 2·6658 | 3·0054 | 3·3863 |
| 26 | 2·1566 | 2·4459 | 2·7724 | 3·1406 | 3·5557 |
| 27 | 2·2213 | 2·5315 | 2·8833 | 3·2820 | 3·7334 |
| 28 | 2·2879 | 2·6201 | 2·9987 | 3·4297 | 3·9201 |
| 29 | 2·3566 | 2·7118 | 3·1186 | 3·5840 | 4·1161 |
| 30 | 2·4273 | 2·8067 | 3·2433 | 3·7453 | 4·3219 |
| 31 | 2·5000 | 2·9050 | 3·3731 | 3·9138 | 4·5380 |
| 32 | 2·5751 | 3·0067 | 3·5080 | 4·0899 | 4·7649 |
| 33 | 2·6523 | 3·1119 | 3·6483 | 4·2740 | 5·0032 |
| 34 | 2·7319 | 3·2208 | 3·7943 | 4·4663 | 5·2533 |
| 35 | 2·8139 | 3·3335 | 3·9460 | 4·6673 | 5·5160 |
| 36 | 2·8983 | 3·4502 | 4·1839 | 4·8773 | 5·7918 |
| 37 | 2·9852 | 3·5710 | 4·2680 | 5·0968 | 6·0814 |
| 38 | 3·0748 | 3·6960 | 4·4388 | 5·3262 | 6·3855 |
| 39 | 3·1670 | 3·8253 | 4·6163 | 5·5658 | 6·7047 |
| 40 | 3·2620 | 3·9592 | 4·8010 | 5·8163 | 7·0400 |
| 41 | 3·3599 | 4·0978 | 4·9930 | 6·0781 | 7·3920 |
| 42 | 3·4607 | 4·2412 | 5·1927 | 6·3516 | 7·7616 |
| 43 | 3·5645 | 4·3897 | 5·4004 | 6·6374 | 8·1497 |
| 44 | 3·6714 | 4·5433 | 5·6165 | 6·9361 | 8·5571 |
| 45 | 3·7816 | 4·7023 | 5·8411 | 7·2482 | 8·9850 |
| 46 | 3·8950 | 4·8669 | 6·0748 | 7·5744 | 9·4342 |
| 47 | 4·0119 | 5·0372 | 6·3178 | 7·9152 | 9·9060 |
| 48 | 4·1322 | 5·2135 | 6·5[illegible]05 | 8·2714 | 10·401 |
| 49 | 4·2562 | 5·3960 | 6·8333 | 8·6436 | 10·921 |
| 50 | 4·3839 | 5·5849 | 7·1066 | 9·0326 | 11·467 |
| 60 | 5·8916 | 7·8781 | 10·520 | 14·027 | 18·679 |
| 70 | 7·9178 | 11·113 | 15·572 | 21·784 | 30·426 |
| 80 | 10·640 | 15·676 | 23·050 | 33·830 | 49·561 |
| 90 | 14·300 | 22·112 | 34·119 | 32·537 | 80·730 |
| 100 | 19·219 | 31·191 | 50·505 | 81·588 | 131·50 |

## To find "m" the Amount.

**Ex. 601.**—What will £450 amount to in 6 years, at 4 per cent. (Here $m = p \times R^t = 450 \times 1{\cdot}04^6$.

The log. of $1{\cdot}04^6$ is found as in Ex. 586 to be ·1022, and the natural number under it on D is 1·265, which is $R^t$. Multiply by 450, and the "Amount" (including interest) is £569. The *true* answer is 569·38 or an error of only $\frac{1}{1500}$.

N.B. With a "Table" such as I. page 260 which is $R^t$ for numbers of years, the Slide Rule will solve several cases at *one setting*, as follows :—

**Ex. 602.**—What would £450 amount to in 6, 7, 8, and 9 years, at 4 per cent. ?

| A | 1 | 1·265 | 1·316 | 1·423 = $R^t$ |
|---|---|---|---|---|
| B | 450 | £569 | £392 | £640 |

N.B. When the Interest is converted into Principal *half-yearly*, $m = p \times \left(1 + \frac{r}{2}\right)^{2t}$ : and if *quarterly*, $m = p \times \left(1 + \frac{r}{2}\right)^{4t}$.

**Ex. 603.**—What would £1 at $2\frac{1}{2}$ per cent. for 11 years amount to (including interest) if calculated quarterly ? (Here $m = \left(1 + \frac{\cdot 035}{4}\right)^{44}$.

Now $\frac{\cdot 035}{4} = \cdot 00875$ ; whence $m = 1{\cdot}00875^{44}$. The log. of ·00875 = ·0037837 ; and this multiplied by 44 = ·1664828, the *natural number* of which (*i.e.* the answer) is 1·4672. Had the Interest been added in *yearly* for the 11 years, the "amount" would have been $1{\cdot}035^{11}$ = £1·460.

---

## To find "p" the principal.

**Ex. 604.**—A certain sum amounted to £704 in 6 years, at 5 per cent. compound interest. What was that sum ?

$\left(\text{Here } p = \frac{R^t}{m}\right)$. Find $R^t$ or $1{\cdot}05^6$ as in Ex. 586, to be 1·34 (or take it from Table I.). Then

$$\frac{704}{1{\cdot}34}\ \frac{A}{B}\ \frac{1}{1{\cdot}34}\ \overline{\qquad\qquad}\ \frac{525\pounds = p}{704}$$

---

## To find the "Present value" $= \frac{1}{R^t} \times m$.

**Ex. 605.**—Find the "present value" of £3000 due 20 years hence, at 5 per cent. compound interest?

$R^t$ or $1{\cdot}025^{20}$ is found as in Ex. 586, (or Table I.) to be 2·653, and $\frac{1}{2{\cdot}653} = {\cdot}3769$. Hence the "present value" $= {\cdot}3769 \times 3000 =$ £1130⅔.

N.B. The "reciprocals" of the values in Table I. are the *present values* for £1.

---

## To find "t" the Time in years.

**Ex. 606.**—In how many years will £327 amount to £467 at 3 per cent. compound interest? $\left(\text{Here } t = \sqrt[1{\cdot}03]{\frac{467}{328}}.\right)$ The Slide Rule as in page 19, gives $\frac{467}{328} = 1{\cdot}42$. Then as in Ex. 586, the log. of 1·42 is ·153; and as the log. of 1·03 is shown to be ·0128, we have $\frac{{\cdot}153}{{\cdot}0128} = 12$ years the answer.

N.B. The time in years in which any sum will *double* itself, is $\frac{\text{Log. } 2}{\text{Log. R}}$. To *treble* itself $\frac{\text{Log. } 3}{\text{Log. R}}$, &c. Thus to double at 5 per cent.

takes $\frac{\cdot 30103}{\cdot 0212} = 14\cdot 2$ years. At $4\frac{1}{2}$ per cent. it takes $\frac{\cdot 30103}{\cdot 01911} = 15\cdot 75$ years. To *treble* itself at $3\frac{1}{2}$ per cent., it takes $\frac{\cdot 47712}{= 01494} = 31\cdot 9$ years.

---

## To find "R" or the amount of £1 at the end of the first year.

**Ex. 607.**—What will R be when £225 amounts to £260 10*s.* in three* years? $\left(\text{Here R} = \sqrt[3]{\frac{m}{p}}.\right)$

The Slide Rule, as in page 19, shows $\frac{m}{p}$ or $\frac{260\cdot 5}{225}$ to be $1\cdot 16$; and $\sqrt[3]{1\cdot 16}$ is found by dividing the log. of $1\cdot 16$ by 3, as in Ex. 587, or $\frac{\cdot 0636}{3} = \cdot 0212$. This is a logarithm of which the natural number on the line D (page 253) is $1\cdot 05$. Therefore the answer is $1\cdot 05 = \text{R}$.

---

## To find "r" the rate of Interest.

Find R, as above, and deduct 1. Thus in Ex. 607 R being $1\cdot 05$ the rate of interest (or "r") is $1\cdot 05 - 1$, or $\cdot 05$, or 5 per cent.

### *Population.*

The calculations are made as in "Compound Interest."

Let $p$ = the population when a former census was made.

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* When the number of years is *three*, as in Ex. 607, $\sqrt[3]{\frac{m}{p}}$ may be found as in Ex. 314, to be $1\cdot 05$.

$m$ = the population at a subsequent census.

$t$ = time in years between $p$ and $m$.

$r$ = per centage of annual measure.

$R = 1 + r$.

Then $m \times p = R^t$; $R = \sqrt[t]{\frac{m}{p}}$; $r = R - 1$.

**Ex. 608.**—The population of a county in 1861, was 140140; and in 1871, it was 153196. What will be its population in 1881? What its per centage of increase annually? In how many years will it double itself? *Work by the Slide Rule alone.*

1st. $R = \sqrt[10]{\frac{153196}{140140}}$ log. 153196 = 5·185
log. 140140 = 5·146

$10\ )\ 0·039 = R^t$

$·0039 = R = 1·009$

Or nearly 1 per cent. annually.

2d. $m = p \times R^t$ log. of $R^t$ as above 0·039
log. of 153196 as above 5·185

5·224

And 224 by line N = 167500 = $m$.

(*True* answer is 167468.)

3d. Years to double $= \frac{\text{log. } 2}{\text{log. } R} = \frac{·301}{·0039} = 77$ years.

(The *true* answer is 77·815; for log. R = ·0038685.)

**Ex. 609.**—The population of Great Britain in 1861 was 23,200,000. In 1871 it was 26,200,000. What was [I.] the annual rate of increase? [II.] What would the population be in 1881? [III.] In how many years will it double itself?

[I.] $R = \sqrt[t]{\dfrac{26{,}200{,}000}{23{,}200{,}000}}$ 

| | | |
|---|---|---|
| log. 26·2 mill. | = | 7·415 |
| log. 23·2 mill. | = | 7·365 |
| | 10 ) | 0·050 = log. $R^t$. |
| Log. R | = | ·005 |
| Natural number | = | 1·012 |

Or an increase of 1·2 per cent. annually

[II.] $m = p \times R^t$

| | |
|---|---|
| log. $R^t$ as above | ·050 |
| log. $p$ as above | 7·415 |
| | 7·465 = 29,180,000 |

[III.] Years to double $= \dfrac{\text{log. } 2}{\text{log. R}} = \dfrac{301}{\cdot 005} = 60\cdot 2$ years.

## [V.] ANNUITIES AND LEASES.*

(For "forborne" Annuities, see page 272.

$r$ = Interest of £1 for 1 year; thus at 5 per cent., $r$ = ·05.

R = 1 + $r$, or the amount of £1, with interest for 1 year.

$$p = \text{Present value} = a \times \frac{1 - \frac{1}{R^t}}{r}$$

$$a = \text{Annuity} = p \div \frac{1 - \frac{1}{R^t}}{r}$$

$$t = \text{Time in years} = \frac{\text{Log. } a - \text{Log. } (a - rp)}{\text{Log. R}}.$$

* Those who use the Slide Rule for these calculations should have inscribed on some blank part of the Rule, the Logarithms of 1·035, 1·04, 1·045, &c., from page 259.

## TABLE II. $\left(1 - \frac{1}{R^t}\right) \div r.$

### Present Value of an "Annuity" of £1.

| Years. | 3½ per cent. | 4 per cent. | 5 per cent. | 6 per cent. | 7 per cent. |
|---|---|---|---|---|---|
| 1 | 0·9662 | 0·9615 | 0·9524 | ·9434 | ·9346 |
| 2 | 1·8997 | 1·8861 | 1·8594 | 1·8334 | 1·8080 |
| 3 | 2·8016 | 2·7751 | 2·7232 | 2·6730 | 2·6243 |
| 4 | 3·6731 | 3·6299 | 3·5459 | 3·4651 | 3·3872 |
| 5 | 4·5150 | 4·4518 | 4·3295 | 4·2124 | 4·1002 |
| 6 | 5·3285 | 5·2421 | 5·7057 | 4·9173 | 4·7665 |
| 7 | 6·1145 | 6·0020 | 5·7864 | 5·5824 | 5·3893 |
| 8 | 6·8739 | 6·7327 | 6·4632 | 6·2098 | 5·9713 |
| 9 | 7·6077 | 7·4353 | 7·1078 | 6·8017 | 6·5152 |
| 10 | 8·3166 | 8·1109 | 7·7217 | 7·3601 | 7·0236 |
| 11 | 9·0015 | 8·7605 | 8·3064 | 7·8869 | 7·4987 |
| 12 | 9·6633 | 9·3851 | 8·8632 | 8·3838 | 7·9427 |
| 13 | 10·303 | 9·9856 | 9·3936 | 8·8527 | 8·3576 |
| 14 | 10·920 | 10·563 | 9·8986 | 9·2495 | 8·7455 |
| 15 | 11·517 | 11·118 | 10·380 | 9·7122 | 9·1079 |
| 16 | 12·094 | 11·562 | 10·838 | 10·106 | 9·4466 |
| 17 | 12·651 | 12·166 | 11·274 | 10·477 | 9·7632 |
| 18 | 13·190 | 12·659 | 11·689 | 10·828 | 10·059 |
| 19 | 13·710 | 13·134 | 12·085 | 11·158 | 10·335 |
| 20 | 14·212 | 13·590 | 12·462 | 11·470 | 10·594 |
| 21 | 14·698 | 14·029 | 12·821 | 11·764 | 10·835 |
| 22 | 15·167 | 14·451 | 13·163 | 12·041 | 11·061 |
| 23 | 15·620 | 14·857 | 13·489 | 12·303 | 11·272 |
| 24 | 16·058 | 15·247 | 13·799 | 12·550 | 11·469 |
| 25 | 16·481 | 15·622 | 14·094 | 12·783 | 11·653 |
| 26 | 16·890 | 15·983 | 14·375 | 13·003 | 11·826 |
| 27 | 17·285 | 16·329 | 14·643 | 13·210 | 11·987 |
| 28 | 17·667 | 16·663 | 14·898 | 13·406 | 12·137 |
| 29 | 18.036 | 16·984 | 15·141 | 13·591 | 12·278 |
| 30 | 18·392 | 17·292 | 15·372 | 13·765 | 12·409 |
| 31 | 18·736 | 17·588 | 15·593 | 13·929 | 12·531 |
| 32 | 19·069 | 17·873 | 15·803 | 14·084 | 12·646 |
| 33 | 19·390 | 18·148 | 16·002 | 14·230 | 12·754 |
| 34 | 19·700 | 18·411 | 16·193 | 14·368 | 12·854 |
| 35 | 20·000 | 18·665 | 16·374 | 14·498 | 12·948 |
| 36 | 20·290 | 18·908 | 16·547 | 14·621 | 13·035 |
| 37 | 20·570 | 19·142 | 16·711 | 14·737 | 13·117 |
| 38 | 20·841 | 19·368 | 16·868 | 14·846 | 13·193 |
| 39 | 21·102 | 19·584 | 17·017 | 14·949 | 13·265 |
| 40 | 21·355 | 19·793 | 17·159 | 15·046 | 13·332 |
| 41 | 21·599 | 19·993 | 17·294 | 15·138 | 13·394 |
| 42 | 21·835 | 20·186 | 17·423 | 15·224 | 13·452 |
| 43 | 22·063 | 20·371 | 17·546 | 15·306 | 13·506 |
| 44 | 22·283 | 20·549 | 17·663 | 15·383 | 13·557 |
| 45 | 22·495 | 20·720 | 17·774 | 15·456 | 13·605 |
| 46 | 22·701 | 20·885 | 17·880 | 15·524 | 13·650 |
| 47 | 22·899 | 21·043 | 17·981 | 15·589 | 13·691 |
| 48 | 23·091 | 21·195 | 18·077 | 15·650 | 13·730 |
| 49 | 23·276 | 21·341 | 18·169 | 15·707 | 13·767 |
| 50 | 23·455 | 21·482 | 18·256 | 15·762 | 13·800 |

## TABLE II. *continued.*

| Years. | 3½ per cent. | 4 per cent. | 5 per cent. | 6 per cent. | 7 per cent. |
|---|---|---|---|---|---|
| 51 | 23·629 | 21·617 | 18·339 | 15·813 | 13·832 |
| 52 | 23·796 | 21·747 | 18·418 | 15·861 | 13·862 |
| 53 | 24·957 | 21·873 | 18·493 | 15·907 | 13·890 |
| 54 | 24·113 | 21·993 | 18·565 | 15·950 | 13·916 |
| 55 | 24·264 | 22·109 | 18·633 | 15·990 | 13·940 |
| 56 | 24·410 | 22·220 | 18·698 | 16·029 | 13·962 |
| 57 | 24·550 | 22·227 | 18·760 | 16·065 | 13·984 |
| 58 | 24·686 | 22·429 | 18·819 | 16·099 | 14·003 |
| 59 | 24·818 | 22·528 | 18·876 | 16·131 | 14·022 |
| 60 | 24·945 | 22·623 | 18·929 | 16·161 | 14·039 |
| 61 | 25·667 | 22·715 | 18·980 | 16·190 | 14·055 |
| 62 | 25·186 | 22·803 | 19·029 | 16·217 | 14·070 |
| 63 | 25·300 | 22·887 | 19·075 | 16·242 | 14·084 |
| 64 | 25·411 | 22·968 | 19·119 | 16·266 | 14·098 |
| 65 | 25·518 | 23·047 | 19·161 | 16·289 | 14·110 |
| 66 | 25·621 | 23·122 | 19·201 | 16·310 | 14·121 |
| 67 | 25·721 | 23·194 | 19·239 | 16·331 | 14·132 |
| 68 | 25·817 | 23·263 | 19·275 | 16·350 | 14·142 |
| 69 | 25·910 | 23·330 | 19·310 | 16·368 | 14·152 |
| 70 | 26·000 | 23·394 | 19·343 | 16·384 | 14·160 |
| 71 | 26·087 | 23·456 | 19·374 | 16·400 | 14·168 |
| 72 | 26·171 | 23·516 | 19·404 | 16·415 | 14·176 |
| 73 | 26·252 | 23·573 | 19·432 | 16·430 | 14·182 |
| 74 | 26·331 | 23·627 | 19·459 | 16·443 | 14·190 |
| 75 | 26·407 | 23·680 | 19·485 | 16·456 | 14·196 |
| 76 | 26·480 | 23·731 | 19·509 | 16·468 | 14·202 |
| 77 | 26·551 | 23·780 | 19·533 | 16·479 | 14·208 |
| 78 | 26·619 | 23·827 | 19·55[illegible] | 16·490 | 14·213 |
| 79 | 26·685 | 23·872 | 19·576 | 16·500 | 14·217 |
| 80 | 26·749 | 23·915 | 19·596 | 16·509 | 14·222 |
| 81 | 26·810 | 23·957 | 19·616 | 16·518 | 14·226 |
| 82 | 26·870 | 23·997 | 19·634 | 16·526 | 14·230 |
| 83 | 26·927 | 24·036 | 19·651 | 16·534 | 14·234 |
| 84 | 26·983 | 24·073 | 19·668 | 16·542 | 14·237 |
| 85 | 27·037 | 24·108 | 19·684 | 16·549 | 14·240 |
| 86 | 27·087 | 24·143 | 19·699 | 16·556 | 14·243 |
| 87 | 27·139 | 24·176 | 19·713 | 16·562 | 14·246 |
| 88 | 27·187 | 24·207 | 19·727 | 16·568 | 14·249 |
| 89 | 27·234 | 24·238 | 19·740 | 16·573 | 14·251 |
| 90 | 27·279 | 24·267 | 19·752 | 16·579 | 14·253 |
| 95 | 27·483 | 24·398 | 19·806 | 16·601 | 14·263 |
| 100 | 27·655 | 24·505 | 19·848 | 16·617 | 14·269 |
| Perp. | 28·571 | 25·000 | 20·000 | 16·667 | 14·286 |

## To find "p" the "Present value" of an Annuity.

**Ex. 610.**—What is the "Present value" (*i.e.* what is required to purchase) an Annuity of £320 for 18 years at $3\frac{1}{2}$ per cent.?

$$\left(p = 320 \times \frac{1 - \frac{1}{1\cdot035^{18}}.}{r}\right)$$

First find what would be the value of £1, or $\dfrac{1 - \frac{1}{R^{18}}.}{\cdot035}$ $R^{18}$ is found as in Ex. 586, as follows: Log. R $\times$ 18 gives a number which is the *logarithm* of $R^{18}$. Log. R or 1·035 = ·01494.* This $\times$ 18 gives ·269, which the Slide Rule shows to be the *logarithm* of 1·858 = $R^t$. Then $\frac{1}{R^t} = \frac{1}{1\cdot858}$ = ·538 †; and 1 − ·538 = ·462; whence $\dfrac{1 - \frac{1}{R^t}}{r} = \dfrac{\cdot462}{\cdot035}$ = 13·2 for £1 (Table II. shows it to be 13·19); and 13·2 $\times$ 320 = £4220 the *answer*.

**Ex. 611.**—What sum ought to be given for the Lease of a house for 75 years, producing a net rental of £280 per annum, so as to obtain 5 per cent. on the outlay? Work by Slide Rule alone, supposing the Log. of 1·05 (page 259) to be written on the Rule.

First find $\dfrac{1 - \frac{1}{1\cdot05^{75}}.}{\cdot05}$ Log. 1·05 = ·0212. Then ·0212 $\times$ 75 = 1·59. Then 10 − 1·59 = 8·41 which is the Log. (as shown in page 253) of ·0257. Then 1·0000 — ·0257 = ·9743; and $\frac{\cdot9743}{\cdot05}$, on the

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* Those who use the Slide Rule for Annuities, should have the Logs. of 1·035, 1·04, &c., written on a blank part of the Rule. See page 259.

† It is not necessary to find the Nat. number of which ·269 is the Logarithm; for $\frac{1}{1\cdot858}$ is the same as Log. 1 — Log. 1·858 and we have Log. 1·858 already as ·269. Whence 1 — ·269 = ·731 which, as the Slide Rule shows, is the *logarithm* of ·538. Then 1 — ·538 as before.

Slide Rule, would read 19·5 ; and $19·5 \times 280 = £5460$. The *true* answer is £5,455 16*s*.

N.B. The "*number of years purchase*" is $\frac{5456}{280 \text{ years}} = 19·5$ years purchase. The learner may practise examples of "Present value" for different numbers of years for £1, comparing Slide Rule results with Table II.

**Ex. 612.**—What would be the "Present value" of the Annuity in Ex. 576, if paid *half-yearly?* (This is the same as an Annuity of half the amount, at half the rate of interest, for twice the number of years, or $p = \frac{1 - \frac{1}{1·0175^{36}}}{·0175}$).

Log. of 1·0175 = 0·0075 ; and ·0075 × 36 = ·260
Log. compl. of ·260 = 9·740 = ·550

$$\frac{·450 \times 160}{·0175} = £4110.$$

The *true* answer is £4109·7.

**Ex. 613.**—If I purchase furniture, and calculate that it will fetch at the end of 20 years what will just cover the expense of repairs and replacement during the 20 years, what *yearly interest* ought to be charged to the furniture, supposing the money paid for it had otherwise been invested in an Annuity for 20 years at 4 per cent.?

*Answer.*—At 4 per cent. an Annuity of £1, for 20 years, has a "present value" of £13·6. (See Table II.) Then $\frac{100}{13·6} = 7\frac{1}{3}$ per cent. *yearly* interest to be charged on the furniture. (Ex. 615 will show that £2000 will purchase £147 for 20 years, or £1000 will purchase £73·5, or for each £100, £7·35).

*Renewal of Leases.*

The fine to be required for renewing any number of years expired in a Lease, will be the "present value" of an Annuity *deferred* for

A A 2

the unexpired term of the Lease, and then to continue for the period renewed.

**Ex. 614.**—I have a 60 years' Lease, of which 50 have expired. What sum must I pay for renewing the Lease for 50 years more, supposing the net rental to be £90, and interest 5 per cent.?

£1 for 60 years at 5 per cent. Table II. = 18·93
£1 for 10 years, the unexpired term — 7·72

$$11{\cdot}21 \times 90 = £1009.$$

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## To find "a" the Annuity.

**Ex. 615.**—What Annuity for 20 years, can be purchased for £2000, reckoning interest at 4 per cent.? (Here $x = 2000 \div \left.\dfrac{1 - \dfrac{1}{1{\cdot}04^{20}}}{{\cdot}04}\right)$.

$$\text{Log. of } 1{\cdot}04 = {\cdot}017\text{; and } {\cdot}017 \times 20 = {\cdot}340$$
$$\text{Log. compl. of } {\cdot}340 = 9{\cdot}660 = {\cdot}4575$$
$$\overline{{\cdot}5425} = 1 - \frac{1}{1{\cdot}04^{20}}.$$

$$a = 2000 \div \frac{{\cdot}5425}{{\cdot}04} \quad \frac{A}{B} \quad \frac{£147}{{\cdot}04} \quad \text{———} \quad \frac{2000}{{\cdot}5425}$$

N.B. 1st. With such a "Table" as II. we find $\dfrac{1 - \dfrac{1}{1{\cdot}04^{20}}}{{\cdot}04} = 13{\cdot}59$; and $2000 \div 13{\cdot}59 = £147$.

N.B. 2d. It will be observed by Table I. that £2000 at 4 per cent. Compound interest, would in 20 years have amounted to $2000 \times 2{\cdot}191 = £4382$; and by Table III. an *Annuity* of £147 a year, *forborne* for 20 years, would amount to $147 \times 29{\cdot}78 = £4382$ also.

**Ex. 616.**—I am asked £5400 for the 70 years' *Lease* of a house. What should be the clear annual Rent, to enable me to make 5 per

cent. yearly interest on my outlay? (*i.e.* 5 per cent. for the 70 years.)

$$\left(\text{Here } x = 5400 \div \frac{1 - \frac{1}{1{\cdot}05^{70}}}{{\cdot}05}.\right)$$

$$\begin{array}{rl} \text{Log. of } 1{\cdot}05 = & {\cdot}0212 \\ & \times\ 70 \\ \hline & 1{\cdot}484 \end{array}$$

Log. compl. of $1{\cdot}484 = 8{\cdot}516 = \dfrac{{\cdot}0328}{{\cdot}9672}$. Then $\dfrac{5400 \times {\cdot}05}{{\cdot}9672} = £280$.

See N.B. 1st to previous Example. With a "Table" such as II. we find in the 5 per cent. column, of 70 years, the number 19·343, which is $\frac{{\cdot}0328}{{\cdot}9672}$, or as it is often called, the *number of years' purchase*. Then $5400 \div 19{\cdot}343 = £279{\cdot}2$.

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## To find "t" the Time in years.

**Ex. 617.**—A man builds a cottage for £460, and gets from it after paying all expenses and repairs, a clear rent of £27. Assuming interest at 4 per cent., in how many years will he have cleared his money? (Here $a = 27$. $r = {\cdot}04$. $p = 460$. $rp = 18{\cdot}04$.)

$$t = \frac{\log.\ a\ . - \log.\ (a - rp}{\log.\ \mathrm{R}} = \frac{\log.\ 27 - \log.\ (27 - 18{\cdot}4)}{\log.\ 1{\cdot}04}$$

$$= \frac{\log.\ 27 - \log.\ 8{\cdot}6}{\log.\ 1{\cdot}04} = \frac{1{\cdot}431 - {\cdot}934}{{\cdot}017} = \frac{{\cdot}497}{{\cdot}017} = 29{\cdot}2 \text{ years.}$$

N.B. If we have a "Table" such as II., look for 17 $\left(i.e.\ \frac{460}{27}\right)$ in the 4 per cent. column, and it will be found alongside of 29·2 years. (If the net rent were £18·4, and interest 4 per cent., he would never clear his money, for $460 \times {\cdot}04 = 18{\cdot}4$, which is a *perpetuity*. At 5 per cent. it would be 39 years (the rent £27 remaining the same); that is, at 17 years purchase, 4 per cent. would be repaid in 29·2 years, but at 5 per cent. it would take 39 years.

### To find "r" the Interest of £1 Annuity for 1 year.

This is only to be found by reference to a "Table" such as II.

**Ex. 618.**—An annuity of £120 for 18 years was purchased for £1403. What was the rate of interest calculated at?

Here $r = \frac{1403}{120} = 11 \cdot 7$, which in Table (II), and with 18 years, we find to be in the *five per cent.* column.

*Perpetuities.*

An Annuity to continue *for ever,* or the annual value of a *Freehold,* is called a "Perpetuity."

1st. The "annuity" or yearly rent = Present value $\times$ $r$.

2d. The "present value" = Annuity or yearly rent $\div$ $r$.

3d. The rate per cent. or $r$ = Annuity $\div$ present value.

**Ex. 619.**—What is the yearly income of a Freehold bought for £4000, assuming interest at $3\frac{1}{2}$ per cent.? (Here $a = 4000 \times \cdot 035 =$ £140.

**Ex. 620.**—What should be given for a Freehold of £250 a year, to make 5 per cent. interest? (Here $p = \frac{250}{\cdot 05} =$ £5000.)

**Ex. 621.**—A Freehold of £135 a year, is bought for £3000. What interest does the purchaser get for his money? (Here $r = \frac{135}{5000} = \cdot 045$, or $4\frac{1}{2}$ per cent.)

N.B. 20 years' purchase is 5 per cent.; $16\frac{2}{3}$ years' purchase is 6 per cent.; and so on, as the Slide *inverted* (page 25) will show.

Annuities "*Forborne.*"

$r$ = Interest of £1 for 1 year: then at 5 per cent., $r$ = ·05.

R = 1 + $r$, or the amount of £1 with interest, for 1 year.

$a$ = Annuity. $t$ = time in years.

$m$ = Amount to which a "forborne" Annuity has accumulated. $\Big\} = a \times \dfrac{R^t - 1}{r}$.

$$a = m \div \frac{R^t - 1}{r}.$$

$$t = \left(\text{Log. } 1 + \frac{mr}{a}\right) \div \text{Log. R.}$$

Persons using these Logarithmic calculations should have the Logs. in page 259 inscribed on their Rules (see footnote, page 265).

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## To find "m" the accumulated amount.

**Ex. 622.**—What will an annuity of £300, forborne for 23 years, have amounted to, reckoning 5 per cent.? (Here $m = 300 \times \dfrac{1{\cdot}05^{23} - 1}{\cdot 05}$.)

$$\begin{array}{rl} \text{Log. } 1{\cdot}05 = & 0{\cdot}0212 \\ & \times\ 23 \\ \hline \text{Log. of } 3{\cdot}07 = & 0{\cdot}4875 \\ -\ 1{\cdot} & \end{array}$$

$$\frac{2{\cdot}07 \times 300}{\cdot 05} = \frac{A}{B} \quad \frac{2{\cdot}07}{\cdot 05} \quad \frac{\pounds 12420}{300}$$

N.B. Table III. is $\dfrac{R^t - 1}{r}$ for £1, at various rates of Interest, and gives $\dfrac{2{\cdot}07}{\cdot 05}$ = 41·43. Whence the *true* answer is 41·43 × 300 = £12429. In like manner an Annuity of £1 forborne for 5 years at $2\frac{1}{2}$ per cent. would accumulate to $\dfrac{\cdot 45}{\cdot 025}$ = £18.

## To find "a" the Annuity that has been forborne.

**Ex. 623.**—A person wishes to provide £350 to be ready for use 10 years hence. What sum must he lay by annually, allowing 4 per cent. interest? $\left(\text{Here } a = 350 \div \frac{1{\cdot}04^{10} - 1}{{\cdot}04}.\right)$

$1{\cdot}04^{10}$ is found by the line N, as in Ex. 586, to be 1·48; and $\frac{0{\cdot}48}{350 \times {\cdot}04} = £29{\cdot}1$.

N.B. With a "Table" such as III. we find in the 4 per cent. column for 10 years, the number 12. Then 350 ÷ 12 = 29·1

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## To find "t" the Time in years.

**Ex. 624.**—How many years' forbearance of an Annuity of £60 will produce £500? interest $3\frac{1}{2}$ per cent.

$$t = \frac{\log.\left(\frac{500 \times {\cdot}035}{60}\right)}{\log.\ 1{\cdot}035} = \frac{\log.\ 1 + {\cdot}2917}{\log.\ 1{\cdot}035} = \frac{{\cdot}111}{{\cdot}015} = 7{\cdot}4 \text{ years.}$$

N.B. With a "Table" such as III. we find that $\frac{500}{60}$, or 8·33, corresponds in the $3\frac{1}{2}$ per cent. column, with about 7·4 years.

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## TABLE III. $= \dfrac{R^t - 1}{r}$.

### Amount of a "forborne" Annuity.

| Years. | 3 per cent. | 3½ per cent. | 4 per cent. | 4½ per cent. | 5 per cent. |
|---|---|---|---|---|---|
| 1 | 1·0000 | 1·0000 | 1·0000 | 1·0000 | 1·0000 |
| 2 | 2·0300 | 2·0350 | 2·0400 | 2·0450 | 2·0500 |
| 3 | 3·0909 | 3·1062 | 3·1216 | 3·1370 | 3·1525 |
| 4 | 4·1836 | 4·2149 | 4·2465 | 4·2782 | 4·3101 |
| 5 | 5·3091 | 5·3625 | 5·4163 | 5·4707 | 5·5256 |
| 6 | 6·4684 | 6·5501 | 6·6330 | 6·7169 | 6·8019 |
| 7 | 7·6625 | 7·7794 | 7·8983 | 8·0191 | 8·1420 |
| 8 | 8·8923 | 9·0517 | 9·2142 | 9·3800 | 9·5491 |
| 9 | 10·159 | 10·368 | 10·583 | 10·802 | 11·026 |
| 10 | 11·464 | 11·731 | 12·006 | 12·288 | 12·578 |
| 11 | 12·808 | 13·142 | 13·486 | 13·841 | 14·207 |
| 12 | 14·192 | 14·602 | 15·026 | 15·464 | 15·917 |
| 13 | 15·618 | 16·113 | 16·627 | 17·156 | 17·713 |
| 14 | 17·086 | 17·677 | 18·292 | 18·932 | 19·599 |
| 15 | 18·599 | 19·296 | 20·023 | 20·784 | 21·579 |
| 16 | 20·157 | 20·971 | 21·824 | 22·719 | 23·657 |
| 17 | 21·761 | 22·705 | 23·697 | 24·742 | 25·840 |
| 18 | 23·414 | 24·500 | 25·645 | 26·855 | 28·132 |
| 19 | 25·117 | 26·357 | 27·671 | 29·063 | 30·539 |
| 20 | 26·870 | 28·280 | 29·778 | 31·371 | 33·066 |
| 21 | 28·676 | 30·269 | 31·969 | 33·783 | 35·719 |
| 22 | 30·537 | 32·329 | 34·247 | 36·303 | 38·505 |
| 23 | 32·453 | 34·460 | 36·618 | 38·937 | 41·430 |
| 24 | 34·426 | 36·666 | 39·083 | 41·689 | 44·502 |
| 25 | 36·459 | 38·950 | 41·646 | 44·565 | 47·727 |
| 26 | 38·553 | 41·313 | 44·312 | 47·571 | 51·113 |
| 27 | 40·710 | 43·759 | 47·084 | 50·711 | 54·669 |
| 28 | 42·931 | 46·291 | 49·967 | 53·993 | 58·403 |
| 29 | 45·219 | 48·910 | 52·966 | 57·423 | 62·323 |
| 30 | 47·575 | 51·623 | 56·085 | 61·007 | 66·439 |
| 31 | 50·003 | 54·429 | 59·328 | 64·752 | 70·761 |
| 32 | 52·503 | 57·334 | 62·701 | 68·666 | 75·299 |
| 33 | 55·078 | 60·341 | 66·209 | 72·756 | 80·064 |
| 34 | 57·730 | 63·453 | 69·858 | 77·030 | 85·067 |
| 35 | 60·462 | 66·674 | 73·652 | 81·497 | 90·320 |
| 36 | 63·276 | 70·008 | 77·598 | 86·163 | 95·836 |
| 37 | 66·174 | 73·458 | 81·702 | 91·041 | 101·63 |
| 38 | 69·159 | 77·028 | 85·970 | 96·138 | 107·71 |
| 39 | 72·234 | 80·725 | 90·409 | 101·46 | 114·09 |
| 40 | 75·401 | 84·550 | 95·025 | 107·03 | 120·80 |
| 41 | 78·663 | 88·509 | 99·826 | 112·85 | 127·84 |
| 42 | 82·023 | 92·607 | 104·82 | 118·92 | 135·23 |
| 43 | 85·484 | 96·849 | 110·01 | 125·28 | 142·99 |
| 44 | 89·048 | 101·24 | 115·41 | 131·91 | 151·14 |
| 45 | 92·720 | 105·78 | 121·03 | 138·85 | 159·70 |
| 46 | 96·501 | 110·48 | 126·87 | 146·10 | 168·68 |
| 47 | 100·40 | 115·35 | 132·94 | 153·67 | 178·12 |
| 48 | 104·41 | 120·39 | 139·26 | 161·59 | 188·02 |
| 49 | 108·54 | 125·60 | 145·83 | 169·86 | 198·43 |
| 50 | 112·80 | 131·00 | 152·67 | 178·50 | 209·35 |
| 60 | 163·05 | 196·52 | 237·99 | 289·50 | 353·58 |
| 70 | 230·59 | 288·94 | 364·29 | 461·87 | 588·53 |
| 80 | 321·36 | 419·31 | 551·24 | 729·56 | 791·23 |
| 90 | 443·35 | 603·20 | 827·98 | 1145·3 | 1594·6 |
| 100 | 607·29 | 862·61 | 1237·6 | 1790·8 | 2610·0 |

## LIFE ANNUITIES.

The previous pages refer to "Annuities *certain*" for a fixed number of years; but a "*Life* annuity" is not calculated as an Annuity for a certain number of years, but on the *expectation of life* as shown in Table IV.

The present value of an Annuity *certain*, for a term of years equal to the "expectation of life," is *greater* than the value of the same Annuity to cease on failure of that person's existence; because in a number of Life Annuities many of the payments would not be made till a much more remote period than the term noted as *expectation*. Thus, at the age of 46, the "expectation of life" by the Carlisle Tables would be 24 years; and at 3½ per cent. interest, the value of a *Life* Annuity of £1 is £14·7; whereas by Table II. the present value of an Annuity of £1 for 24 years *certain*, at 3½ per cent., is £16.

Table IV. supposes the Annuity paid at the end of each year. If paid *half-yearly*, add £·25, and if *quarterly*, add £·375, to the sums in the Table.*

If payable *in advance*, add to the Tabular values £1. If payable half-yearly in advance, add £·75. If quarterly in advance, add ·625.

---

* Thus, if the "Present value" of a Life annuity for £1, for 20 years, at 4 per cent. is £13·965, if paid *annually*, the same would be worth £14·44 if paid *quarterly*.

# TABLE IV.

## LIFE ANNUITIES.

| Age. | Expectation. | | Value of £1 Annuity by Carlisle rate. | |
|---|---|---|---|---|
| | Carlisle. | Males.* | $3\frac{1}{2}$ per cent. | 4 per cent. |
| 12 | 47·27 | 44·07 | 21·09 | 19.33 |
| 14 | 45·75 | 42·53 | 20·79 | 19·08 |
| 16 | 44·27 | 41·01 | 20·49 | 18·84 |
| 18 | 42·87 | 39·61 | 20·21 | 18·61 |
| 20 | 41·46 | 38·39 | 19·91 | 17·3 |
| 22 | 40·04 | 37·34 | 19·59 | 17·1 |
| 24 | 38·59 | 36·39 | 19·24 | 17 |
| 26 | 37·14 | 35·41 | 18·87 | 16·8 |
| 28 | 35·69 | 34·31 | 18·48 | 16·7 |
| 30 | 34·34 | 33·17 | 18·12 | 16·4 |
| 32 | 33·03 | 32·00 | 17·77 | 16·2 |
| 34 | 31·68 | 30·79 | 17·38 | 15·9 |
| 36 | 30·32 | 29·54 | 16·95 | 15·6 |
| 38 | 28·96 | 28·28 | 16·51 | 15·2 |
| 40 | 27·61 | 27·02 | 16·05 | 14·9 |
| 42 | 26·34 | 25·74 | 15·62 | 14·5 |
| 44 | 25·09 | 24·42 | 15·17 | 14 |
| 46 | 23·82 | 23·07 | 14·70 | 13·5 |
| 48 | 22·50 | 21·68 | 14·17 | 13 |
| 50 | 21·11 | 20·30 | 13·55 | 12·4 |
| 52 | 19·68 | 18·97 | 12·88 | 11·8 |
| 54 | 18·28 | 17·73 | 12·19 | 11·3 |
| 56 | 16·89 | 16·57 | 11·47 | 10·8 |
| 58 | 15·55 | 15·47 | 10·73 | 10·2 |
| 60 | 14·34 | 14·39 | 10·06 | 9·7 |
| 62 | 13·31 | 13·28 | 9·49 | 9·1 |
| 64 | 12·30 | 12·17 | 8·91 | 8·5 |
| 66 | 11·27 | 11·10 | 8·28 | 7·9 |
| 68 | 10·23 | 10·14 | 7·62 | 7·3 |
| 70 | 9·18 | 9·22 | 6·91 | 6·8 |
| 72 | 8·16 | 8·37 | 6·19 | 6·2 |
| 74 | 7·33 | 7·54 | 5·60 | 5·7 |
| 76 | 6·69 | 6·69 | 5·15 | 5·1 |
| 78 | 6·12 | 5·78 | 4·73 | 4·4 |
| 80 | 5·51 | 4·94 | 4·27 | 3·8 |

The "Northampton Tables," at age 22, give 32·4 years "expectation;" at 40 they give 23; at 50 they give 18; at 64 they give 11¾; so that the value of an Annuity is from £¾ to ¼ a £ less.

The value of Annuities by the Government Tables is 6 or 7 shillings less than the Carlisle.

* In the lives of "females" about 4 years longer, till 54. Then about 2 years.

B B

## APPENDIX C.

### WHERE TO PLACE THE DECIMAL POINT IN DECIMAL DIVISION.

In using the Slide Rule for Division of *Decimals*, it is very useful to know *beforehand* where to place the Decimal point in the Quotient. It is true this may be seen from a careful inspection of the run of the figures on the Slide, as in examples *a*, *b*, *c*, &c. page 9: still, the following rules will be easily remembered and found useful.

### DECIMAL DIVISION.

CASE I. $\frac{162{\cdot}54}{18}$, $\frac{25517}{422{\cdot}5}$, $\frac{57075{\cdot}8}{43{\cdot}56}$. The number of *integral* figures in the Quotient will be known beforehand by the rule at the lower part of p. 22 (the decimals being disregarded).

CASE II. $\frac{5}{16}$, $\frac{1}{625}$. *Rule.* Multiply the Numerator by as many tens as will make it divisible by the Denominator. If adding *one* cipher (*i.e.* multiplying by ten) is enough, the Quotient will commence with a single dot. If two ciphers are required (*i.e.* multiplying by 100) then the Quotient begins with ·0. If three ciphers, the Quotient begins with ·00. (See Examples next page.)

CASE III. (*a*) $\frac{{\cdot}576}{{\cdot}48}$, $\frac{441}{{\cdot}09}$, $\frac{3028{\cdot}5}{{\cdot}673}$. First *equate the fraction: i.e.* multiply *both* Numerator and Denominator by as many tens as there are decimals in the number (either Numerator or Denominator) that has most decimals, so as to make both of them *whole numbers.* The result is a Fraction coming under CASE I.; as shown in the Examples next page.

CASE III. (*b*) $\frac{{\cdot}213}{{\cdot}284}$, $\frac{13}{20{\cdot}8}$, $\frac{16{\cdot}254}{18}$, $\frac{{\cdot}864}{36}$, $\frac{3{\cdot}9677}{721{\cdot}4}$, $\frac{{\cdot}04147}{71{\cdot}5}$. *Equate the fraction,* as in CASE III. (*a*). The result will be a fraction coming under CASE II.; as shown in the Examples next page.

*The following gives an Example of every case that can occur:*

*m* means a Mixed number; *w*, a Whole number; *d*, a Decimal. Where + is prefixed to the Denominator, it means that it is greater (— that it is less) than the Numerator.

## Case I.

$\frac{m}{w\,-}\ \frac{162{\cdot}54}{18}$

$\frac{w}{m\,-}\ \frac{25517}{422{\cdot}5}$

$\frac{m}{m\,-}\ \frac{57075{\cdot}8}{43{\cdot}56}$

The number of *integral* figures in the Quotient is known beforehand; being just the same in number, as if we had $\frac{162}{18}$, $\frac{25517}{422}$, $\frac{57075}{43}$. The answers are, 9·03, 60·4, 1310·28. (See lower part of p. 22.)

## Case II.

$\frac{w}{w\,+}\ \frac{5}{16}$. Mult. Numerator by 10. $= \frac{5(0)}{16} =$ ·3125.

$\frac{w}{w\,+}\ \frac{1}{625}$. Mult. Numerator by 1000. $= \frac{1(000)}{625} =$ ·0016.

## Case III. (*a*).

$\frac{d}{d\,-}\ \frac{{\cdot}576}{{\cdot}48}$. Mult. by 1000. $= \frac{576}{480} = 1{\cdot}2$.

$\frac{w}{d}\ \frac{441}{{\cdot}09}$. Mult. by 100. $= \frac{44100}{9} = 4900$.

$\frac{m}{d}\ \frac{3028{\cdot}5}{{\cdot}673}$. Mult. by 1000. $= \frac{3082500}{673} = 4500$.

## Case III. (*b*).

$\frac{d}{d\,+}\ \frac{{\cdot}213}{{\cdot}284}$. Mult. by 1000. $= \frac{213}{284} = \frac{213(0)}{284} =$ ·75.

$\frac{w}{m\,+}\ \frac{13}{20{\cdot}8}$. Mult. by 10. $= \frac{130}{208} = \frac{130(0)}{208} =$ ·625.

$\frac{m}{w\,+}\ \frac{16{\cdot}254}{18}$. Mult. by 1000. $= \frac{16254}{18000} = \frac{16254(0)}{18000} =$ ·903.

$\frac{d}{w}\ \frac{{\cdot}864}{36}$. Mult. by 1000. $= \frac{864}{36000} = \frac{864(00)}{36000} =$ ·024.

$\frac{m}{m\,+}\ \frac{3{\cdot}9677}{721{\cdot}4}$. Mult. by 10000. $= \frac{39677}{7214000} = \frac{39677(000)}{7214000} =$ ·0055.

$\frac{d}{m}\ \frac{{\cdot}04147}{71{\cdot}5}$. Mult. by 100,000. $= \frac{4147}{7150000} = \frac{4147(0000)}{7150000} =$ ·00058.

# APPENDIX D.

## 'DUODECIMALS.

So called because each quantity, whether linear, square, or cubic, is $\frac{1}{12}$ of that which precedes it. Also called "Cross multiplication," because we begin with the left hand multiplier.

The Linear inch is divided into 12ths,* called *seconds* (″); each second into 12ths called *thirds* (‴); and each third into 12ths called *fourths* (⁗).

---

### Square Measure.

Feet × feet, make Square feet, written *Sq. ft.*

Feet × inches, make Primes, written ′.

Inches × inches, } make Square inches, written ″.
Feet × twelfths, }

Inches × twelfths, make Thirds, written ‴.

Twelfths × twelfths, make Fourths, written ⁗.

N.B. *Remember* that primes ′ (often miscalled *inches*) are 12ths of a Square foot; and Square inches ″ (sometimes called *parts*) are 12ths of these, or 144ths of a Square foot. Thus, if we have a surface 4 feet long, and 8 inches wide, its area = $4 \times \frac{2}{3}$ Square feet = $2\frac{2}{3}$ Square feet: which is the same $4 \times 8 = 32$ *primes*, or $\frac{32}{12}$ths of a Square foot.

The chief advantage of Duodecimals is, that when prices are given (generally in *s. d.* per Square foot), the value is easily calculated by "Practice," as shown in Ex. 628.

**Ex. 625.**—Multiply 7 feet 9 inches, by 4 feet 6 inches.

---

* Carpenters' Rules in *practice* do not have 12ths of an inch, but *eighths.* Each eighth = $1\frac{1}{2}$ twelfths. Hence $\frac{1}{4}$ inch = 3 twelfths; $\frac{3}{8}$ inch = $4\frac{1}{2}$ twelfths, &c.; but the odd $\frac{1}{2}$ twelfth ($\frac{1}{24}$ inch) may be disregarded. 8 feet $7\frac{3}{4}$ inch = 8 feet 7′ 9″.

| ft. | in. | |
|---|---|---|
| 7 | 9 | |
| 4 | 6 | |
| 31 | 0 | |
| 3 | 10 | 6 |
| 34 | 10 | 6 |

First multiply 7 ft. 9 in. by 4; which makes 31 feet. Then multiply 7 inches 9 *twelfths* by 6; which makes 3 ft. 10 in. 6 twelfths. Total = 34 Square feet,—10 inches,—6 parts. Or $34\frac{126}{144}$ Square feet.

N.B. In using the Slide Rule, we should reduce the inches to decimals of a foot, as in page 86, and then 7·75 × 4·5 = 34·875 Square feet. (Decimals of 12ths should be learnt by heart.)

**Ex. 626.**—

| ft. | | | | | | | Sq. ft. | Sq. in. |
|---|---|---|---|---|---|---|---|---|
| 7 | 5′ | 9″ | | | 25 | | 25 | |
| 3 | 5 | 3 | | | 8′ | . . | | 96 |
| 22 | 5 | 3 | | | 6″ | . . | | 6 |
| 3 | 1 | 4 | 9‴ | | 2‴ | . . | | $0\frac{2}{12}$ |
| | 1 | 10 | 5 | 3⁗ | 3⁗ | . . | | $0\frac{3}{144}$ |
| 25 | 1′ | 6″ | 2‴ | 3⁗ | | Ans. | 25 | $102\frac{27}{144}$ |

Thus 14 ft. 3 in. 6 twelfths × 5 ft. 4 in. = 76 2′ 8″.
5 ft. $4\frac{1}{4}$ in. × 6 ft. $8\frac{1}{2}$ in. = 35 11′ 0″ 1‴ 6⁗, or by the Slide Rule, 5·354 × 8·708 = 35·9 Square feet.

**Ex. 627.**—What is the Square surface of a gilt plate 3ft. $5\frac{3}{8}$ in. by 2 ft. $2\frac{7}{8}$ in.?

With reference to N.B. page 280, this would be 2 5′ $4\frac{1}{2}$″ × 2 2′ $10\frac{1}{2}$″, but omitting the $\frac{1}{2}$ twelfths of inches (or rather omitting one and increasing the other), it would be as follows:

| | | | | |
|---|---|---|---|---|
| 2 | 5′ | 4″ | | |
| 2 | 2 | 11 | | |
| 4 | 10 | 8 | | |
| | 4 | 10 | 8‴ | |
| | 2 | 4 | 10 | 8⁗ |
| 5 | 5′ | 11″ | 6‴ | 8⁗ |

Thus = 5 Sq. ft. $71\frac{80}{144}$ Sq. in. = $791\frac{80}{144}$ Sq. in. If worked accurately by fractions, it would be $29\frac{3}{8}$ in. × $26\frac{7}{8}$ in. = $789\frac{80}{144}$ Square inches.

**Ex. 628.**—At 3*s*. 10*d*. per Square foot, what would be the price of 34 Sq. ft. 10′ 6′?

| | s. | d. |
|---|---|---|
| | 3 | 10 |
| | | × 4 |
| | 15 | 4 |
| | | × 8 |
| | 124 | 8 |
| 2 = | 7 | 8 |
| | 131 | 4 |
| 6′ = ½ Sq. ft. | 1 | 11 |
| 3′ = ½ of 6′ | | 11½ |
| 1½ = ½ of 3′ | | 5¾ |
| | 133 | 8¼ |
| = £7 | 13*s.* | 8¼*d.* |

1 shilling per Square foot is 1*d.* per prime (′), and $\frac{1}{12}$*d.* per ″ Square inch (″).

3*s.* per Square foot is 3*d.* per prime (′) and ¼*d.* per Square inch (″).

Sometimes both measurements are reduced *to inches*, when using the Slide Rule. Thus in Ex. 625, 93 × 54 = 5022 Square inches; and $\frac{5022}{144}$ = 34·875 Square feet, or 34 Square feet, 126 Square inches.

---

## Cubic Measure.

Here ‴ represent *Cubic* inches, and ⁗ are twelfths of Cubic inches.

**Ex. 629.**—What is the Cubic content of a block of steel, measuring 2 feet 9 inches, by 2 feet 3 inches, by 1 foot 6 inches?

| | ft. | in. | | |
|---|---|---|---|---|
| | 2 | 9 | | |
| × | 2 | 3 | | |
| | 5 | 6 | | |
| | | 8 | 3″ | |
| | 6 | 2 | 3 | |
| × | 1 | 6 | 3″ | |
| | 6 | 2 | 3″ | |
| | 3 | 1 | 1″ | 6‴ |
| | 9 | 3′ | 4″ | 6‴ |

| | Cub. ft. | Cub. in. |
|---|---|---|
| 9 Cubic feet . . | 9 | |
| 3′ = 3 × 144 . | | 432 |
| 4″ = 4 × 12 . | | 48 |
| 6‴ . . . . . . | | 6 |
| | 9 C. F. + | 486 Cub. in. |

By Slide Rule, $\frac{33 \times 27 \times 18}{1728} = \frac{891 \times 18}{1728}$ = 9·3 Cub. ft. The true answer being 9·28125.

# APPENDIX E.

## NOTES ON CYLINDERS.

The figure of a Cylinder is probably more commonly met with practically in "Solid Mensuration," than any other figure; and a few notes may be useful.

In page 166, the method of computing contents has been given. The Formula $S = \frac{d^2 \times l}{1{\cdot}2732}$, is derived from the usual Formula $S = d^2 \times {\cdot}7854 \times l$, by using the reciprocal of $l$ as a *divisor*. So $S = \frac{c^2 \times l}{12{\cdot}566}$ is derived from $S = c^2 \times {\cdot}0796 \times l$.

A *Cylindrical inch* is a cylinder 1 inch deep, and 1 inch diameter. It is less than a *cubic inch*, the proportion being 1 Cylindrical to 1·2732 Cubic, or 11 : 14 as shown in Formula IV., page 148. So if a Bushel contains 2218·2 Cubic inches, it contains 2824·3 Cylindrical inches, or 2218·2 × 1·2732. So if the diameter in inches of any cylinder is given, the Square of it multiplied by the length in inches, gives the content in *Cylindrical* inches.

In the TABLE page 196, under PARALLELOPIPEDS I. I. I., are the *cubic* inches in a Bushel, Gallon, &c.; and under CYLINDER I. I. are the Cylindrical inches.

Continuing the "proportion," we find that a Cubic foot contains $1728 \times 1{\cdot}2732$, or $\frac{1728}{{\cdot}7854}$ Cylindrical inches; and a Cylindrical foot contains 1728 × ·7854, or 1357·2 Cubic inches. The number $1{\cdot}1284 = \sqrt{1{\cdot}2732}$ represents the number of inches diameter of a Circle whose area is one Square inch.

**Ex. 630.**—Required the diameter of a Cylinder 4 inches deep, that shall contain as much as a Cube of 4 inches diameter. It is not necessary to find the content of the Cube, the answer being simply 4 × 1·1284 = 4·52 inches diameter. To prove it we may take 64 (the cube of 4) and use the Formula (*a*) in page 167.

| | | |
|---|---|---|
| C | 40 (= 4 × 10) | 64 Cubic inches |
| D | 3·57 (constant) | 4·52 inches diameter |

**Ex. 631.**—Required the length of a Cylinder $4\frac{3}{4}$ inches diameter, equal to a Sphere of the same diameter. *Answer* $\frac{2}{3}$ of $4\frac{3}{4}$, or 3·167 inches.

To *double* the capacity of a Cylinder, preserving the "proportion" of length to diameter, multiply both by 1·26, or $\sqrt[3]{2}$. To *quadruple* multiply both by 1·5875, or $\sqrt[3]{4}$; and so on. Hence

**Ex. 632.**—Suppose a Cylindrical disc, $\frac{1}{4}$ inch deep, and 1·128 inches diameter; its content is $\frac{1}{4}$ Cubic inch. It is required to make another containing 320 times as much material, but the same proportion of depth to diameter. What will [I.] its depth be? and what [II.] its diameter? Here depth = $\cdot25 \times \sqrt[3]{320}$, and diameter = $1\cdot128 \times \sqrt[3]{320}$. Both these are readily solved by the Slide Rule having a Cube line of E, as in Example omitted in page 118.

| | | | |
|---|---|---|---|
| [I.] | E | 1 | 320 |
| | D | ·25 | 1·71 depth |
| [II.] | E | 1 | 320 |
| | D | 1·128 | 7·73 diam. |

To find what is the right proportion between the diameter and depth of a cylindrical vessel when the least quantity of metal is to be used.

[I.] When both ends are closed, diameter equal to height.

[II.] When one end is open, diameter double the height. In the *former* case if $s$ Cubic content, $d = \sqrt[3]{s \times 1\cdot2732}$, or in a form adapted to the Slide Rule (Ex. 314), $d = \sqrt[3]{\frac{s}{\cdot7854}}$. In the *latter* case $d = \sqrt[3]{s \times 2\cdot5464}$, or in a form adapted to the Slide Rule, Ex. 314, $d = \sqrt[3]{\frac{s}{\cdot7854 \times \cdot5}}$.

**Ex. 633.**—Required the diameter and depth of a Cylinder whose diameter and depth are *equal*, to contain 124 Cubic feet. $d = \sqrt[3]{\frac{124}{\cdot7854}}$.

| | | |
|---|---|---|
| E | ·7854 | 124 |
| D | 1 | 5·41 |

Formula (*a*) page 167, will show that a Cylinder whose diameter is 5·41 feet and depth 5·41 feet will contain 124 Cubic feet.

N.B. Whatever be the "proportion" between the diameter and depth of a Cylinder, their measure may be found when *s* the Cubic content is known, as follows :

If $d : h :: 2 : 1$. Then $d = \sqrt{\frac{s}{\cdot 7854 \times \frac{1}{2}}}$, or $\sqrt[3]{\frac{s}{\cdot 3927}}$.

If $d : h :: 1 : 2$. Then $d = \sqrt[3]{\frac{s}{\cdot 7854 \times \frac{2}{1}}}$, or $\sqrt[3]{\frac{s}{1 \cdot 5078}}$.

If $d : h :: 3 : 4$. Then $d = \sqrt[3]{\cdot 7854 \times \frac{4}{3}}$, or $\sqrt[3]{\frac{s}{1 \cdot 0472}}$.

**Ex. 634.**—In the margin are the Cubic inches in a ½ Pint, a Pint, a Quart, a Gallon, and a Bushel. It is required to find the diameters and heights of 5 cylindrical vessels to contain these respectively, but to be so made that the diameter of each shall be to its depth, as 3 to 4. (*s* = content.)

| Cubic inches. |
|---|
| 17·330 = ½ pt. |
| 34·659 = Pt. |
| 69·318 = Qt. |
| 277·274 = Gall. |
| 2218·2 = Bus. |

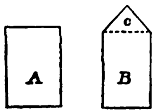
| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1st. | E | 1·0472 | 17·330 | 34·659 | 69·318 | 277·274 | 2218·2 |
| | D | 1 | 2·55 | 3·21 | 4·04 | 6·42 | 12·84 diameters |
| 2d. | A | 3 | 2·55 | 3·21 | 4·04 | 6·42 | 12·84 diams. |
| | B | 4 | 3·40 | 4·29 | 5·39 | 8·57 | 17·12 depths |
| | | | ½ pt. | Pt. | Qt. | Gall. | Bus. |

So that the entire columns of diameters and depths can be filled up in a couple of minutes by the aid of the Slide Rule, and to as much exactness as it is possible to make up the Cylinders. The process by pen and ink would be a tedious one.

A Quart, in the proportion of diameter : depth :: 3 : 4 is as shown above 4·04 : 5·39. If it had been 2 : 3, then 3·89 to 5·82. If as 2 : 1, it would be 3·53 to 7·06, &c. (See also Ex. 430.)

**Ex. 635.**—It is required to make a Cylindrical vessel, which

when *heaped* to a right-angled cone, will contain as much as the "standard" vessel *struck*. The height of the new measure to be the same as the "standard," but the *diameter of course reduced.*




Let A be a "standard" measure 3 inches high,* with a content of 11·72 cubic inches. What must be the diameter in inches of the new vessel B, also 3 inches high, so that it, together with $c$ the right-angled cone, on it, may also contain 11·72 cubic inches?

Let $x$ = the required diameter of B.

Then (p. 166) Content of B $= x^2 \times \cdot 7854 \times 3 = x^2 \times 2{\cdot}3562$.

And (p. 179) Content of $c = \cdot 1309\, x^3$.

Whence B + $c = 2{\cdot}3562\, x^2 + \cdot 1309\, x^3 = 11{\cdot}72$ cubic inches.

Now to find the *exact* value of $x$ from this Equation, is not easy; but the Slide Rule may be usefully employed† in "Trial and Error." A few preliminary trials gave $x$ about $2\frac{1}{10}$ inches, and if we assume $x = 2{\cdot}11$, it gives B = 10·5, and then E will be 1·23; as follows:—

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| C | 2·3562 | 10·2 | 10·3 | 10·4 | $\boxed{10{\cdot}5}$ | 10·6 | 10·7 | 10·8 = B |
| D | 1 | 20·8 | 2·09 | 2·10 | $\boxed{2{\cdot}11}$ | 2·12 | 2·13 | 2·14 = $x$ |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| E | ·1309 | 1·18 | 1·19 | 1·21 | $\boxed{1{\cdot}23}$ | 1·25 | 1·26 | 1·28 = $c$ |
| D | 1 | 20·8 | 2·09 | 2·10 | $\boxed{2{\cdot}11}$ | 2·12 | 2·13 | 2·14 = $x$ |

If we take 2·11 for $x$.

| | | Cub. in. | True. |
|---|---|---|---|
| Then | B = | 10·50 | 10·490 |
| | $c$ = | 1·23 | 1·229 |
| | B + $c$ = | 11·73 | 11·719 |

* In the question, it is not necessary to give the *diameter* of A. Still, as a guide to the diameter of B, (which of course will be *less*,) it may easily be found, as in line 5, p. 167; or $d = \dfrac{\sqrt{11{\cdot}72} \times 1{\cdot}128}{\sqrt{3}}$, solved as in Ex. 257, and found = 2·23. See also Ex. 429.

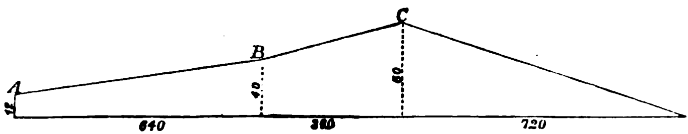
† The above case occurred in practice, and *was* solved by the Slide Rule only.

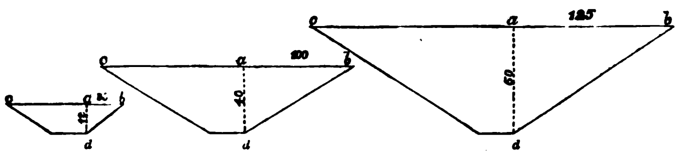
# APPENDIX F.

## CUTTINGS AND EMBANKMENTS.

In practice, it is usual to compute the content of the whole Cutting or Embankment, by one Formula, where the ratio of slope (*i.e.*, *ab* to *ad* in the figure) is known, and is the same throughout, and the perpendiculars taken at equal distances. This is shown in N.B. 2d, page 288, and in the subsequent Examples.

*But if the ratio of slope is not given*, but simply the widths and depths, and the perpendiculars *not* taken at equal distances, the cubic content must be taken for *each section*, as in the following Example.




**Ex. 636.**—Let the upper figure represent the *longitudinal* section of a Railway Cutting or Embankment, and the three smaller figures the *cross* sections at A, B, and C. The width of the Roadway being 30 feet through the whole length 1720 feet.

Taking the *first* section, and computing its Cubic content as in Ex. 470, we have for one *end*, a Trapezoid whose depth is 12 feet, and its upper and lower breadths 90, and 30 feet ; or a *mean breadth* = 60 ft. The other end is a Trapezoid whose depth is 40 feet, and its upper and lower breadths 230, and 30 feet; or a mean breadth = 130 feet. Total length 640 feet.

$$\left.\begin{array}{r} 60 \times 12 = 720 \\ 130 \times 40 = 5200 \\ \hline 190 \times 52 = 9880 \\ \hline 15800 \end{array}\right\} \text{whence } \frac{15800 \times 640}{6} = 1{,}685{,}333 \text{ Cubic feet.}$$

Or if we use the Cubic *yard* divisor 162 (N.B. page 190), we have $\frac{15800 \times 640}{162} = 62420$ Cubic *yards*.

The *second* section may be found in like manner to be $\frac{38600 \times 360}{6}$ $= 2316000$ Cubic feet, or $\frac{38600 \times 360}{162} = 85778$ Cubic yards. The *third* section is solved in Ex. 471, and shown to be 2040000 Cubic feet, or 75555 Cubic yards. Total 6,041,333 Cubic feet, or 223753 Cubic yards.

N.B. 1st. Though the "slope" is not given in the above Example, it is soon found; for comparing *ab* of the cross sections, with *ad*, we see 125 : 50, &c., or 2½ to 1 throughout.

N.B. 2d. Cuttings and Embankments are also sometimes computed as follows: each block or section is considered to consist of [I.] its "core," or parallelopiped, of which $l$ = length, $b$ = breadth at top or bottom, and $m$ = mean of D and $d$ the two opposite depths. [II.] The "two slopes," which *together* constitute a triangular Prismoid, as in Ex. 469.

The Formula, when the measurements are in feet, and the content required in *Cubic yards*, are as follows: "Core" $= m \times b \times l$ (a simple parallelopiped); "Two slopes" $= \frac{[(D + d)^2 - Dd] \times l}{3 \div r}$, as in Ex. 469.

Thus in the previous Example taking the *first* block or section:

$$\text{"Core"} = 26 \times 30 \times 640 = 499200 \text{ Cubic feet.}$$
$$\text{"Two slopes"} = \frac{(2704 - 480) \times 640}{(3 \div 2\tfrac{1}{2}) = 1{\cdot}2} = 1186133 \text{ Cubic feet.}$$
$$1685333 \text{ Cubic feet.}$$

In like manner the *third* section is found as follows:

$$\text{"Core"} = 25 \times 30 \times 720 = 540000 \text{ Cubic feet.}$$
$$\text{"Two slopes"} = \frac{(2500 - 0) \times 720}{(3 \div 2\tfrac{1}{2}) = 1{\cdot}2} = 1500000 \text{ Cubic feet.}$$
$$2040000 \text{ Cubic feet.}$$

If measured in feet, and content required in *cubic yards:*

$$\text{“Core”} = \frac{m \times b \times l}{27}.$$

$$\text{“Two slopes”} = \frac{[(D+d)^2 - Dd] \times l}{81 \div r}. \quad (\text{Here } 81 = 3 \times 27).$$

If measured in *chains*, and content required in *cubic yards:*

$$\text{“Core”} = \frac{m \times b \times l}{\cdot 4091}. \quad (\text{Here } \cdot 4091 = \tfrac{27}{66}).$$

$$\text{“Two slopes”} = \frac{[(D+d)^2 - Dd] \times l}{1 \cdot 227 \div r}. \quad (\text{Here } 1 \cdot 227 = \tfrac{27}{22}).$$

**Ex. 637.**—Required the content in cubic yards of a *Section* of a Railway Cutting, which Section is 594 feet long, Roadway 23 feet wide, Slope $1\frac{1}{2}$ to 1. Depth at one end 20 feet, and at the other 35 feet or a *mean* depth of 27·5 feet.

$$\text{“Core”} = \frac{27 \cdot 5 \times 23 \times 594}{27} = 13915 \text{ Cubic yards.}$$

$$\text{“Two slopes”} \frac{(3025 - 700) \times 594}{54} = 25575 \text{ Cubic yards.}$$

TOTAL 39490 Cubic yards.

N.B. 1st. It has been shewn in Ex. 469, how the content of the “Two Slopes” is computed (as a Triangular cutting) from the depths and upper breadths of each end, without reference to the *ratio* of Slope.

N.B. 2nd. It has been shewn in Ex. 470, how the content of the *whole* of the block given above may be computed, without knowing the ratio of Slope; as long as the depths and upper and lower breadths of each end are known.

**Ex. 638.**—Suppose the *length* in the above Example had been given in *chains* (*i.e.* 9 chains), how would the cubic content be computed?

$$\text{“Core”} = \frac{27 \cdot 5 \times 23 \times 9}{\cdot 4091} = 13915 \text{ Cubic yards.}$$

$$\text{“Two slopes”} = \frac{(3025 - 700) \times 9}{\cdot 8182} = 25575 \text{ Cubic yards.}$$

TOTAL 39490 Cubic yds. as before.

## To compute the content of the whole Cutting or Embankment.

It is supposed that the ratio of Slope is known; and also that the Perpendiculars are taken at *equal* distances.

Let $w$ be the breadth of the roadway; $a$, $b$, $c$, $d$, &c. the perpendiculars taken at $p$ feet apart; and $r$ = ratio of Slope. Then

[I.] Central part $= \dfrac{w \times p \times \{(a + v) + 2(b + c + d \text{ \&c.})\}}{2}$ where $a$ and $v$ are the extreme ordinates or perpendiculars.

[II.] Two slopes

$$\frac{(r \times p) \times \{(a + b)^2 + (b + c)^2 + (c + d)^2 \text{ \&c.} - (ab + bc + cd \text{ \&c.})\}}{3}.$$

**Ex. 639.**—Required the content in Cubic feet, of a Cutting or Embankment where the Slope is 3 to 1, and the perpendiculars, taken at 1 Chain (66 feet) apart, are as follows: 6, 3, 4, 5, 7, 2 feet. The width of the Roadway 30 feet throughout. (Total length 5 chains or 330 feet).

[I.] Central part $= \dfrac{(30 \times 66) \times \{6 + 2 + 2(3 + 4 + 5 + 7)\}}{2 \text{ (constant)}}$.

[II.] Two slopes $(3 \times 66)\{(6 + 3)^2 + (3 + 4)^2 + (4 + 5)^2 + (5 + 7)^2 + (7+2)^2 - (6 \times 3) + (3 \times 4) + (4 \times 5) + (5 \times 7) + (7 \times 2)\} \div 3$ (constant)

[I.] $= \dfrac{(30 \times 66) \times [8 + (2 \times 19)]}{2} = \dfrac{1980 \times 8 + 38}{2} = \dfrac{1980 \times 46}{2}$

= 45540 Cubic feet.

[II.] =

$$\frac{(3 \times 66) \times (9^2 + 7^2 + 12^2 + 9^2 + 12^4 + 9^2 - 18 + 12 + 20 + 35 + 14)}{3}$$

$= \dfrac{198 \times (436 - 99)}{3} = \dfrac{198 \times 337}{3} =$ 22242 Cubic feet.

Therefore Total Content = 67782 Cubic feet.

N.B. If on [I.] we use the Divisor 54 (= 2 × 27) and on [II.] the Divisor 81 = (3 × 27), we get the content in Cubic *yards*, viz.: $1686\frac{6}{9}$, and $823\frac{7}{9}$; or a Total of $2510\frac{4}{9}$ Cubic yards.

# APPENDIX G.

## Fractions to decimals,—Reciprocals,—£ s. d. to decimals of £1. (see pages 10 and 11). Decimals of cwts.—lbs.—acres—bushels, &c.

The learner is supposed to *know* that $\frac{1}{2}$ = ·5, $\frac{1}{4}$ = ·25, $\frac{3}{4}$ = ·75, and that $\frac{2}{10}$ = ·2, $\frac{3}{10}$ = ·3, &c.; also that $\frac{2}{5}$ = ·4, $\frac{3}{5}$ = ·6, $\frac{4}{5}$ = ·8; but it will be of great service if the decimal equivalents of 16ths and 12ths are known also, especially those marked in the following Table with an asterisk.

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\frac{1}{16}$ | = | ·0625 | | | $\frac{1}{12}$ | = | $\cdot 083\dot{3}$ | | | $\frac{1}{24}$ | = | $\cdot 0416\dot{6}$ |
| *$\frac{2}{16}$ | = | ·1250 | = | *$\frac{1}{8}$ | *$\frac{2}{12}$ | = | $\cdot 166\dot{6}$ | = | *$\frac{1}{6}$ | $\frac{5}{24}$ | = | $\cdot 2083\dot{3}$ |
| $\frac{3}{16}$ | = | ·1875 | | | *$\frac{3}{12}$ | = | ·2500 | = | *$\frac{1}{4}$ | $\frac{7}{24}$ | = | $\cdot 2916\dot{6}$ |
| $\frac{4}{16}$ | = | ·2500 | = | *$\frac{1}{4}$ | *$\frac{4}{12}$ | = | $\cdot 333\dot{3}$ | = | *$\frac{1}{3}$ | $\frac{11}{24}$ | = | $\cdot 4583\dot{3}$ |
| $\frac{5}{16}$ | = | ·3125 | | | $\frac{5}{12}$ | = | $\cdot 416\dot{6}$ | | | $\frac{13}{24}$ | = | $\cdot 5416\dot{6}$ |
| *$\frac{6}{16}$ | = | ·3750 | = | *$\frac{3}{8}$ | $\frac{6}{12}$ | = | ·5000 | | | $\frac{17}{24}$ | = | $\cdot 7073\dot{3}$ |
| $\frac{7}{16}$ | = | ·4375 | | | $\frac{7}{12}$ | = | $\cdot 583\dot{3}$ | | | $\frac{19}{24}$ | = | $\cdot 7906\dot{6}$ |
| *$\frac{8}{16}$ | = | ·5000 | = | *$\frac{1}{2}$ | *$\frac{8}{12}$ | = | $\cdot 666\dot{6}$ | = | *$\frac{2}{3}$ | *$\frac{1}{7}$ | = | ·14286 |
| $\frac{9}{16}$ | = | ·5625 | | | *$\frac{9}{12}$ | = | ·7500 | = | *$\frac{3}{4}$ | $\frac{2}{7}$ | = | ·28571 |
| *$\frac{10}{16}$ | = | ·6250 | = | *$\frac{5}{8}$ | *$\frac{10}{12}$ | = | $\cdot 833\dot{3}$ | = | *$\frac{5}{6}$ | $\frac{3}{7}$ | = | ·42857 |
| $\frac{11}{16}$ | = | ·6875 | | | $\frac{11}{12}$ | = | $\cdot 916\dot{6}$ | | | $\frac{4}{7}$ | = | ·57128 |
| *$\frac{12}{16}$ | = | ·7500 | = | *$\frac{3}{4}$ | $\frac{1}{15}$ | = | $\cdot 066\dot{6}$ | | | $\frac{5}{7}$ | = | ·71428 |
| $\frac{13}{16}$ | = | ·8125 | | | $\frac{1}{30}$ | = | $\cdot 033\dot{3}$ | | | $\frac{6}{7}$ | = | ·85714 |
| *$\frac{14}{16}$ | = | ·8750 | = | *$\frac{7}{8}$ | $\frac{1}{60}$ | = | $\cdot 016\dot{6}$ | | | $\frac{1}{9}$ | = | $\cdot 111\dot{1}$ |
| $\frac{15}{16}$ | = | ·9375 | | | $\frac{1}{112}$ | = | ·008928 | | | $\frac{1}{11}$ | = | $\cdot 09\dot{0}\dot{9}$ |
| $\frac{1}{160}$ | = | ·00625 | | | 1/[illegible] | = | ·000194 | | | $\frac{1}{240}$ | = | $\cdot 0041\dot{6}$ |

The 8ths are useful for reducing gallons to decimals of 1 bushel; pints to decimals of 1 gallon; and for other purposes.

The 16ths are useful for reducing oz. avoir. to decimals of 1 lb. avoir.; drams to decimals of 1 oz., and in many other cases.

The 12ths are useful for reducing pence to decimals of 1 shilling; inches to decimals of 1 foot; and double grains to decimals of 1 dwt.

## To reduce s. d. to decimals of £1, and *vice versâ.*

(*a*) *s. d. to dec. of* £1.—As far as regards Slide Rule work, it is quite near enough (nearer in fact than is necessary) to know the decimal parts of £1 to *mils* (or thousandths); and this is easy enough. The *greatest* error that can arise is only about $\frac{1}{2}$ a mil, or half a farthing. To obtain the *exact* decimal, a "TABLE" is given two pages on.

*Rule.*—For every "pair" of shillings 100 mils.* For every odd shilling (if any) 50 mils. Then add as many mils as there are farthings in the pence and farthing column; with 1 extra if the farthings exceed 12, and 2 extra if they exceed 36.

Ex. 1.—Reduce 19*s.* $2\frac{3}{4}$*d.* to the decimal of £1.

| | | |
|---|---|---|
| 18*s.* (9 "pair") | = | 900 mils |
| 1*s.* (odd shilling) | = | 50 mils |
| $2\frac{3}{4}$*d.* (11 farthings) | = | 11 mils |
| | | 961 mils or £·961 (True £·961458$\dot{3}$) |

Ex. 2.—Reduce 14*s.* $9\frac{1}{4}$*d.* to the decimal of £1.

| | | |
|---|---|---|
| 14*s.* (7 "pair") | = | 700 mils |
| $9\frac{1}{4}$*d.* (37 farthings) | = | 37 mils |
| Add for over 36*f.* | | 2 mils |
| | | 739 mils = £·689 (True £·73958$\dot{3}$) |

Ex. 3.—Reduce 1*s.* $1\frac{1}{4}$*d* to the decimal of £1.

| | | |
|---|---|---|
| 1*s.* | = | 50 mils |
| $1\frac{1}{4}$*d.* (5 farthings) | = | 5 mils |
| | | 55 mils = £·055 (True £·055208$\dot{3}$) |

---

* The learner is supposed to know that 5 mils = £·005; 55 mils = £·055; 550 mils = £·55; &c.

(*b*) *Decimals of* £1 to *s. d.*

*Rule.*—Allow "a pair" of shillings for every 100 mils; and 1 shilling for the odd 50 (if any). Consider the remainder — *minus* 1 if it exceed 12,—and *minus* 2 if it exceed 37,—as farthings. In this reduction (from 3 places of decimals) the answer in *s. d.* is never in error to the amount of one farthing.

Ex. 4.—Reduce £·887 to *s. d.*

| | | *s.* | *d.* |
|---|---|---|---|
| 800 | (8 "pair" = | 16 | 0 |
| 50 | = | 1 | 0 |
| 37*f.* | (37 — 1) = | | 9 |
| | | 17*s.* | 9*d.* (True 17*s.* 8*d.* 3·52*f.*) |

Ex. 5.—Reduce £·408 to *s. d.*

| | | *s.* | *d.* |
|---|---|---|---|
| 400 | = | 8 | 0 |
| 8 farthings | = | | 2. |
| | | 8*s.* | 2*d.* (True 8*s.* 1*d.* 3·68*f.*) |

Ex. 6.—Reduce £·093 to *s. d.*

| | | *s.* | *d.* |
|---|---|---|---|
| | 50 = | 1 | 0 |
| 42 — 2 = 41 farthings | | | $10\frac{1}{4}$ |
| | | 1*s.* | $10\frac{1}{4}$*d.* (True 1*s.* 10*d.* 1·25*f.*) |

## TABLE for reducing *s. d.* to decimals of £1.

| *d.* | | *d.* | | *s. d.* | | *s. d.* | |
|---|---|---|---|---|---|---|---|
| ¼ | ·0010416 | 6¼ | ·0260416 | 1 0¼ | ·0510416 | 1 6¼ | ·0760416 |
| ½ | ·0020833 | 6½ | ·0270833 | 1 0½ | ·0520833 | 1 6½ | ·0770833 |
| ¾ | ·0031250 | 6¾ | ·0281250 | 1 0¾ | ·0531250 | 1 6¾ | ·0781250 |
| 1 | ·0041666 | 7 | ·0291666 | 1 1 | ·0541666 | 1 7 | ·0791666 |
| 1¼ | ·0052083 | 7¼ | ·0302083 | 1 1¼ | ·0552083 | 1 7¼ | ·0802083 |
| 1½ | ·0062500 | 7½ | ·0312500 | 1 1½ | ·0562500 | 1 7½ | ·0812500 |
| 1¾ | ·0072916 | 7¾ | ·0322916 | 1 1¾ | ·0572916 | 1 7¾ | ·6822916 |
| 2 | ·0083333 | 8 | ·0033333 | 1 2 | ·0583333 | 1 8 | ·0 83333 |
| 2¼ | ·0093750 | 8¼ | ·0347350 | 1 2¼ | ·0593750 | 1 8¼ | ·0843750 |
| 2½ | ·0104166 | 8½ | ·0354166 | 1 2½ | ·0604166 | 1 8½ | ·0854166 |
| 2¾ | ·0114583 | 8¾ | ·0364583 | 1 2¾ | ·0614583 | 1 8¾ | ·0864583 |
| 3 | ·0125000 | 9 | ·0375000 | 1 3 | ·0625000 | 1 9 | ·0875000 |
| 3¼ | ·0135416 | 9¼ | ·0385416 | 1 3¼ | ·0635416 | 1 9¼ | ·0885416 |
| 3½ | ·0145833 | 9½ | ·0395833 | 1 3½ | ·0645833 | 1 9½ | ·0895833 |
| 3¾ | ·0156250 | 9¾ | ·0406250 | 1 3¾ | ·0656250 | 1 9¾ | ·0906250 |
| 4 | ·0166666 | 10 | ·0416666 | 1 4 | ·0666666 | 1 10 | ·0916666 |
| 4¼ | ·0177083 | 10¼ | ·0427083 | 1 4¼ | ·0677083 | 1 10¼ | ·0927083 |
| 4½ | ·0187500 | 10½ | ·0437500 | 1 4½ | ·0687500 | 1 10½ | ·0927500 |
| 4¾ | ·0197916 | 10¾ | ·0447916 | 1 4¾ | ·0697916 | 1 10¾ | ·0947916 |
| 5 | ·0208333 | 11 | ·0458333 | 1 5 | ·0708333 | 1 11 | ·0958333 |
| 5¼ | ·0218750 | 11¼ | ·0468750 | 1 5¼ | ·0718750 | 1 11¼ | ·0968750 |
| 5½ | ·0229166 | 11½ | ·0479166 | 1 5½ | ·0729166 | 1 11½ | ·0979166 |
| 5¾ | ·0239583 | 11¾ | ·0489583 | 1 5¾ | ·0739583 | 1 11¾ | ·0989583 |
| 6 | ·0250000 | 12 | ·0500000 | 1 6 | ·0750000 | 2 0 | ·1000000 |

For every "pair" of shillings, write ·1000000; and then fill in from the above Table. Thus, reduce 14*s.* 9½*d.* to the decimal of £1.

14*s.* (7 pair) = ·7000000
9*d.* (from Table) = ·039583
·7395833£

So again, reduce 19*s.* 2¾*d.* to the decimal of £1.

18*s.* (9 pair) = ·9000000
1*s.* 2¾*d.* (from Table = ·0614583
·9614583£

| "Aliquot parts" of £1. | | | | | | | |
|---|---|---|---|---|---|---|---|
| *s.* | *d.* | | | *s.* | *d.* | | |
| 6 | 8 | = | $\frac{1}{3}$ | 12 | 6 | = | $\frac{5}{8}$ |
| 13 | 4 | = | $\frac{2}{3}$ | 17 | 6 | = | $\frac{7}{8}$ |
| 3 | 4 | = | $\frac{1}{6}$ | 1 | 8 | = | $\frac{1}{12}$ |
| 16 | 1 | = | $\frac{5}{6}$ | 1 | 4 | = | $\frac{1}{15}$ |
| 2 | 6 | = | $\frac{1}{8}$ | 1 | 3 | = | $\frac{1}{16}$ |
| 7 | 6 | = | $\frac{3}{8}$ | | 8 | = | $\frac{1}{30}$ |

Any one who has in memory a few of the decimals corresponding to fractions of 6ths and 8ths, &c. on page 291, may find the marginal Table useful.

It is not difficult to commit to memory the decimals for *s. d.* to £, to the very last decimal place, if we begin with learning those for ¼*d.* ½*d.* ¾*d.* and 1*d.* Then for 6*d.* = £·25 ; and from this, 3*d.* 9*d.* 1½*d.* 7½*d.* 1¼*d.* Then for 8*d.* = £·33$\dot{3}$ ; and from this 4*d.* 2*d.* Then from 10*d.* = £·416$\dot{6}$, we get 5*d.* 2½*d.* 1¼*d.* If we add the decimals for 7*d.* and 11*d.*, the whole Table is learnt.

---

## Lbs. to decimals of 1 Cwt.

The Slide Rule itself will give the decimal, as in Ex. 85, as near as is necessary for Slide Rule work.

Another Rule is as follows : Multiply the lbs. by 9 ; deduct 1 when the number of lbs. is from 8 to 21, both inclusive ; and deduct 2 if the number exceeds 21.

**Ex. 640.**—3 qrs. 8 lbs. Here [(8 × 9) — 1] = 71 ; and as 3 qr. = 750, we have 750 + 71 = ·821 cwt.

**Ex. 641.**—1 qr. 22 lbs. Here [(22 × 9) — 2] = 196 ; and as 1 qr. = ·250, we have 250 + 196 = ·446 cwt.

Strictly speaking, 1 lb. or $\frac{1}{112}$ cwt. = ·00893 cwt.; whence the following TABLE.

| lbs. | cwt. | lbs. | cwt. |
|---|---|---|---|
| 1 | ·00893 | 15 | ·13393 |
| 2 | ·01786 | 16 | ·14286 |
| 3 | ·02679 | 17 | ·15178 |
| 4 | ·03571 | 18 | ·16071 |
| 5 | ·04464 | 19 | ·16964 |
| 6 | ·05357 | 20 | ·17857 |
| 7 | ·06250 | 21 | ·18750 |
| 8 | ·07146 | 22 | ·19643 |
| 9 | ·08036 | 23 | ·20536 |
| 10 | ·08923 | 24 | ·21429 |
| 11 | ·09821 | 25 | ·22321 |
| 12 | ·10714 | 26 | ·23214 |
| 13 | ·11607 | 27 | ·24107 |
| 14 | ·12500 | 28 | ·25000 |
| 4lbs. | = $\frac{1}{7}$ qr. | 16lbs. | = $\frac{1}{7}$ cwt. |
| $3\frac{1}{2}$lbs. | = $\frac{1}{8}$ qr. | 7lbs. | = $\frac{1}{8}$ of 2 qr. |
| 14lbs. | = $\frac{1}{8}$ cwt. | 8lbs. | = $\frac{1}{7}$ of 2 qr. |

If we wish to reduce cwts. and lbs. to decimals of 1 *Ton*, we have only to divide by 20. Thus 7 cwt. 3 qrs. 8 lbs. = 7·821 cwt. or $\frac{7·821}{20}$ = ·39105 *Ton*.

If "pence per lb." be given, the £. per ton, or shillings per cwt., is found by reducing the pence to farthings, and multiplying by $2\frac{1}{3}$ or $\frac{7}{3}$. Thus $8\frac{3}{4}$ per lb. = £$81\frac{2}{3}$ per ton, or $81\frac{2}{3}$*s.* per cwt. (see Formula III., p. 72). 3*d.* per lb. is £28 per ton, or 28*s.* per cwt., and $\frac{3}{4}$*d.* per lb. is £7 per ton, or 7*s.* per cwt.

## Oz. Avoir. to decimals of 1 Lb. Avoir.

Since there are 16 oz. in 1 lb. Avoir., a knowledge of the decimals of 16ths as given in p. 291 will suffice.

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## Dwts. and Grains Troy to decimals of 1 Oz. Troy.

24 grs. make 1 dwt.; and 20 dwts. make 1 oz. Troy. All we have to do therefore in order to get the decimal of 1 oz. is to consider the dwts. as *halfpence*, and proceed as in reducing *s. d.* to decimals of £1 p. 292. Then 16 dwts. 8 grs. is (like 16*s.* 4*d.*) = ·8166 oz. Troy.

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## Pecks to decimals of 1 Bushel, Gallons to decimals of 1 Bushel, Bushels to Decimals of 1 Quarter.

1 peck, or 2 gallons, = $\frac{1}{4}$ = ·25 bushel. 1 gallon, = $\frac{1}{8}$ = ·125 bushel. 1 bushel = $\frac{1}{8}$ = ·125 quarter: so that a knowledge of the decimals of 8ths, p. 291, will be useful.

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## Inches to decimals of 1 Foot.

Since 1 inch = $\frac{1}{12}$ foot, the decimals to 12ths (p. 291) should be kept in memory.

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## Feet to decimals of 1 Mile.

1 foot is $\frac{1}{5280}$ = ·0001894 mile. The values may best be taken out on the Slide Rule itself, as follows $\frac{A}{B}\frac{\text{·72 miles}}{\text{3800 feet}}$. Strictly, ·72 miles = 3801·6 feet.

---

## Yards to decimals of 1 Mile.

1 yard is $\frac{1}{1760}$ = ·00056818 mile. The decimals are easily taken out by the Slide Rule, in the form $\frac{A}{B}\frac{880}{\cdot5}\frac{1760}{1}$, which is exact.

## Poles (or Perches) to decimals of 1 Acre.

1 pole $= \frac{1}{160} =$ ·00625 acre, or $\frac{A}{B} \frac{40 \text{ poles}}{\cdot 25 \text{ acre}}$ is near enough for all Slide Rule work, but if greater nicety is required, the following Table may be useful.

| Po. | | Acre. | Po. | | Acre. | Po. | | Acre. | Po. | | Acre. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | = | ·00625 | 11 | = | ·06875 | 21 | = | ·13125 | 31 | = | ·19375 |
| 2 | = | ·01250 | 12 | = | ·07500 | 22 | = | ·13750 | 32 | = | ·20000 |
| 3 | = | ·01875 | 13 | = | ·08125 | 23 | = | ·14375 | 33 | = | ·20625 |
| 4 | = | ·02500 | 14 | = | ·08750 | 24 | = | ·15000 | 34 | = | ·21250 |
| 5 | = | ·03125 | 15 | = | ·09075 | 25 | = | ·15625 | 35 | = | ·21875 |
| 6 | = | ·03750 | 16 | = | ·10000 | 26 | = | ·16250 | 36 | = | ·22500 |
| 7 | = | ·04375 | 17 | = | ·10625 | 27 | = | ·16875 | 37 | = | ·23125 |
| 8 | = | ·05000 | 18 | = | ·11250 | 28 | = | ·17500 | 38 | = | ·23750 |
| 9 | = | ·05625 | 19 | = | ·11875 | 29 | = | ·18125 | 39 | = | ·24375 |
| 10 | = | ·06250 | 20 | = | ·12500 | 30 | = | ·18750 | 40 | = | ·25000 |

Now since 1 rood $= \frac{1}{4} =$ ·25 acre, the reduction would be as follows : *Example.*—Reduce 8 ac. 2 ro. 8 po. to acres and decimals.

| | | |
|---|---|---|
| 8 ac. 2 ro. | = | 8·50 |
| 18 po. | = | ·11250 |
| | | 8·61250 acres. |

N.B. It may be useful to remember that 10 po. $= \frac{1}{16}$ ac., 20 po. $= \frac{1}{8}$ ac., 16 po. $= \frac{1}{10}$ ac., 32 po. $= \frac{1}{5}$ ac.

## Square Feet to decimals of 1 Acre.

1 Square foot $= \frac{1}{43560}$ acre ; and for Slide Rule work, the formula $\frac{A}{B} \frac{\cdot 62 \text{ Acres}}{27000 \text{ Sq. ft.}}$ given in p. 223, is sufficient (strictly ·62 acres = 27007·2 Square feet). If greater nicety be required, the

following multiples of 43560 may be useful, on dividing the Square feet by that number.

| | Sq. feet. |
|---|---|
| 43560 × 1 = | 43560 |
| 43560 × 2 = | 87120 |
| 43560 × 3 = | 130680 |
| 43560 × 4 = | 174240 |
| 43560 × 5 = | 217800 |
| 43560 × 6 = | 261360 |
| 43560 × 7 = | 304920 |
| 43560 × 8 = | 348480 |
| 43560 × 9 = | 392040 |

Ex. Reduce 117682 Sq. feet to acres and decimals.

43560 ) 117682 ( 2·7016 acres
87120
305620
304920
700000
435600
264400
261360
3040 over

## Square Yards to decimals of 1 Acre.

1 Square yard = $\frac{1}{4840}$ = ·0008264 acre ; and for Slide Rule work the formula $\frac{A}{B}\frac{\cdot 62 \text{ Acres}}{3000 \text{ Sq. yds.}}$ given in p. 223 is sufficient. If great nicety is required the following multiples of 4840 may be useful.

| | |
|---|---|
| 4840 × 2 = | 9680 |
| 4840 × 3 = | 14520 |
| 4840 × 4 = | 19360 |
| 4840 × 5 = | 24200 |
| 4840 × 6 = | 29040 |
| 4840 × 7 = | 31880 |
| 4840 × 8 = | 38720 |
| 4840 × 9 = | 43560 |

Ex. Reduce 14822·5 Sq. yds. to acres.

4840 ) 14822·5 ( 3·0625 acres
14520
30250
29040
12100
9680
24200
24200

## Hours to decimals of 1 Day.

As 1 hour = $\frac{1}{24}$ = ·04166 day, the decimals of 24ths given on page 291 will be useful to know. Minutes are easily reduced to decimals of 1 hour, mentally, by dividing by 6, and cutting off 1 decimal point. Thus 42 min. = ·7 hour.

---

## Days to decimals of 1 Year.

For Slide Rule work, Ex. 91, or the formula $\frac{A}{B}\frac{73 \text{ days}}{\cdot 2 \text{ year}}$ is sufficient; but if greater nicety is required, the following Table will be useful.

| Day. | | Year. |
|---|---|---|
| 1 | = | ·0027397 |
| 2 | = | ·0054794 |
| 3 | = | ·0082192 |
| 4 | = | ·0109589 |
| 5 | = | ·0136986 |
| 6 | = | ·0164384 |
| 7 | = | ·0191781 |
| 8 | = | ·0219178 |
| 9 | = | ·0246575 |
| 10 | = | ·0273973 |

Ex. Reduce 219 days to the decimal of 1 year.

| | |
|---|---|
| 200 days | ·54794 |
| 10 days | ·0273973 |
| 9 days | ·0246575 |
| | ·5999948 |

It is *exactly* ·6 year.

73 days is $\frac{1}{5}$ of a year, and 146 days $\frac{4}{10}$ of a year.

# APPENDIX H.

## LINE "D" ON CARPENTERS' RULES.

In many of those two-feet folding Rules used by carpenters, builders, timber-merchants, &c. the "D" line, or, as it is usually called, the *girt* line, begins with 4 instead of 1.

The reason of this arrangement is, that in those mensuration cases which *most frequently occur* to the classes above named, it saves the use of a "Check number;" as explained in p. 90. The "Gauge-points" most in use by artizans, run between 1 and 1·3, or 10 to 13, and the object is to get to use these on the Rule, instead of their check numbers.

For instance, under "Cylinders" p. 167, we see that, where the diameter is given, the "Gauge-point" is 1·128, and consequently we are often obliged in the ordinary Slide Rule to use its "Check number" 3·568 on D (as in Ex. 424); but if the line D is made to begin with 4 instead of 1, the Gauge-point 1·128 comes about the middle of the line D, and it is hardly ever necessary to use the "Check number"; so in the above Example we should at once read what is over 4·75 on D,* if the D line began with 4.

So again in *Timber measuring* (whence the line D is termed *girt* line), Ex. 517; if we put the 24 of line C over the 12 (the Gauge point) of D, we cannot read what is over the 9·75 on D; and must use the "Check number" 37·95 on D; over which we find 10 times the answer on C. But if D is made to begin with 4, we can at once read what is over 9·75.

So again in *Parallelopipeds*, where in many cases 12 is a Gauge point (column IV., p. 163, 4th Formula), the same remarks apply as in the above case of timber-measuring.

It is however to be observed that for *general use* it is better to begin the D line with 1. If it begins with 4, it cannot solve such Examples as 259 to 262, and should the Gauge point happen to lie between ·25 and ·42, we should have constantly to use a "Check-number."

---

* In line 7, page 168, 3·568 is put erroneously for 4·75.

# APPENDIX I.

## MANIPULATION AND FORMULÆ.

The accuracy of a result worked out on the Slide Rule, depends not altogether on the accuracy with which the instrument is graduated, but also on the way in which the Slide is *set*. The following hints may be useful.

1st. See if the same figures correspond in *both* Radii. Thus in Ex. 1, see if the 1 on B is under the 15 of A, on *both* Radii; also see if the *answer* is the same on *both* Radii.

2d. When the numbers given will at sight admit of it, see if their doubles or halves correspond on other parts of the Rule. Thus in Ex. 2, see if 2 on B is under 17 on A. In Ex. 38, see if 120 on B is under 17 on A. In Ex. 41, see if 23 on A coincides with 34 on B. *In setting Formulæ*, as in p. 74, 123, 233, &c. this should be strictly attended to.

3d. In such an Equation as $x = \dfrac{bc}{a}$, if either $b$, or $c$, be a number cut or engraved on the Rule, such as 4·1, 86, 11·4, 205, &c., and not an *estimated* number, as 207, 35·1, 105, &c. (see p. 6), reserve the cut or engraved number to find the answer with. Thus if we have $x = \dfrac{16 \times 323}{74}$, the setting $\dfrac{\text{A } x}{\text{B } 16}$ ——————— $\dfrac{323}{74}$ is preferable to $\dfrac{\text{A } 16}{\text{B } 74}$ ——————— $\dfrac{x}{323}$, because in the latter setting we leave 323, itself an *estimated* number, to find what *may* be also an estimated number in the answer.

4th. In using the line D with a number on it which, before the answer can be found, requires either the Slide to be shifted (p. 89) or else a *check number* (p. 90) to be used, it is better,—unless the number on D is one which is likely *constantly* to occur,—not to trouble to calculate the check number, but to *shift the Slide*, as in Exs. 247, 248, 249, (though in those three Examples the D numbers, 12, 7·4, 1·128, *are* constantly required, and therefore their check numbers 37·95, 23·4, and 3·568 are pretty well known.) Even with the often required Gauge-point 13·54, in (*e*) p. 170, it is perhaps more accurate to *shift*

*the Slide* than to use the check number 42·82, which is an *estimated* value, and not on a cut division of the Rule. Thus in Ex. 482, having set the Rule $\frac{C\ 8}{D\ 13{\cdot}54} \quad \frac{}{5}$, we shall see that 26 on D coincides with 29·5 on C; which, as shown on p. 86, is the same at $\frac{C\ 2{\cdot}6}{D\ {\cdot}295 \qquad 5}$; whence we easily and accurately read what is over 5.

Independently of the above hints, there are several little points in which an *experienced hand* shows a facility which can only be acquired by practice.

For instance, he is ready with his "reciprocals" (APPENDIX G), so that if he has $x = 42 \times 15 \times 192$, he at once sets it at $x = \frac{45 \times 192}{{\cdot}0\dot{6}66}$ (Ex. 59). If he has $x = \frac{405}{37 \times {\cdot}0625}$, he at once sets it as $x = \frac{405 \times 16}{37}$. If he has the *proportion* $a : x :: b : c$, where $a$, $b$, $c$ are known, and $x$ required, he at once puts it $x = \frac{a \times c}{b}$, (N.B. 1, p. 14). If he has $\frac{a \times b}{c}, \frac{a \times b}{d}, \frac{a \times b}{e}$, all three together, he will *invert* the Slide, as in Ex. 77. If he has 1*s.* $2\frac{1}{2}d. \times 723$ and the answer to be in £, he will not set the Rule $\frac{A\ {\cdot}0604£ \qquad x}{B\ \ 1 \qquad 723}$, but attain greater certainty by reducing mentally the *s. d.* to pence, and setting it $\frac{A\ 14{\cdot}5d. \qquad x}{B\ 240 \qquad 723}$.

If he has $x = \frac{350}{6\frac{1}{7}}$, he will not set the Rule $\frac{A\ 1 \qquad x}{B\ 6{\cdot}143 \qquad 350}$, but $\frac{A\ x \qquad 350}{B\ 7 \qquad 43}$, as in Ex. $58\frac{1}{2}$. If in such an Example as 126, the price of the Stock is constant (say $90\frac{5}{8}$), and the amount to be purchased varies, he will set the Rule $\frac{A\ 725 \qquad \text{Stock} \times 100}{B\ 8 \qquad £ \text{ to pay}}$. If the price of the Stock varies, and the amount to be paid is constant (say buy £4000 Stock at $90\frac{3}{8}$, $90\frac{5}{8}$, $91\frac{1}{8}$), he sets the Rule

$$\frac{A\ 4000}{B\ \ 8 \qquad 723 \qquad 725 \qquad 759},$$

and finds £3615, £3625, and £3645 respectively, on C. If he has

$x = \left(\frac{17}{3}\right)^2 \div 8$, he, knowing that ·125 is the Reciprocal of 8, transposes it to $x = \frac{17^2 \times \cdot 125}{3^2}$, and finds $x = 4{\cdot}014$, as in Ex. 256. If he has (as is very common in Solid Mensuration) $x = \frac{a \times b^2}{c}$, and must work it on the *two* lines C D only, he finds $\sqrt{c} = R$, and brings it into the form $x = \frac{a \times b^2}{R^2}$, as in (I) p. 91. If he has $x = \frac{a \times c}{d^2}$, and must work it on the two lines C D only, he finds $\sqrt{c} = R$, and transposes it to $x = \frac{c \times R^2}{d^2}$, as in Ex. 256. If he has $x = \sqrt{\frac{a \times c}{b}}$, and must use the lines C D only, he finds $\sqrt{c} = R$, and transposes it to $x = \frac{\sqrt{a} \times \sqrt{R}}{\sqrt{b}}$ as in Ex. 257.

If he has $x = 12 \times \sqrt{7 \times 9}$, he sees at once that $12^2 = 144$, and $7 \times 9 = 63$, and adapts it to the Slide Rule as $x = \sqrt{144 \times 63}$, as in Ex. 258. If he has $x = \frac{1}{4}\sqrt{169}$, he sees at once that $4^2$ being 16, it may be set as $x = \sqrt{\frac{169}{16}}$, as in Ex. 251. If he has $x = 1{\cdot}75 \times \sqrt{15 \times 14{\cdot}2}$, he, knowing that the Reciprocal of 15 is $\cdot 066\dot{6}$, sets it as $x = \frac{1{\cdot}75 \times \sqrt{14{\cdot}2}}{\sqrt{\cdot 066\dot{6}}}$ as in Ex. 257. If he has $x = \frac{d^3}{m}$, and no line of Cubes (E p. 115) on his Rule, he at once sets it as $x = \frac{d^2 \times d}{m}$, as in Ex. 259.

If he has $x = \sqrt{m^2 - o^2}$, he sees at once that he can better work it out as $x = \sqrt{(m + o) \times (m - o)}$, as in Ex. 277, and as in the Formula, p. 155, for Circular Rings.

The Slide Rule is very useful when we have such equivalents given, as 1 metre = 3·281 feet; or 1 cubic foot = 6·232 gallons; or diameter 1 = circumference = 3·1416. In such cases the Slide Rule becomes a "Table" of Equivalents, as in pages 73, 76, 123, with the lines A and B. Then again, with the line D. For example, if the area of a circle is equal to ·7854 times the square of its diameter, the Rule set as $\frac{\text{C}\ \ \cdot 7854}{\text{D}\ \ 1} \text{-----} \frac{43\ \text{Area}}{7{\cdot}4\ \text{Diam.}}$, gives a "Table" of areas and diameters

as in pages 148, 149, 167. These "Tabular" Formulæ are often *better* than many pages of "Tables," because they give the equivalents for *any* number or fraction of a number. For instance a Table may give the diameter for an area of 15·9, but not for 16·5; whereas by the Slide Rule set as in page 148, $\frac{\text{C } 43}{\text{D } 7\cdot4} \frac{\text{Area}}{\text{Diam.}}$, one is as easy as the other.

---

*Preparation of Formulæ.*

A little practice in preparing the more simple Formulæ adapted to the Slide Rule, will soon show the learner how to prepare others for himself.

Ex. 1.—Wanted a Formula to show how many square feet there are of a material whose length is given in *yards,* and breadth in inches. Here sq. ft. $= \frac{(l \times 36) \times b}{144} = \frac{l \times b}{4}$. So we get

$$\frac{\text{A} \qquad \text{Inches wide} \qquad \text{Square feet}}{\text{B} \qquad 4 \qquad \text{Yards long}}.$$

This also serves the purpose if the area and length in *yards* are given and we want the breadth in *inches.* (XXI. p. 74, and Ex. 225.)

Ex. 2.—Wanted a Formula for "£ per year" = "shillings for any number of days."

$$\text{Here shillings} = \frac{\pounds \times 20 \times \text{No. of days}}{365} = \frac{\pounds \times \text{No. of days}}{18\cdot25},$$

$$\text{or } \frac{\text{A} \qquad \pounds \qquad \text{Shillings}}{\text{B} \qquad 18\cdot25 \qquad \text{No. of days}}.$$

This is better in many cases than XX. p. 74.

Again, in computing the content of a Cylinder whose height $= l$, and diameter $= d$. The content is by the ordinary formula $S = d^2 \times \cdot7854 \times h$. Now if we want a Formula for the two lines C and D only, we get first $S = \frac{d^2 \times h}{1\cdot2732}$ (which does for the four lines A B C D) *and thence* $\frac{d^2 \times h}{1\cdot128^2}$ which does for the *two* lines C and D. See Formula (*a*) and (*b*) p. 167.

Again, how is Formula iv. p. 229 found? The divisor 17·42 is $\frac{43560 \text{ square feet in an acre}}{50^2}$.

Again: The Content of a conical frustum, where A = area at bottom, and $a$ = area at top, and $l$ = height, is $\frac{(A+a+\sqrt{A \times a}) \times l}{3}$.
Now suppose we have not got either area, but only the greater and lesser *diameters* D and $d$, and we want to avoid the extraction of a Square Root, we proceed as follows:

$$\sqrt{A \times a} = \sqrt{D^2 \times {\cdot}7854 \times d^2 \times {\cdot}7854} = D \times d \times {\cdot}7854.$$

$$\text{Then add } \left.\begin{array}{r} A = D^2 \times {\cdot}7854 \\ a = d^2 \times {\cdot}7854 \\ D \times d \times {\cdot}7854 \end{array}\right\} \begin{array}{l} \text{These three added} \\ = (D^2 + d^2 + Dd) \times {\cdot}7854, \end{array}$$

whence we have for the whole Formula

$$\frac{(D^2+d^2+Dd)+{\cdot}7854 \times l}{3} = \frac{(D^2+d^2+Dd) \times l}{3 \div {\cdot}7854} = \frac{(D^2+d^2+Dd) \times l}{3{\cdot}819}$$

as in page 185.

Another useful Example of a Formula, as well as of the *inverted slide*, will be found in page 218, under "Timber Measuring." Here the customary Formula of Content = $\frac{(\frac{1}{4}\text{ Girt})^2 \times \text{length}}{144}$ which is only suitable to the four lines A B C D, is transposed to $\frac{\text{Girt}^2 \times \text{length}}{48^2}$, which does not require a "quartering of the Girt," and can be used with the lines C and D only; and *both* the Customary and the True content *in Cubic* feet can be found at once, by *inverting* the Slide as under:

| | | | |
|---|---|---|---|
| Ͻ | Length in ft. | True | Customary |
| D | Girt in inches | 42·54 | 48 |

So again, in page 219, under "Weight of Cattle," where the weight in both kinds of Stone, and in Cwt., can be found at one setting, by *inverting* the Slide:

| | | | | |
|---|---|---|---|---|
| Ͻ | Stones of 8 lb. | St. of 14 lbs. | Cwt. | length in ft. |
| D | 1·55 | 2·05 | 5·79 | girt in ft. |

Formulæ for Foreign coins, weights, and measures, as compared with English, may be constructed from a Table previously prepared, giving the chief equivalents.

Thus for French weights, &c. if we take the franc at the rate of 25 francs for £1, we have the following:

| | | | | | |
|---|---|---|---|---|---|
| 1 Franc | = | ·8 sh. | 1 Kilogramme | = | 2·205 lbs av. |
| 1 Franc | = | 9·6 d. | 1 Mètre | = | 1·0936 yds. |
| 1 Shilling | = | 1·25 fr. | 1 oz. Avoir. | = | 23·348 grs. |
| 1 Penny | = | ·1042 | 1 lb. Avoir. | = | ·4536 kilogr. |
| | | | 1 yard | = | ·9144 inches. |

Then, suppose we want a Formula for "Francs per Kilogramme" to be converted to "pence per lb." (or *vice versâ*).

Here pence per lb.

$$= \frac{\text{Fr. per kilo.} \times \text{No. of lbs. given}}{2{\cdot}205 \times {\cdot}1042} = \frac{\text{Fr. per kilo.} \times \text{No. of lbs.}}{{\cdot}22976}$$

which is XLV. of page 76.

So if a Formula is required to convert francs paid for any number of mètres, to "shillings per yard" (or *vice versâ*).

Here shillings per yard

$$= \frac{\text{No. of fr.} \times {\cdot}8 \times {\cdot}9144}{\text{No. of mètres}} = \frac{\text{No. of fr.} \times {\cdot}73152}{\text{No. of mètres}},$$

as in XLIII. p. 76.

If the rate of exchange is $25\frac{1}{4}$ francs for £1 we have:

| | | | | | |
|---|---|---|---|---|---|
| 1 Franc | = | ·7921 sh. | 1 Shilling | = | 1·2625 fr. |
| 1 Franc | = | 9·505 d. | 1 Penny | = | ·1052. |

For further details on this head, see Appendix K.

# APPENDIX K.

## "DIVISORS" AND "GAUGE POINTS."

[Pages 163, 166, 196, 298.]

### Cubic Contents.

In finding the content of Solids, we have generally to multiply by two of the dimensions given, and to divide the product by a third number, which is *constant* for certain figures, and is called a "Divisor." [Its Square Root is a *Gauge Point* (see p. 310)].

For instance, in a PARALLELOPIPED,—if the length $l$ is given in feet, and the breadth $b$, and depth $d$, in inches,—and the content required in Cubic *feet*;—we have $x = \frac{(b \times d) \times (l \times 12)}{1728}$. But as, in similar cases, the numbers 12 and 1728 are *constant*, they can be combined, and the Formula becomes $\frac{(b \times d) \times l^*}{144}$. Here 144 is the "Divisor," whenever $b$ and $d$ are given in inches, and $l$ in feet, and the content required in Cubic feet. See page 165, and Ex. 371.

So again in a CYLINDER, if the length $l$ is given in feet, and the circumference $c$ in inches,—and the content required in Cubic feet; —we have $x = \frac{c^2 \times \cdot 0796 \times (l \times 12)}{1728}$, or $c^2 \times l \div \frac{1728}{\cdot 0796 \times 12}$ $= \frac{c^2 \times l}{1809 \cdot 5}$. Here 1809·5 is the "Divisor" for similar cases. See page 172. and Ex. 383.

---

* Where $b$ and $d$ are *equal*, as in many cases in Squared timber, Square iron, &c., we have content in Cubic feet $= \frac{b^2 \times l}{144}$.

## Contents in Gallons, Bushels, &c. (Page 196.)

If in a CYLINDER whose length $l$ is given in *feet*, and diameter $d$ in *inches*, we require the content in *Imperial Gallons* of 277·274 cubic inches; we have $d^2 \times \cdot 7854 \times (l \times 12)$ = the content in cubic inches; and this divided by 277·274, or $\frac{d^2 \times \cdot 7854 \times (l \times 12)}{277 \cdot 274}$, gives the content in *gallons*. But the above may be reduced (since ·7854, 12, and 277·274 are *constant*) to $d^2 \times l \div \frac{277 \cdot 274}{\cdot 7854 \times 12}$, or $\frac{d^2 \times l}{29 \cdot 419}$. Here 29·419 is the "Divisor" for Gallons, in Cylinders whose lengths are given in *feet*, and diameters in *inches*.

---

## Lbs. weight of Metal, &c.*

27·7274 cubic inches of water weigh 1 lb. If then we know the number of *cubic inches* in a mass of metal, &c., that number of cubic inches divided by 27·724, gives the weight in lbs. of an equal bulk of *water*. This weight multiplied by the specific gravity of the metal, &c., gives its weight in lbs.

For instance, in a Cylinder of cast iron (Sp. Gr. = 7·207) whose length in feet $l$, and the diameter in inches $d$, are given.

(I.) $d^2 \times \cdot 7854 \times l \times 12$ = Cubic inches.

(II.) $\frac{d^2 \times \cdot 7854 \times l \times 12}{27 \cdot 7274}$ = lbs. weight of water.

(III.) $\frac{d^2 \times \cdot 7854 \times l \times 12 \times 7 \cdot 207}{27 \cdot 7274}$ lbs. weight of cast iron.

---

* To find the pounds weight of *Water*; use the same Divisors as for "Gallons" (see page 196), and multiply the result by 10; for 1 Gallon of water weighs 10 lbs. Avoir.

But as ·7854, 12, 27·7274, and 7·207 are *constant* for cast iron cylinders whose dimensions are given as above, we have $d^2 \times l \div \left(\frac{27{\cdot}7274}{{\cdot}7854 \times 12} \div 7{\cdot}207\right)$ or $\frac{d^2 \times l}{{\cdot}4082}$, where ·4082 is the "Divisor," as seen in page 196. In this way the Tables in page 196 have been computed; but the Divisor might also be found from the weight of a Cubic *foot* of water, viz. 62·321 lbs., as follows:

$$\frac{d^2 \times {\cdot}7854 \times (l \times 12)}{1728} = \text{Content in Cubic feet.}$$

$$\frac{d^2 \times {\cdot}7854 \times (l \times 12) \times 69{\cdot}321}{1728} = \text{Weight in lbs. of } \textit{water}.$$

$$\frac{d^2 \times {\cdot}7854 \times l \times 12 \times 62{\cdot}321 \times 7{\cdot}207}{1728} = \frac{d^2 \times l}{{\cdot}4082} = \text{Weight in}$$

lbs. of cast iron.

---

# APPENDIX L.

## Gauge-Points.

The term "Gauge-point" is well known to all who use the Slide Rule. It is in fact a *constant* Divisor; but the term "Divisor" is generally used in cases where the two lines A and B, or the four lines A, B, C, D are used together; whereas the term "Gauge-point" is limited to a constant on the line D, when that line is used with C only. Gauge-points are, in fact, the Square Roots of Divisors, and thus become useful where the line D is used with C only. In the lower part of page 166, we have 1·2732, 12·566 as "Divisors," where the *four* lines are used, and their Square Roots 1·128, 3·545 (in p. 167) as "Gauge-points," when *two* lines only are to be used. In p. 94 we see that $\frac{a^2 \times b}{\text{Divisor}}$ is suitable for the *four* lines, whilst its equivalent $\frac{a^2 \times b}{(\sqrt{\text{Divisor}})^2}$, is suitable when *two* lines, C, D, only, are used.

The Gauge-point of a *Cylinder*, where its measurements are in inches, is the diameter, in inches, of a Cylinder one inch high, capable of containing *one* of the Measure specified. Thus 18·79 is the well-known Gauge-point for *Gallons* (Ex. 475), because a Cylinder one inch deep, but 18·79 inches diameter, is just capable of holding one gallon.

---

### To compute Divisors and Gauge-points.

As Gauge-points are simply (as stated in page 310) the Square Roots of the Divisors, it will only be necessary to show the *modus operandi* of finding *Divisors.*

---

### Parallelopipeds (p. 163).

Here, 1728 being the number of Cubic inches in a Cubic foot, we have $144 = \frac{1728}{12}$ ; and $\cdot0833 = \frac{1}{12}$. (In Col. IV. are their Square Roots, or Gauge-points.)

---

### Cylinders (p. 166).

Here 1·2732 is the reciprocal of ·7854, and 12·566 is the reciprocal of ·0796 : the content of a Cylinder whose measurements are given all in inches, or all in feet, being $d^2 \times \cdot7854 \times h$, or $c^2 \times \cdot0796 \times h$. The Square Roots of 1·2732, and ·0796, are the Gauge-points 1·128 (check number 3·568) and 3·545 (check number 11·21), as shown in the Formulæ on p. 167.

But when measurements of Cylinders are given, the length in *feet*, and diameter, or circumference, in *inches*, as in p. 170, we have first the "Divisor" 183·35, thus obtained. Content $= \frac{d^2 \times \cdot7854 \times (l \times 12)}{1728}$ $= \frac{d^2 \times l}{183\cdot35}$ fitted for the use of the *four* lines A, B, C, D, or content

$= \frac{c^2 \times \cdot 0796 \times (l \times 12)}{1728} = \frac{c^2 \times l}{1809 \cdot 5}$, also fitted for the four lines; and for the *two* lines' use, we have $\sqrt{183 \cdot 35} = 13 \cdot 54$ and $\sqrt{1809 \cdot 5} = 42 \cdot 54$. (See Formula *g*, p. 170, and the *Note* following Ex. 436.)

---

## Spheres (p. 173).

Here the "Divisor" 1·909, is the reciprocal of ·5236, and its Square Root or 1·582, is the Gauge-point. (Content of a Sphere $= d^3 \times \cdot 5236$.)

---

## Liquid or Grain Measures.

N.B.—If weight in *Water* is required, find the "Gallons," and multiply by 10.

For "Bushels" use 2218·2* instead of 277·274, or multiply the "Gallon" Divisors by 8. For "Quarts" use 69·318 instead of 277·274, or multiply the "Gallon" Divisors by 4.

---

## Gallons (p. 196).

There are 277·274* *Cubic* inches, and 353·06 *Cylindrical* inches, in 1 Gallon; and 1728 Cubic inches in a Cubic foot. Whence

*Parallelopipeds.* F.F.F. $\cdot 1606 = \frac{277 \cdot 274}{1728}$. F.I.I. $23 \cdot 106 = \frac{277 \cdot 274}{12}$.

*Cylinders.* F.I. $29 \cdot 419 = \frac{277 \cdot 274}{\cdot 7854 \times 12}$.

*Spheres.* $\cdot 30645 = \frac{277 \cdot 274}{\cdot 5236 \times 1728}$.

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* In the Excise Slide Rules there is a brass pin mark at 277·274 on the line D, with I M : G (Imperial Gallons), and another at 2218·2 marked I M : B (Imperial Bushels). There is also a mark M S (Malt Square) at 47·1, as in Table, p. 196. This is the gauge-point for Bushels, where the measurements of a Rectangular vessel are in inches.

## To find the Divisors, as in p. 208.

The *weight in pounds* being required, we start with 27·7274, since 27·7274 cubic inches of water weigh 1 lb. The "constants" ·7854, ·0796 for Cylinders, and ·5236, ·0169 for Spheres, have been before referred to, in connection with Solid Mensuration.

The specific gravity of the material is supposed to be known. In the case of "Cast Iron" it is 7·207; and it will be sufficient to show how the Divisors, Gauge-points, and Check-numbers (p. 90) of the Table in page 208, are found for that metal.

"Divisors."

PARALLELOPIPEDS.

$$\left\{\begin{array}{llll} \text{F. F. F.} & \dfrac{27{\cdot}7274}{1728} \div 7{\cdot}207 = \dfrac{{\cdot}016046}{7{\cdot}207} & = & {\cdot}00223. \\ \text{F. I. I.} & \dfrac{27{\cdot}7274}{12} \div 7{\cdot}207 = \dfrac{2{\cdot}3106}{7{\cdot}207} & = & {\cdot}3206. \\ \text{I. I. I.} & \dfrac{27{\cdot}7274}{1} \div 7{\cdot}207 = \dfrac{27{\cdot}7274}{7{\cdot}207} & = & 3{\cdot}847. \end{array}\right.$$

CYLINDERS.

$$\left\{\begin{array}{ll} \text{diam.} & \left\{\begin{array}{lll} \text{F. I.} & \dfrac{27{\cdot}7274}{{\cdot}7854 \times 12} \div 7{\cdot}207 = \dfrac{2{\cdot}9419}{7{\cdot}207} & = {\cdot}4082. \\ \text{I. I.} & \dfrac{27{\cdot}7274}{{\cdot}7854} \div 7{\cdot}207 = \dfrac{35{\cdot}3036}{7{\cdot}207} & = 4{\cdot}898. \end{array}\right. \\ \text{circf.} & \left\{\begin{array}{lll} \text{F. I.} & \dfrac{27{\cdot}7274}{{\cdot}0796 \times 12} \div 7{\cdot}207 = \dfrac{29{\cdot}036}{7{\cdot}207} & = 4{\cdot}029. \\ \text{I. I.} & \dfrac{27{\cdot}7274}{{\cdot}0796} \div 7{\cdot}207 = \dfrac{348{\cdot}43}{7{\cdot}207} & = 48{\cdot}35. \end{array}\right. \end{array}\right.$$

As has been stated before, if we get the "Divisors" we can get the "Gauge-points;" for they are the Square Roots of the "Divisors."

"Divisors."

SPHERES.

$$\text{diam.}\begin{cases}\text{F. I.} & \dfrac{27{\cdot}7274}{{\cdot}5236\times1728}\div7{\cdot}207=\dfrac{{\cdot}030645}{7{\cdot}207}={\cdot}00425.\\[2ex]\text{I. I.} & \dfrac{27{\cdot}7274}{{\cdot}5236}\div7{\cdot}207=\dfrac{52{\cdot}955}{7{\cdot}207}=7{\cdot}348.\end{cases}$$

$$\text{circf.}\begin{cases}\text{F. I.} & \dfrac{27{\cdot}7274}{{\cdot}0169\times1728}\div7{\cdot}207=\dfrac{{\cdot}9502}{7{\cdot}207}={\cdot}1319.\\[2ex]\text{I. I.} & \dfrac{27{\cdot}7274}{{\cdot}0169}\div7{\cdot}207=\dfrac{1642}{7{\cdot}207}=227{\cdot}8.\end{cases}$$

A "Divisor" (and thence its Square Root or Gauge-point) may also be found by the direct but more lengthy process shown in p. 309.

In Slide Rule "Tables," these Divisors are often worked out from doubtful Specific Gravities. To know what Specific Gravity has been used,* refer to N.B. page 209, and *Remarks* in p. 213. For instance, under I. I. I. *Brass*, we sometimes find 3·390 (Table in p. 208, shows 3·302). The Specific Gravity used is therefore $\dfrac{27{\cdot}7274}{3{\cdot}39} = 8{\cdot}18$. So if we find 3·878 for Cast Iron, the Specific Gravity assumed is 7·15.

The Slide Rule may be made very useful in checking the "Divisors," when computing Tables like the above. We have only to set 1 on B, under the Specific Gravity on A, and under ·016046, 2·3106, &c., to find the "Divisors" for the *whole horizontal line.*

| | | Parallelopipeds. | | | Cylinders. | | | | Spheres. | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| A | Spec. Gr. | ·01605 | 2·31 | 27·73 | 2·94 | 35·3 | 29·1 | 348 | ·03064 | 52·96 | ·95 | 1642 |
| B | ·1 | F. F. F. | F. I. I. | I. I. I. | F. I. | I. I. | F. I. | I. I. | F. I. | I. I. | F. I. | I. I. |

* In some Tables "Gauge-points" only are given. In this case square them, and they become "Divisors."

# APPENDIX M.

## WEIGHTS and MEASURES (ENGLISH and FOREIGN).

### LINEAR.

| Inches. | | | | | |
|---|---|---|---|---|---|
| 12 = | 1 foot | | | | |
| 36 = | 3 feet = | 1 yard | | | |
| 198 = | 16·5 feet = | 5·5 yards = | 1 pole | | |
| 7920 = | 660 feet = | 220 yards = | 40 poles = | 1 furlong | |
| 63360 = | 5280 feet = | 1760 yards = | 320 poles = | 8 furlongs = | 1 mile. |

The Surveyor's "Chain" consists of 100 links, of 7·92 inches each, or a total of 22 yards or 66 feet. 80 chains = 1 mile. The "line" used by opticians is $\frac{1}{12}$ inch.

A "Fathom" is 6 feet, and 100 fathoms or 200 yards is a "cable's length," (sometimes reckoned 120 fathoms). The French "Brasse" or fathom is ·888 English, or 5·33 feet. The Spanish "Braza" = ·927 fathom. The Dutch "Palm" = 1·0547 fathoms.

The Military "Pace" = $2\frac{1}{2}$ feet. A "Hand" for horses = 4 inches. A "Span" = 9 inches. A "Cubit" = 18 inches.

The Degrees of LATITUDE are not all of the same length (*though nearly so*), owing to the spheroidal form of the earth. The 1st degree from the Equator is 69·05 statute miles; and at the Pole it is 69·41 miles.

In Lat. 52° (London), a degree of Latitude is 365060 feet = 69·14 statute miles. In Lat. 56° (Edinburgh) it is 365308 feet = 69·187 statute miles.

The *Geographical* mile is $\frac{1}{60}$ of a Geographical degree. The Equatorial circumference of the earth being 24900 statute miles, $\frac{1}{360}$ of this is a Geographical degree, or 365200 feet = 69·167 statute miles. The Geographical mile is therefore $\frac{365200}{60}$ = 6086 6 feet, or 1·153 statute miles.

The *Nautical* mile, or "Knot," is in fact the Geographical mile, though, for convenience, seamen consider it 6080 feet. For all practical purposes it may be considered as exceeding a statute mile as 23 exceeds 20.

The Degrees of LONGITUDE vary considerably in length, owing to the meridians converging to the Pole. As shown above, a degree of Longitude at the *Equator* is 60 Geographical, or 69·167 statute miles, whereas in Lat. 52° it is 37 Geographical, or 42·6 statute miles. In Lat. 60°, it is 30 Geographical, or *one half* its length at the Equator.

The Equatorial diameter of the Earth is 7925·7 miles; and its Polar diameter is 7899·2 miles: being a difference of 26½ miles.

A *Scotch* mile is 1977 yards = 1·123 statute miles.

An *Irish* mile is 2242 yards = 1·273 statute miles.

The *Russian* verst is 1167 yards, or ·663 English miles; the *Spanish* league 7416 yards; the *Prussian* mile 8237; the *German* "long" mile = 10126; the *German* "short" mile = 6859; the *Italian* 1467; the *French* kilomètre = 1093·63 yards, or 6·2138 statute miles.

## CLOTH MEASURE.

2¼ inches = 1 nail. 4 nails = 1 quarter (of a yard,) or 9 inches. 1 foot = 5⅓ nails. A *Dutch* or Flemish ell = 27 inches. A *Scotch* ell = 37 inches. An *English* ell = 45 inches. A *French* "aune" = 47 inches. An *Austrian* ell = 30⅝ inches.

## SUPERFICIAL MEASURE.

| Sq. inches. | | | | |
|---|---|---|---|---|
| 144 = | 1 sq. ft. | | | |
| 1296 = | 9 sq. ft. = | 1 sq. yd. | | |
| 39204 = | 272¼ sq. ft. = | 30¼ sq. yds. = | 1 perch or *pole*. | |
| 1568160 = | 10890 sq. ft. = | 1210 sq. yds. = | 40 perches = | 1 rood. |
| 6272640 = | 43560 sq. ft. = | 4840 sq. yds. = | 160 perches = | 40 rds. = 1 acre. |

A square mile contains 640 acres.
An acre is 10 square chains, or 10 chains long and 1 chain wide.

| | Sq. ft. | Sq. in. | |
|---|---|---|---|
| $\frac{1}{4}$ sq. yard. | = 2 | „ 36. | = $2\frac{1}{4}$ sq. feet. |
| $\frac{1}{2}$ sq. yard. | = 4 | „ 72. | = $4\frac{1}{2}$ sq. feet. |
| $\frac{3}{4}$ sq. yard. | = 6 | „ 108. | = $6\frac{3}{4}$ sq. feet. |

The *Dutch* morgen is 2·11 acres : generally reckoned 2 acres.

### Solid.

1728 cubic inches. = 1 cubic foot.
46656 cubic inches or 27 cubic feet = 1 cubic yard.

A cubic foot contains 6·232 gallons of water, weighing 62·32 lbs. Avoir.

### Fluid Measure.

| | | | Cubic inches. |
|---|---|---|---|
| 60 minims (♏) | = 1 drachm | = | ·2166 |
| 8 drachms | = 1 fl. ounce | = | 1·733 |
| 20 ounces | = 1 pint | = | 34·659 |
| 2 pints | = 1 quart | = | 69·318 |
| 8 ounces | = 1 gallon | = | 277·274 |

1 fluid ounce of distilled water, at 62°, weighs 1 oz. Avoir. ; and 1 pint weighs $1\frac{1}{4}$ lbs. Avoir. A cubic foot of water weighs 997·137 oz. Avoir., or 62·3211 lbs.

The ordinary (so-called) Quart bottle is only $\frac{2}{3}$ of a quart, or $1\frac{1}{3}$ pint. It is usual to reckon therefore 6 such bottles to an imperial gallon, which would give to each bottle $26\frac{2}{3}$ fluid ounces, or 46·213 cubic inches. The mean of several trials (filled to the cork) gave $27\frac{1}{2}$ fluid ounces, or 47·66 cubic inches.

In old medicine books, we have as follows : 1 tea-spoon = 1 dram. 1 dessert-spoon = 3 drams. 1 table-spoon = 5 drams. 3 table-spoons or 5 dessert-spoons = 1 wine-glass = 15 drams, or nearly 2 fluid oz. A tumbler = 8 oz.

## Measures of Capacity.

A gill is ¼ pint.

Pecks, bushels, and quarters are "Grain" measures.

| Cubic inches. | | | | | | |
|---|---|---|---|---|---|---|
| 34·659 = | 1 pt. | | | | | |
| 69·318 = | 2 pts. = | 1 qt. | | | | |
| 277·274 = | 8 pts. = | 4 qts. = | 1 gall. | | | |
| 554·548 = | 16 pts. = | 8 qts. = | 2 galls. = | 1 pk. | | |
| 2218·19 = | 64 pts. = | 32 qts. = | 8 galls. = | 4 pks = | 1 bushel. | |
| 17745·5 = | 512 pts. = | 250 qts. = | 64 galls. = | 32 pks. = | 8 bush. = | 1 qr. |

| Wine. | Galls. | Imp. galls. | Beer. | Galls. | Imp. galls. |
|---|---|---|---|---|---|
| 1 anker | = 10 | = 8·33 | 1 firkin | = 9 | = 9·15 |
| 1 rundlet | = 18 | = 15·00 | 1 kilderkin | = 18 | = 18·31 |
| 1 tierce | = 42 | = 34·99 | 1 barrel | = 36 | = 36·62 |
| 1 hogshead | = 63 | = 52·48 | 1 hogshead (= 1½ brl.) | = 54 | = 54·92 |
| 1 puncheon | = 84 | = 69·98 | | | |
| 1 pipe | = 126 | = 104·97 | 1 butt (= 3 brls.) | = 108 | = 109·84 |
| 1 tun | = 252 | = 209·94 | | | |

The above Tables for Wine and Beer were framed with reference to the old Wine Gallon of 231 cubic inches, and old Ale Gallon of 282 cubic inches. The old Wine Quart was 57·75 cubic inches, (see Formula IX. p. 73), and the old Ale Quart was 70·5 cubic inches.*

In *Corn Measure*, the Winchester bushel was 2150·42 cubic inches, and ⅛ of it, or Gallon, was 268·8 cubic inches (Formula VIII. p. 73). The *heaped* Bushel was 2815 cubic inches, or 2·7 per cent. greater than the present imperial: but heaped measure was abolished by Act. 5 & 6 William IV.

A Sack of Corn = 3 bushels; and 40 bushels to a load. In some places the Coomb is 4 bushels.

In *Scotland*, the Standard pint, or "Stirling jug" = 3 imperial pints. The "Firlot" was 4 pecks, and 4 firlots = 1 boll: but the firlot for wheat, beans, peas, rye, and salt, differed from that for barley, oats, and malt: the former being ·998 imperial bushels, and the latter 1·456 imperial bushels.

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* See farther on, under *Wines*.

A bushel of best wheat should weigh 63 lbs.,* rice 64½ lbs., oats 40 lbs, barley 55 lbs., peas 65 lbs., beans 63 lbs. 6 bushels of wheat should produce a sack of flour sufficient for 100 quartern loaves of 4 lb. each.

At the Cape of Good Hope, a "muid" or 4 schepels of corn = 3 imperial bushels.

### *Wines.*

A Pipe of *Port* is about 114 imperial gallons, and turns out about 56 doz. bottles. The ½ Butt, or Hogshead, of *Sherry* is about 53 imperial gallons, and turns out about 26 dozen bottles; the ¼ Cask or Octave, 13 dozen.

A Puncheon of *Rum* contains 72 to 80 imperial gallons. A Pipe of Madeira, or Cape, about 92 imperial gallons. A Cask or "Barrique" of Claret, about 47 gallons, and turns out about 22½ dozen English bottles. The Cape "Leaguer" is 126 imperial gallons, and the "Aum" is ¼ Leaguer, or 31½ imperial gallons. A Tun of fish oil is 252 gallons, or 4 hogsheads of 63 gallons. A Tun of seed oil is 216 gallons, or 4 hogsheads of 54 gallons. 1 Butt of whale oil = 330 imperial gallons.

### *Flour.*

Is sometimes sold by measure, but generally by weight. The Peck or 2 gallons = 1 stone of 14 lbs.; and 4 pecks or 56 lbs. = 1 bushel; also 5 bushels or 280 lbs., or 2½ cwt. to a sack. The Boll is ½ a sack, and the Barrel 196 lbs. The Quartern is ½ a gallon, or 3½ lbs., or ¼ of a stone.

### *Coals.*

The Act. 5 & 6 William IV. cap. 63 (9th Sept. 1835) obliges coals to be sold *by weight.* (In London, the ton is 10 sacks of 224 lbs. each). Formerly, coals were sold by the chaldron of 36 Winchester bushels, or 12 sacks. A local Act of 1831 had previously fixed the weight of a chaldron at 25½ cwt.

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* Wheat is sometimes sold at 36 stones, or 504 lbs., the quarter. This gives 63 lbs. per bushel.

### Troy Weight.

| Grains | | | |
|---|---|---|---|
| 24 = | 1 dwt. | | |
| 480 = | 20 dwts. = | 1 oz. | |
| 5760 = | 240 dwts. = | 12 oz. = | 1 lb. |

(Carat for diamonds, $3\frac{1}{2}$ grains.)

### Apothecaries' Weight.

| Grains | | | | |
|---|---|---|---|---|
| 20 = | 1 scruple (℈) | | | |
| 60 = | 3 scrs. = | 1 dram. (ʒ) | | |
| 480 = | 24 scrs. = | 8 drs. = | 1 oz. (℥) | |
| 5760 = | 288 scrs. = | 96 drs. = | 12 oz. = | 1 lb. |

The lb. and **oz.** are exactly the **same** both in Troy **and** Apothecaries' **weight.** But the oz. is ***divided*** differently.

### Avoirdupois Weight.

| Grains | | | | | |
|---|---|---|---|---|---|
| $27\frac{11}{32}$ or 7·34375 | = 1 dram. | | | | |
| 437·5 | = 16 dr. = | 1 oz. | | | |
| 7000 | = 256 dr. = | 16 oz. = | 1 lb. | | |
| 19600 | = 7168 dr. = | 448 oz. = | 28 lb. = | 1 qr. | |
| 784000 | = 28672 dr. = | 1792 oz. = | 112 lb. = | 4 qr. = | 1 cwt. |
| | | 35840 oz. = | 2240 lb. = | 80 qr. = | 20 cwt. = 1 ton. |

N.B. The Avoirdupois lb. is 7000 grains, whilst the Troy lb. is 5760 grains, or 65 lbs. Avoirdupois = 79 lbs. Troy. But the Avoirdupois oz. is only $437\frac{1}{2}$ grains, whilst the Troy oz. is 480 grains, or 79 oz. Avoirdupois = 72 oz. Troy (pp. 72, 73).

The "Stone" is 14 lbs. Avoirdupois, but a Stone of *meat* is generally 8 lbs. A stone of *hay* is 22 lbs.; of glass, 5 lbs. The *Dutch* 91·8 lbs. = 100 lbs. English; 1 Dutch lb. = 1·089 English.

### Wool.

| | | | |
|---|---|---|---|
| 7 lbs. | = | 1 clove | |
| 14 lbs. | = | 1 stone | |
| 28 lbs. | = | 2 stones | = 1 tod. |
| 13 stones | = | 152 lbs. | = 1 wey. |
| 2 weys | = | 364 lbs | = 1 sack. |
| 12 sacks | = | 1 last, or load; also | |
| 240 lbs. | = | 1 pack. | |

Woolstaplers often allow 30 lbs. to a tod.

A bale of East India cotton varies from 320 to 360 lbs.; Egyptian 180 to 280 lbs.; Virginia 300 to 310; New Orleans 400 to 500.

## HAY AND STRAW.

*Straw.* 36 lbs. = 1 truss. 36 trusses = 1 load (11·571 cwt.)

*Hay.* { 60 lbs. = 1 truss *new* hay. 36 trusses = 1 load (19·286 cwt.)
56 lbs. = 1 truss *old* hay. 36 trusses = 1 load (18 cwt.)

A "stone" of hay is 22 lbs. New hay weighs about 6 stones per cubic yard, or 17 cubic yards to a ton. Old hay weighs about 10 stones to a cubic yard, or 11·3 cubic feet to a ton.

## TIME.

The "Solar Year" is 365 days, 5 hrs. 43 min. 49 sec., or 365·2422 days. 1 hour = 3600 seconds. 1 day (of 24 hours) = 1440 minutes, or 86400 seconds. 1° of longitude makes a difference of 4 minutes of time. As shown in p. 315, the degrees of longitude *vary*, according to the latitude. In Lat. 52° (N. or S.) a difference of 42·6 statute miles makes a difference of 4 minutes of time. In Lat. 60°, $34\frac{2}{3}$ statute miles make a difference of 4 minutes. In the latitude of Dublin $41\frac{1}{6}$ statute miles make a difference of 4 minutes of time. Dublin is 6° 20·5′ west of Greenwich; so when it is 5 o'clock in Dublin it is 5 h. 25 m. 22 s. at Greenwich. Any Star comes to the position it occupied in the heavens the night before, 3 m. 55·9 s. earlier; or 3·93 m. in 9 days; or 4 hours sooner in 61 days.

### *Arc into Time.*

Multiply by 4. This turns degrees of Arc into minutes of Time; minutes into seconds; and seconds into 60ths of a second.

Example. 80° 14′ 19·5″
4

320 m. 57 s. 18-60ths.
= 5 h. 20 m. $57\frac{18}{60}$ s.

### *Time into Arc.*

Reduce all to minutes and seconds; and any decimals of seconds into 60ths. Then divide by 4. The answer is in Degrees and parts.

Example.
2 h. 24 m. 46·4 s. = 144 m. $46\frac{24}{60}$ s.

Divided by 4 = 36° 11′ 36″

## FRENCH "MEASURES AND WEIGHTS."

The MÈTRE is the Standard of *Length*. It is supposed to be a ten-millionth of a quarter of the distance from the Pole to the Equator. The English equivalent is 39·3708 inches, or 3·2809 feet, or 1·09363 yards (p. 74). The centimètre is $\frac{1}{100}$ of a mètre, or ·393708 inch, or 30½ centimètres = 1 foot. The millimètre is $\frac{1}{1000}$ of a mètre, or ·03937 inch, or 25·4 millimètres to an inch. The kilomètre is ·62138 miles, or 1093·63 yards. The marine league is $\frac{1}{20}$ of a degree of the Equator, or 3 English *geographical* (or nautical) miles (p. 315).

The GRAMME is the Standard of *Weight*, being the weight of a cubic centimètre of water at 3·945° Cent. (or 39·1° Fahr.). According to the Schedule appended to Act 27 & 28 Vic. cap. 117 (29th July, 1864), it is equivalent to 15·4323487 grains. Also, 1000 grammes = 1 *kilogramme* = 2·20462146 lbs. Avoir.; or 1 lb. Avoir. = ·45359265 kilogrammes; or 1 cwt. = 50·8023769 kilogrammes (see p. 74). 1 quintal is 100 kilogrammes, or ·9842 tons. 1 Tonneau = 2204·6 lbs.

The LITRE is the Standard of *Capacity*. It is that which will contain 1 kilogramme of water at 3·945° Cent. (39·1° Fahr.). According to the Act above referred to, the Litre is 1·76077 imperial pints (see *note*, p. 323), or ·2209 gallons, or 61·027 cubic inches. The Litre is 28 per cent. more than the ordinary quart bottle.

The *Stere* or cubic mètre = 35·316 cubic feet, or 1·308 cubic yards. (It is the "kilolitre" of capacity.) The *Hectolitre*, or 100 litres = 2·7512 imperial bushels.

The ARE is the Standard for *Land Measure*. It is a square, each side of which is 1 decamètre (10 mètres) long. It is 119·6033 square yards; or 1 *Hectare* (100 Ares) = 2·47115 acres. The *Square Kilomètre* is ·3861 square miles (see p. 75), or 247·1 acres.

According to the "système usuel," which was a kind of compromise between the "ancient" and "metrical" systems, and which was only pronounced illegal in 1840, the *pied* was ⅓ mètre, or 13·1236 inches, and the *pouce* 1·0936 inches. The *toise* was 2 mètres, and the *aune*

120 centimètres, or 47·245 inches. The *livre* was ½ a kilogramme, and the *once* $\frac{1}{16}$ livre (or 8 gros of 24 grains each) = 482·3 English grains. The *boisseau* 12½ litres, or 2¾ imperial gallons. The "*lieue*" or Post league, was 2000 toises, or 4 kilomètres, or about 2·48 English miles.

According to the "ancient" system, previous to 1795, the *pied* was 12·79 inches; the pouce 1·0658 inches, and the *ligne* ·8881 inch, which is the value yet assigned to it by opticians. The old *boisseau* was about 2·86 imp. galls. The *arpent* d'ordonnance = 1·262 acre; the Paris arpent = ·84 acre; the common arpent = 1·044 acre. The old "poids de marc" or *livre* = 1·08 lbs. Avoir., and the quintal 108 lbs. The old "minot" of 1·0736 imp. bushels, and the arpent of ·84 acre, are in use in Lower Canada.

NOTE.—According to the Act of 1824 introducing "Imperial" measures, the Gallon of water weighs 10 lbs. Avoir., whence 1·76077 pints would weigh 15406·74 grains. But according to the French law, a "Litre" of water weighs one kilogramme, or 15432·35 grains. The discrepancy is owing to the difference of temperature of the water. The French temperature of water was at its greatest density, or 39·1° Fahrenheit: the English, 62° Fahrenheit; causing a difference in the weight of a similar bulk of water.

## *Thermometers.*

1. If Centigrade is given, and Fahrenheit wanted. Multiply by 2, subtract $\frac{1}{10}$, and to the remainder add 32°. Thus 20° Cent. = 68° Fahr.

2. If Fahrenheit is given, and Centigrade wanted. Subtract 32°, add $\frac{1}{9}$, halve the sum. Thus 86° Fahr. = 30° Cent.

3. If Réaumur is given, and Fahrenheit wanted. Consider 8° Réaum. = 50° Fahr.; and for every 4° more of Réaum. add 9° of Fahr. Thus 20° Réaum. = 77° Fahr.

4. If Fahrenheit is given, and Réaumur wanted. Subtract 32°, and multiply the remainder by $\frac{4}{9}$. Thus 68° Fahr. = 16° Réaum.

(See also p. 77.)

## British Coinage.

The fineness of Gold, is estimated in *carats*, or 24ths. Thus English "Standard" gold, being that from which our gold coins are made, is 22 carats fine, or $\frac{22}{24}$ ; this gives $\frac{11}{12}$ of *pure* gold, or what is called a "touch" of $\cdot 91\dot{6}$, being 9,166 parts in 10,000, pure gold. The carat is subdivided into quarters called grains. Gold of 70 per cent. pure would be $\frac{70 \times 24}{100} = 16\cdot8$ carats, or 16 carats 3·2 grains. Assayers report from "Standard." Thus gold 18 carats fine, is reported 4 W, or 4 carats worse.

*Coins.*—40 lbs. Troy of "Standard" gold is coined into 1869 sovereigns, which gives the weight of a sovereign = 5 dwts. $3\frac{171}{183}$ grs. or 123·2745 grains ; of which 113·0165 grains *pure* gold, and 10·258 grains alloy (of copper). It is seen from the above, that 1 oz. Troy of "Standard" gold is coined into 3·89375 sovereigns, or 1 oz. "Standard" gold is valued at the Mint at £3 17*s.* 10½*d.* 1 lb. *pure* gold would, at this rate, be worth £509$\frac{8}{11}$, and 1 oz. = £4 4*s.* 11½*d.* The Mint passes all gold coins whose weight does not vary from that prescribed, more than 12 grains in 1 lb. Troy, *i.e.* ·20835 per cent. or ·26 grains in a sovereign: but a sovereign that weighs less than 122½ grains (owing to wear or clipping) is not a legal tender. As to *fineness*, the "remedy" or Mint allowance, is ¼ carat grain, so that a fineness of $21\frac{15}{16}$, or $\cdot 914\dot{6}$ will pass ; or a difference of ·235 per cent., or £2 17*s.* in £1000.

The fineness of Silver is estimated in dwts. (pennyweights) of a pound Troy, *i.e.* in 240ths. "Standard" silver is 11 oz. 2 dwts. fine, or $\frac{222}{240}$, or a "touch" of ·925. Perfectly pure silver would be reported 18 B, or 18 carats better, being $\frac{240}{240}$. Silver of 9 oz. 14 dwts. fine would be reported 28 W. or 1 oz. 8 dwts. ; or 28 dwts. worse than "Standard" *coins.* 1 lb. Troy of "Standard" silver is coined into 66 shillings, making the weight of a shilling $87\cdot\dot{2}\dot{7}$, or $87\frac{3}{11}$ grains ; of which $80\frac{8}{11}$ (or $80\cdot\dot{7}\dot{2}$) are *pure* silver, and $6\frac{6}{11}$ (or 6·546) copper ; or 37 parts of pure silver, with 3 of copper.

Silver coin passes at the Mint if within the *weight* "remedy" of $\frac{1}{240}$ of the standard weight, or $\frac{4}{11}$ of a grain in a shilling. The "remedy" allowed for *fineness* is 1 dwt. in 1 lb. Troy, so that Silver I. W. or fineness $\frac{221}{240}$ = ·921 is passed.

1 lb. of Gold coin = 14·2878 of Silver *as coin.* Standard gold coin is worth about 1·9 per grain: if quite pure without alloy, 2·1*d.* per grain. Silver, *as coin*, is worth about $7\frac{3}{4}$ grains for a penny: as *bullion*, its value varies; at 50*d.* per oz. standard silver would be worth 9·6 grains for a penny.

5*s.* worth of newly coined silver weighs $1\frac{3}{22}$ grains less than 1 oz. Avoir.; and 80 shillings weigh $18\frac{2}{11}$ grains less than 1 lb. Avoir.

In the *bronze* coinage (consisting of 95 copper + 4 tin + 1 zinc), 1 lb. Avoir. is coined into 48 pence; or 3 pence to 1 oz. Avoir.

Silver coins are a legal tender up to 40*s.* Pence are a legal tender up to 1*s.*: halfpence to 6*d.*

## *Silver Plate.*

It must be of the same fineness as Standard silver. The "marks" are as follows: I. **A lion passant,* certifying its correct fineness. II. †*A leopard's head crowned,* or "Hall mark," showing where it was assayed. III. *The letter,* showing the year of its manufacture. IV. ‡*The Sovereign's head,* showing that the duty has been paid.

N.B. The Date mark or "Letter," runs in series of 20 years from letter "a" to "u," both inclusive,—"i" and "j" being considered as one letter. Beginning at 1756 "a" (small Roman letters).—1776 begins with "𝔞" (old English).—1796 with "A" (Roman capitals).—1816 with "a" (small Roman).—1836 with "𝔞" (old English).—1856 with "𝔄" (old- English capitals).—1876 with "A" (Roman capitals). Thus "𝔪" is the 12th letter, and is either 1788 or 1848. "M" would be 1808. "k" = 1825.

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* The Standard mark in Edinburgh, is a thistle. In Dublin a harp crowned. For *gold,* this mark is everywhere a crown with the carat's fineness under it.

† The "Hall mark" in Edinburgh is a castle. In Dublin, a figure of Hibernia.

‡ The "Duty mark" is not required for watch cases.

## French Coinage.

The old money of account was the *livre Tournois*, the value of which was a trifle less than the present franc (81 livres = 80 francs). It was subdivided into 20 *sous*, and each sou into 4 *liards*. The *écu* was 3 livres, and the *louis d'or* was 24 livres.

The present Money of account is the Franc, subdivided into 100 centimes. The copper 5 centime piece is commonly called a *sou*, and is worth very nearly $\frac{1}{2}d.$ English.

The franc weighs 5 grammes (or 77·165 English grains), of which $\frac{9}{10}$ or 69·45 English grains are pure silver. The Gold coin is the Napoleon of 20 francs, weighing 6·4516 grammes (or 99·54 English grains), of which $\frac{9}{10}$ are pure gold. The standard fineness of both gold and silver, is $\frac{9}{10}$ pure or ·900; so that an English assayer would report the gold at 21 carats, 2·4 grains; or 0 carats, 1·6 grains W. The silver he would report 216 dwts., or 10 oz. 16 dwts.; or 6 W,

A kilogramme of gold at French standard $\frac{9}{10}$ pure, is coined into pieces equivalent to 3100 francs. English standard gold is $\frac{11}{12}$. Therefore $\frac{9}{10} : \frac{11}{12} :: 3100 : 3157·40$ francs to a kilogramme of *English* standard, or £1 = 25·22 francs. But when a kilogramme of gold of French standard, is brought to their Mint, the Government keep back a "retenue" of 6·7 francs, for "seigniorage," and return the merchant 3093·3 francs. Therefore $\frac{9}{10} : \frac{11}{12} :: 3093·3 : 3150·56$ francs to a kilogramme of *English* standard, or £1 = 25·16 francs at Commercial par. The calculation is as follows: the data given being 20*s.* = £1; 77·875*s.* = 1 oz. Troy. English standard gold.—1 oz. Troy = 31·1 grammes. 1000 grammes of English standard gold = 3150·58 francs.

$$\frac{20 \times 1 \times 31{\cdot}1 \times 3150{\cdot}56}{77{\cdot}875 \times 1 \times 1000} = 25{\cdot}16 \text{ francs.}$$

## Indian Coinage.

Indian Act XVII. of 1835 enacted that there should be henceforward only one kind of Rupee, to be called the "Company's rupee." The same coin, but with the Queen's head on it is now current. The weight was fixed at 180 grains, of whi ch $\frac{11}{12}$, or $9\frac{1}{6}$ per cent. are

pure silver. Its value as *bullion*, taking silver of English standard to be worth 50*d*. per Troy ounce, would be $\frac{165 \times 50}{444} = 18{\cdot}58d.$ (See below).

Accounts are kept in rupees, annas, and pies; and 12 pies = 1 anna; 16 annas = 1 rupee. A pie is less than $\frac{1}{8}$ penny. The silver coins are the rupee, the $\frac{1}{2}$ R., the $\frac{1}{4}$ R. (or 4 annas), and the $\frac{1}{8}$ R. or double anna. The copper coins are pieces of 1 pie, 3 pies ($\frac{1}{4}$ anna, or called in Bengal *pysa*), and 6 pies (or $\frac{1}{2}$ anna.) The gold coin is the gold mohur or 15 rupee piece. It weighs the same as the rupee, *i.e.* 180 grains, and has the same fineness, *i.e.* $\frac{11}{12}$ or ·91667. Silver is the only legal tender in India, and gold coin is seldom seen. Indian silver coin, being $\frac{11}{12}$ or $\frac{220}{240}$, would by English assay be 2 dwts. Worse. (English standard being $\frac{222}{240}$, p. 325.) Gold coin has the same fineness as English. The old "Sicca" rupee formerly in use in Calcutta weighed 192 grains, of which $\frac{11}{12}$ or 176 grains were pure silver. 100 Siccas are considered = 106$\frac{1}{2}$ Company's rupees. Till 1835, the Bengal gold mohur (which weighed 204·71 grains, of $\frac{11}{12}$ fineness, or 187·65 pure gold) was a legal tender for 16 Rs.

### *Valuation of Foreign Silver Coins.*

One ounce Troy of English *standard* silver, contains 444 grains of *pure* silver.

Let $w$ = weight in grains of *pure* silver* in any foreign coin: and let $d$ = number of pence per Troy oz. at which English standard silver is valued *as bullion.*

Then to find the *pence value* of the foreign silver coin, *as bullion:* $x = \frac{w \times d}{444}$. The Indian Rupee weighs 180 grains, of which 165 are pure silver. Suppose the price of standard silver to be 55*d*. per Troy oz. Then $\frac{165 \times 55}{444}$ (solved as in Ex. 37) gives 20·44*d*. the value of the Rupee *as bullion.*

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* If the weight of a coin is known, and its "fineness," multiply one by the other, and we have the weight of *pure* metal in it.

## APPENDIX N.

### Figures engraved on the Back of the Rule.

It is useful, as a help to the memory, to write on a slip of paper and paste on the back of the Rule—or to engrave on it,—such formulæ as the person who uses the Rule is *most likely to want*, such as :

(*a*) (*c*) from page 167. (*f*) and (*g*) from page 170. That given in Ex. 476. Divisor F. I. I. gallons, page 196. Formulæ XXIV. and XXVI. page 74, XXIX. page 75. Formulæ I. and III. page 148, and X. page 149, &c. Also "27·73 : Sp. Gr. :: cub. inches : lbs.," (where if any three terms are given, the fourth may be found, as in N.B. p. 14,) &c. &c.

### *Woollgar's Pocket Calculator.*

This Rule, which from its size (6 or 8 inches) is very handy, does not seem to be made now.

By an ingenious arrangement (continuing as it were the A line to the line on the lower stock generally marked D), a *six*-inch Rule, has its divisions as wide as *the ordinary twelve-inch Rule.* On the back of the Slide, is a line C, which when set over the line on the lower stock, makes the latter a D or "girt" line. The defect is, that the Rule cannot solve Exs. such as 259, 260, 261, 262, as the D line can only be used with C. The construction of this Rule is explained with the third figure in the article "Slide Rule," Knight's Encyclopædia. [Arts and Sciences.] The *circular* Slide Rules mentioned in that Article, have (besides their being of so awkward a shape to carry) the great defect of not being able to *invert* the Slide. This inversion is often of the greatest use. See pages 25, 34, 111, 113, *b d* p. 167, &c.

Messrs. Elliot (449 Strand) have also a "Double" Slide Rule, with a *Trigonometrical* Slide, arranged by the Author of this Work, who hopes soon to bring out a small Treatise on its use.

W. H. B.